PERSATUAN SAINS MATEMATIK MALAYSIA (PERSAMA)
Malaysian Mathematical Sciences Society

SAMARKAND STATE UNIVERSITY, SAMARKAND, UZBEKISTAN

PROCEEDINGS

INTERNATIONAL TRAINING-SEMINARS ON MATHEMATICS

IN CONJUNCTION WITH

THE JOINT MATHEMATICS MEETING, 2011

BETWEEN

SAMARKAND STATE UNIVERSITY AND

MALAYSIAN MATHEMATICAL SCIENCES SOCIETY

3-5 JUNE 2011, SAMARKAND STATE UNIVERSITY

INTERNATIONAL TRAINING-SEMINARS ON MATHEMATICS

IN CONJUNCTION WITH

THE JOINT MATHEMATICS MEETING, 2011

BETWEEN

SAMARKAND STATE UNIVERSITY AND

MALAYSIAN MATHEMATICAL SCIENCES SOCIETY

Organized by

SAMARKAND STATE UNIVERSITY
&
MALAYSIAN MATHEMATICAL SCIENCES SOCIETY

EDITORS:

Maslina Darus (Universiti Kebangsaan Malaysia)
Arsmah Ibrahim (Universiti Teknologi MARA)
Saiful Hafizah Hj Jaaman (Universiti Kebangsaan Malaysia)
Zainidin Eshkuvatov (Universiti Putra Malaysia)
Abdumalik Rakhimov (Universiti Putra Malaysia)
Gafurjan Ibragimov (Universiti Putra Malaysia)
Haydar Ruzimuradov (Samarkand State University)

ISBN 978-967-5878-48-0

FORE WORDS

Dear colleagues,

It is a great pleasure and privilege for me to welcome all participants of the Joint Meeting of the Samarkand State University and the Malaysian Mathematical Sciences Society (PERSAMA) 2011 together with the International Training and Seminars on Mathematics (ITSM2011).

This is indeed a meaningful event for PERSAMA since this is the first time the society has co-organized a meeting outside our country. What better country to start off with if not Uzbekistan – a country known for its beauty, historical places and of course excellent mathematical output.

It is hoped that this meeting shall be a starting point for further collaboration between mathematician from Malaysia and Uzbekistan. PERSAMA is always willing to facilitate this matter.

Finally, PERSAMA would like to thank Samarkand State University for hosting this meeting and also to all committee members who have worked so hard to make this meeting a success. We wish everyone an enjoyable and fruitful meeting.

Thank you all.

Best Regards,

Mohd. Salmi Md. Noorani
President
Malaysian Mathematical Sciences Society (PERSAMA)

FORE WORDS

Dear colleagues,

The main aim of this International Training and Seminars on Mathematics is to bring together specialists, researchers and students working on various aspects of mathematics to discuss and further develop the interactions among these subjects.

Samarkand is one of the oldest inhabited cities in the world, prospering from its location on the trade route between China and the Mediterranean (Silk Road). UNESCO added the city to its World Heritage List as Samarkand - Crossroads of Cultures. In the Middle centuries the first astronomical observatory was built by the grandson of Temur - Ulugbek which went down to history as the "scientist on throne", the patron of science and enlightenment. Being the greatest astronomer he compiled star tables "Zidji Gurgani" with his companions which contains the exact positions of more than thousand stars. The tables preserve their scientific significance till nowadays.

The History of Samarkand State university passed to the history of Temurids. Scientific sciences and education, passed through the censures of Temurids, in Samarkand regions and in Mavaraunahr which was key position of sciences.

Samarkand State University is replaced in the excellent and enjoyable area. Therefore, our wish to organize the Joint Meeting of the Samarkand State University and the Malaysian Mathematical Sciences Society (PERSAMA) together with the International Training and Seminars on Mathematics (ITSM-2011) in Samarkand was not incidental.

This volume contains the abstracts of papers presented at the ITSM-2011. The collection covers various topics of mathematics, algorithms and software applications which were submitted by the Malaysian and Uzbekistan scientists.

We hope that collaboration between mathematician from Malaysia and Uzbekistan will continue. Samarkand State University is always willing to facilitate this matter.

Finally, Samarkand State University would like to thank PERSAMA for publishing of this proceedings and also to all committee members who have worked so hard to make this meeting a success.

Thank you all.
Best Regards,

Ahmadjon Soleev
Professor, Vice- Rector of Samarkand State University

TABLE OF CONTENTS

EXTENDED CUBIC B-SPLINE FOR TWO-POINT BOUNDARY VALUE PROBLEMS

Ahmad Abd. Majid, Nur Nadiah Abd Hamid and Ahmad Izani Md. Ismail
School of Mathematical Sciences, Universiti Sains Malaysia
majid@cs.usm.my

INTRODUCTION

Generally, it is difficult to find the exact solution of two-point boundary value problems. As of today, there are a number of numerical methods developed to approximate the solutions to these problems such as finite difference, shooting, Adomian decomposition and homotopy perturbation methods. The application of cubic splines in solving these problems was first suggested by Bickley in 1968 (Bickley 1968). Since then, the method has been upgraded and analyzed up until 2006 when Caglar et al. proposed on replacing cubic spline by a more stable representation of spline, called cubic B-spline (Albasiny and Hoskins 1969; Fyfe 1969; Al-Said 1998; Khan 2004; Caglar, Caglar et al. 2006). This study adopted the idea by trying out an extended version of cubic B-spline to solve the problems.

Spline essentially means piecewise polynomials. Cubic B-spline is a spline of degree three, constructed by taking a linear combination of its basis. Three types of extended cubic B-spline were introduced by Xu and Wang (Xu and Wang 2008). The bases were constructed from extending cubic B-spline basis by increasing the degree of the polynomial and adding a free parameter, λ. Therefore, extended cubic B-spline is more flexible because of the increased degree of freedom. For a start, extended cubic B-spline basis of degree four was selected to replace cubic B-spline.

Suppose that

$$x_i = a + ih, \qquad h = \frac{b-a}{n}, \qquad n \in \mathbb{Z}^+, \qquad i \in \mathbb{Z}.$$

Then, extended cubic B-spline basis of degree four, $E_i^4(x)$, can be defined as follows,

$$\frac{1}{24h^4}\begin{cases} -4h(\lambda-1)(x-x_i)^3 + 3\lambda(x-x_i)^4, & x\in[x_i, x_{i+1}] \\ (4-\lambda)h^4 + 12h^3(x-x_{i+1}) & \\ \quad +6h^2(2+\lambda)(x-x_{i+1})^2 - 12h(x-x_{i+1})^3 - 3\lambda(x-x_{i+1})^4, & x\in[x_{i+1}, x_{i+2}], \\ (16+2\lambda)h^4 - 12h^2(2+\lambda)(x-x_{i+2})^2 & \\ \quad +12h(1+\lambda)(x-x_{i+2})^3 - 3\lambda(x-x_{i+2})^4, & x\in[x_{i+2}, x_{i+3}], \\ -(h+x_{i+3}-x)^3\left[h(\lambda-4)+3\lambda(x-x_{i+3})\right], & x\in[x_{i+3}, x_{i+4}], \\ 0, & \text{elsewhere.} \end{cases}$$

Extended cubic B-spline basis degenerates into cubic B-spline basis when $\lambda = 0$.
Analogous to B-spline function, extended cubic B-spline function, $S(x)$ is a linear combination of the extended cubic B-spline basis, as in (2).

$$S(x)=\sum_{i=-3}^{n-1} C_i E_i^4(x), \quad x\in[x_0,x_n], \quad C_i\in\mathbb{R}, \quad n\geq 1. \quad (2)$$

As a result, $S(x)$ is a piecewise polynomial function of degree 4. The properties and behaviors of this function are discussed further in (Xu and Wang 2008).

In order to solve the problem, extended cubic B-spline function in (2) is presupposed to be the solution of the problems. The derivatives of $S(x)$ are calculated and simplified at the collocation points by using some properties of the spline function. These simplifications are very useful to generate a system of linear equation that can be used to solve for the unknowns. However, since there is one free parameter involved in the equations, namely λ, an optimization of this value can be carried out.

RESULTS

This approach has been tested on some linear problems. As an example, let us take the following problem,

$$u''(x)-u'(x)=-e^{x-1}-1, \quad x\in[0,1], \quad u(0)=0, \quad u(1)=0,$$

where the exact solution being $u(x)=x\left(1-e^{x-1}\right)$ (Caglar, Caglar et al. 2006). Table 1 shows the numerical results when $n=10$, with

$$\text{Max-norm}=\max_{i=1}^{n-1}\left|S(x_i)-u(x_i)\right|, \quad \text{and} \quad L^2\text{-norm}=\sqrt{\sum_{i=1}^{n-1}\left[S(x_i)-u(x_i)\right]^2}.$$

From the table, extended cubic B-spline produced more accurate approximations than that of cubic B-spline. The details of the approach can be found in (Abd Hamid, Abd. Majid et al. 2010) and (Abd Hamid 2010).

Table 1: A comparison of norms

Method	Max-Norm	L^2-Norm
Cubic B-spline (Caglar, Caglar et al. 2006)	2.8996E-04	6.6089E-04
Extended Cubic B-spline (λ = 2.9097E-03)	7.9187E-06	1.6711E-05

CONCLUSIONS

The study focuses on solving two-point boundary value problems using extended cubic B-splines. Up to this point, the results have been promising. We will continue testing the method on the more complicated problems like nonlinear problems, heat and wave equations. Furthermore, the analysis of the error is also in progress.

REFERENCES

Abd Hamid, N. N. (2010). Splines for Linear Two-Point Boundary Value Problems. School of Mathematical Sciences.Universiti Sains Malaysia. **Master of Science:** 101.

Abd Hamid, N. N., A. Abd. Majid, et al. (2010). Extended cubic B-spline interpolation method applied to linear two-point boundary value problems. Penang, Malaysia, World Academy of Science, Engineering and Technology (WASET). **62:** 1663-1668.

Al-Said, E. A. (1998). "Cubic spline method for solving two-point boundary-value problems." Journal of Applied Mathematics and Computing **5**(3): 669-680.

Albasiny, E. L. and W. D. Hoskins (1969). "Cubic spline solutions to two-point boundary value problems." The Computer Journal **12**(2): 151-153.

Bickley, W. G. (1968). "Piecewise Cubic Interpolation and Two-Point Boundary Problems." The Computer Journal **11**(2): 206-208.

Caglar, H., N. Caglar, et al. (2006). "B-spline interpolation compared with finite difference, finite element and finite volume methods which applied to two-point boundary value problems." Applied Mathematics and Computation **175**(1): 72-79.

Fyfe, D. J. (1969). "The use of cubic splines in the solution of two-point boundary value problems." The Computer Journal **12**(2): 188-192.

Khan, A. (2004). "Parametric cubic spline solution of two point boundary value problems." Applied Mathematics and Computation **154**(1): 175-182.

Xu, G. and G.-Z. Wang (2008). "Extended Cubic Uniform B-spline and [alpha]-B-spline." Acta Automatica Sinica **34**(8): 980-984.

COEFFICIENT INEQUALITIES FOR A CLASS OF MULTIVALENT ANALYTIC FUNCTIONS OF BAZILEVIC TYPE DEFINED BY RUSCHEWEYH DERIVATIVES

[1]Ajab Akbarally, [2]Maslina Darus
[1]Department of Mathematics, Faculty of Computer and Mathematical Sciences,
Universiti Teknologi MARA Malaysia
Email: ajab@tmsk.uitm.edu.my
[2]School of Mathematical Sciences, Faculty of Science and Technology
Universiti Kebangsaan Malaysia
Email: maslina@ukm.my

INTRODUCTION

Let S denote the family of functions of the form

$$f(z) = z + \sum_{k=2}^{\infty} a_k z^k \tag{1.1}$$

which are analytic and univalent in the open unit disk $U = \{z : |z| < 1\}$.

We denote by $S(p)$ the class consisting of functions of the form

$$f(z) = z^p + \sum_{k=1}^{\infty} a_{p+k} z^{p+k} \qquad (p \in N) \tag{1.2}$$

which are *p*-valently analytic in *U*.

The function $f \in S$ is a Bazilevič (1955) function if it satisfies

$$\operatorname{Re}\left\{ f'(z)\left(\frac{f(z)}{z}\right)^{\alpha+i\gamma-1}\left(\frac{g(z)}{z}\right)^{-\alpha}\right\} > 0$$

where $z \in U$, $\alpha > 0$ and γ is a real number. $g(z)$ is a starlike function with

$$\operatorname{Re}\left\{ {zg'(z)}\middle/{g(z)} \right\} > 0.$$

The case where $\gamma = 0$ was also widely studied. Thomas (1972) defined the class $B(\alpha)$ where $f \in B(\alpha)$ if

$$\operatorname{Re}\left\{ \frac{zf'(z)}{f(z)^{1-\alpha} g(z)^{\alpha}} \right\} > 0$$

with $z \in U$ and $\alpha > 0$.

This subclass was later extended by Eenigenburg and Silvia (1973) to a subclass of $f \in S$ which satisfy the condition

$$\operatorname{Re}\left\{ \frac{zf'(z)}{f(z)^{1-\alpha} g(z)^{\alpha}} \right\} > \beta$$

where $z \in U$, $\alpha > 0$ and $0 \le \beta < 1$.

In this paper we consider the class $M(n,p,\alpha,\beta)$ which was defined by Akbarally and Darus (2009). A function $f \in S(p)$ is said to be in the subclass $M(n,p,\alpha,\beta)$ if it satisfies

$$\operatorname{Re}\left(\frac{p\ D^{n+p} f(z)}{D^{n+p-1} f(z)} \left(\frac{D^{n+p-1} f(z)}{z^{p}} \right)^{\alpha} \right) > \beta \tag{1.3}$$

where $z \in U$, $\alpha > 0$ and $0 \le \beta < p$. $D^{n+p} f(z)$ and $D^{n+p-1} f(z)$ are extensions of the familiar operator $D^{n} f(z)$ of Ruscheweyh Derivatives (1975), $n \in N_0 = N \bigcup \{0\}$. These operators were considered by Sekine, Owa and Obradovic (1992) where

$$D^{n+p} f(z) = z^{p} + \sum_{k=1}^{\infty} C_{p,k}(n) a_{p+k} z^{p+k}$$

with

$$C_{p,k}(n) = \frac{(n+p+k)...(1+k)}{(n+p)!}$$

and

$$D^{n+p-1}f(z)=z^p+\sum_{k=1}^{\infty}C_{p-1,k}(n)a_{p+k}z^{p+k}$$

with

$$C_{p-1,k}(n)=\frac{(n+p-1+k)...(1+k)}{(n+p-1)!}.$$

Notice that $M(0,1,1,\beta)=C(\beta)$ is a class of close-to-convex functions of order β where a function $f\in S$ is said to be in the class $C(\beta)$ if it satisfies

$$\mathrm{Re}\left(\frac{f'(z)}{z^{p-1}}\right)>\beta\ .$$

The objective of this paper is to prove a Fekete-Szegö (1933) theorem for this subclass.

RESULTS

Using some lemmas we prove the Fekete-Szego Theorems for functions in the class $M(n,p,\alpha,\beta)$ for complex and real μ.

Theorem 1 *If* $f\in M(n,p,\alpha,\beta)$ *then for a complex number* μ,

$$\left|a_{p+2}-\mu a_{p+1}{}^2\right|\le\frac{4(p-\beta)}{p(n+p+1)(\alpha(n+p)+2)}\times\max\left[1,\frac{\left|p(\alpha(n+p)+1)^2+(p-\beta)(\alpha(n+p)+2)[(n+p)(1-\alpha)-\mu(n+p+1)]\right|}{p(\alpha(n+p)+1)^2}\right]$$

For each μ *there's a function in* $M(n,p,\alpha,\beta)$ *such that equality holds.*

Theorem 2 *If* $f\in M(n,p,\alpha,\beta)$ *then for a real number* μ,

$$\left|a_{p+2}-\mu a_{p+1}{}^2\right|\le\begin{cases}A & \text{if } \mu\le\dfrac{(n+p)(1-\alpha)}{n+p+1}\\ B & \text{if } \dfrac{(n+p)(1-\alpha)}{n+p+1}\le\mu\le\dfrac{2p(\alpha(n+p)+1)^2+(p-\beta)(\alpha(n+p)+2)(n+p)(1-\alpha)}{(p-\beta)(\alpha(n+p)+2)(n+p+1)}\\ C & \text{if } \mu\ge\dfrac{2p(\alpha(n+p)+1)^2+(p-\beta)(\alpha(n+p)+2)(n+p)(1-\alpha)}{(p-\beta)(\alpha(n+p)+2)(n+p+1)},\end{cases}$$

$$A=\frac{4(p-\beta)\left|p(\alpha(n+p)+1)^2+(p-\beta)(\alpha(n+p)+2)[(n+p)(1-\alpha)-\mu(n+p+1)]\right|}{p^2(n+p+1)(\alpha(n+p)+2)(\alpha(n+p)+1)^2},$$

$$B=\frac{4(p-\beta)}{p(n+p+1)(\alpha(n+p)+2)},$$

$$C=\frac{4(p-\beta)[\mu(n+p+1)(p-\beta)(\alpha(n+p)+2)}{p^2(n+p+1)(\alpha(n+p)+2)(\alpha(n+p)+1)^2}$$
$$-\frac{p(\alpha(n+p)+1)^2+(p-\beta)(\alpha(n+p)+2)(n+p)(1-\alpha)}{p^2(n+p+1)(\alpha(n+p)+2)(\alpha(n+p)+1)^2}.$$

For every μ there exist a function in $M(n,p,\alpha,\beta)$ such that equality is attained.

CONCLUSIONS

Both the results are sharp and equality is obtained for for functions $f\in M(n,p,\alpha,\beta)$ given by

$$\frac{p\quad D^{n+p}f(z)}{D^{n+p-1}f(z)}\left(\frac{D^{n+p-1}f(z)}{z^p}\right)^{\alpha}=\frac{1+z(p-2\beta)}{1-z}$$

and

$$\frac{p\quad D^{n+p}f(z)}{D^{n+p-1}f(z)}\left(\frac{D^{n+p-1}f(z)}{z^p}\right)^{\alpha}=\frac{1+z^2(p-2\beta)}{1-z^2}.$$

Acknowledgement: This work is supported by Universiti Teknologi MARA and MOHE with the research grant 600-RMI/ST/FRGS 5/3/Fst (21/2008).

REFERENCES

Akbarally, A. & Darus, M. 2009. A certain subclass of P-valently analytic functions of Bazilevič Type, *Journal of Mathematics, Physics and Astronomy (Vol 3).*

Bazilevič, I.E. 1955 Über einen fall der integrierbarkeit in der gleichung von Löwner-Kufarev, *Maths. Sb*. **37** : 471-476.

Eenigenberg, P.J. & Silvia, E.M. 1973. A coefficient inequality for Bazilevič functions. *Annales Univ. Mariae Curie-Sklodowska*, **27** : 5-12.

Fekete, M. & Szegö, G. 1933 Eine bemerkung über ungerade schlichte funktionen., *J. Lond. Math. Soc*.,**8** : 85-89.

Ruscheweyh, S. 1975. New criteria for univalent functions. *Proc. Amer.Math. Soc*. **49**: 109-115.

Sekine, T., Owa S. & Obradovic, M. 1992. A certain class of p-valent functions with negative coefficients, *Current Topics in Analytic Function Theory*.

Srivastava, H.M. & Owa, S. (Eds.) : World Scientific Publishing Co. Pte Ltd.

Thomas, D. K. 1972. On Bazilevič functions. *Quart J. Maths. Oxford Ser*. **23**(90): pp.135.

ON SOME GOODNESS OF FIT TESTS FOR TESTING LOGISTIC DISTRIBUTION UNDER SIMPLE AND RANKED SET SAMPLING

S.A. Al-Subh[a], K. Ibrahim[b] , M.T. Alodat[c] & A.A. Jemain[d]
[a,b,d]*School of Mathematical Sciences, Universiti Kebangsaan Malaysia, Selangor, Malaysia*
[c] *Department of Statistics, Yarmouk University, Irbid, Jordan*
Email: salsubh@yahoo.com[a], kamarulz@ukm.my[b],
alodatmts@yahoo.com, azizj@ukm.my [d]

INTRODUCTION

Let $X_1, X_2,..., X_n$ be a random sample of size n from the distribution function $F(x)$ and $X_{(1)} < X_{(2)},... < X_{(n)}$ be the associated order statistics. Assume that the objective is to test the statistical hypotheses $H_o : F(x) = F_o(x) \quad \forall x$, vs. $H_1 : F(x) \neq F_o(x)$ for some x, where $F_o(x)$ is the logistic distribution.

In this paper, $F_o(x)$ of interest is a logistic distribution and the goodness of fit tests considered are the chi-square test and empirical distribution function (EDF) tests such as Kolmogorov D, Kuiper statistic V, Cramer-von Mises W^2, Watson statistics U^2 and Anderson-Darling A^2. We are interested in comparing the performance of these test statistics for testing the logistic distribution against several alternative distributions such as normal (N), Laplace (L), Cauchy (C), student t (T) and lognormal when data are generated under simple random sampling (SRS) and ranked set sampling (RSS).

The chi-square test method under SRS is described. One of the well known tests is the χ^2 test statistics which can be described as follows. Let $I_1, I_2,..., I_{k+1}$ be a partition of the support of $F_o(x)$ and N_j = number of X_i's that fall in I_j, $j = 1, 2,..., k+1$. Under H_o, N_j is a binomial distribution with parameters n and P_j. For large n, the null hypothesis is rejected if

$$\chi^2 = \sum_{j=1}^{k+1} \frac{(N_j - nP_j)^2}{nP_j} > \chi^2_{1-\alpha^*,\, k} \tag{1}$$

where $P_j = P_{F_o}(X_i \in I_j),\ j = 1, 2,..., k+1$ and $\chi^2_{1-\alpha^*,\, k}$, is the $(1-\alpha^*)100^{th}$ quantile of the chi-square distribution with k degrees of freedom. The EDF tests considered are the Kolmogorov statistics which is given by

$$D = \max \left[D^+, D^-\right]$$

where

$$D^+ = \max_{1\le i\le n} \left[\left(\frac{i}{n}\right) - F\left(\frac{x_{(i)} - \alpha}{\beta}\right)\right] \text{and } D^+ = \max_{1\le i\le n} \left[F\left(\frac{x_{(i)} - \alpha}{\beta}\right) - \left(\frac{i-1}{n}\right)\right]$$

the Cramer-von Mises statistics which is given by

$$W^2 = \sum_{i=1}^{n}\left[F\left(\frac{x_{(i)} - \alpha}{\beta}\right) - \left(\frac{2i-1}{2n}\right)\right]^2 + \left(\frac{1}{12n}\right),$$

the Kuiper statistics as denoted by

$$V = D^+ + D^-,$$

the Watson statistics as given by

$$U^2 = W^2 - n\left(\bar{F} - \frac{1}{2}\right)^2, \text{ where } \bar{F} = \frac{1}{n}\sum_{i=1}^{n} F\left(\frac{x_{(i)} - \alpha}{\beta}\right),$$

and the Anderson-Darling statistics which is defined by

$$A^2 = -\frac{1}{n}\left\{\sum_{i=1}^{n}(2i-1)\left[\ln F\left(\frac{x_{(i)} - \alpha}{\beta}\right) + \ln\left(1 - F\left(\frac{x_{(n-i+1)} - \alpha}{\beta}\right)\right)\right]\right\} - n, \tag{2}$$

where $i = 1, 2, ..., n$ and n is the sample size.

The basic idea behind selecting a sample under RSS can be described as follows. Select m random samples each of size mn. Using a visual inspection mn, rank the units within each sample with respect to the variable of interest. Then select, for actual measurement, the i^{th} smallest unit from the i^{th} sample, $i = 1, ..., m$. In this way, we obtain a total of m measured units, one from each sample. The procedure could be repeated r times until a sample of mr measurements are obtained. These mr measurements form RSS (McIntyre 1952). Takahasi and Wakimoto (1968) gave the theoretical background for RSS. They showed that the mean of an RSS is the minimum variance unbiased estimator for the population mean. Dell and Clutter (1972) showed that the RSS mean remains unbiased and more efficient than the SRS mean for estimating the population even if ranking is not perfect. Let $X_{(1)1}, X_{(2)1}, \dots, X_{(m)1}, X_{(1)2}, \dots, X_{(m)2}, \dots, X_{(1)r}, \dots, X_{(m)r}$ be a ranked set sampling of size $n = mr$ from a logistic distribution function. The test described is the upper-tail test.

ALGORITHM

Let T denote a test in (2) based on SRS and T^* be the same test but based on RSS. First, we introduce the following algorithm to calculate the percentage points:

1. Let $x_{(i)j}$ be a random sample from $F_o(x)$.
2. We estimate the parameters α and β from the sample by maximum likelihood.
3. Find the EDF, denoted as $F^*(x)$, as follows:

$$F^*(x) = \frac{1}{mr}\sum_{j=1}^{r}\sum_{i=1}^{m} I(x_{(i)j} \le x), \quad I(x_{(i)j} \le x) = \begin{cases} 1 & , x_{(i)j} \le x, \\ 0 & , \quad \text{o.w.} \end{cases} \tag{3}$$

4. Use $F^*(x)$ in (3) instead of $F(.)$ in (2) for calculating the value of T^*.
5. Repeat the steps (1)-(4) 10, 000 times to get $T_1^*, ..., T_{10,000}^*$.
6. The percentage point d_{α^*} of T^* is approximated by the $(1-\alpha^*)100$ quantile of $T_1^*, ..., T_{10,000}^*$.

We design the following algorithm to obtain the power of T^*.

1. Let $x_{(i)j}$ be a random sample from $F(x)$ defined under H_1.
2. We estimate the parameters α and β from the sample by maximum likelihood.
3. Find the EDF, $F^*(x)$ as in (2).
4. Use $F^*(x)$ in (3) instead of $F(.)$ in (2) for calculating the value of T^*.
5. Repeat the steps (1)-(4) 10, 000 times to get $T_1^*,\ldots, T_{10,000}^*$.
6. Power of $T^* \approx \frac{1}{10,000}\sum_{t=1}^{10,000} I(T_t^* > d_{\alpha^*})$, where $I(.)$ stands for indicator function.

The same procedure can be repeated to find the power of the test statistics under SRS. To compare the power of a particular test statistic under SRS against RSS, we compute the efficiency of the test statistics as a ratio of powers given by $eff(T^*, T) = \frac{\text{power of } T^*}{\text{power of } T}$. The test statistic T^* is more powerful than T if $eff(T^*, T) > 1$.

RESULTS AND DISCUSSION

From the simulation, we make the following remarks.

a) For symmetric distribution, we have

1. The chi-square test is found to be more powerful under selective order RSS when compared to SRS, particularly for minimum and maximum.
2. The power increases as the sample size *n* increases.
3. The powers of all EDF tests are all equal to one when the uniform distribution is considered under the alternative hypothesis.
4. When data of the same size are compared, the EDF tests based on data collected via the RSS are more powerful than the EDF tests based on SRS.
5. The EDF tests based on the i^{th} order statistic (i=1, 2, 3) are more efficient than the EDF tests based on the SRS case (m=1) of the same size except for the A^2 test in case of the Cauchy distribution.
6. Most values of the efficiency are greater than 1, which means that the test statistics based on EDF performed better under RSS when compared to SRS.
7. In general, the efficiency decreases when the cycle size *r* increases.

b) For asymmetric distribution, we have

1. When the samples of the same size are compared, the chi-square tests based on the minimum and maximum order statistics are found to be more powerful than their counterparts in SRS for the distributions considered. When the exponential and lognormal distributions are chosen under the alternative hypothesis, the power of the chi-square tests is about one.
2. For lognormal distribution, the efficiency are all equal to one for all sample sizes considered.

ACKNOWLEDGEMENTS

This work is partially supported by University Research Grant: UKM-GUP-TK-08-16-061.

REFERENCES

Dell, D. R. & Clutter, J. L. 1972. Ranked set sampling theory with order statistics background. *Biometrics*. 28: 545-555.

McIntyre, G. A. 1952. A method of unbiased selective sampling using ranked sets. *Australian Journal of Agricultural Research*. 3: 385-390.

Takahasi, K. & Wakitmoto, K. 1968. On unbiased estimates of the population mean based on the sample stratified by means of ordering. *Annals of the Institute of Statistical Mathematics*. 20: 1-31.

EDGE DETECTION OF ABNORMALITIES IN MAMMOGRAMS USING LOW-COST DISTRIBUTED COMPUTING SYSTEM

Arsmah Ibrahim and Hanifah Sulaiman
Faculty of Computer and Mathematical Sciences, Universiti Teknologi MARA, Malaysia
Email: arsmah@salam.uitm.edu.my, hanifah@salam.uitm.edu.my

INTRODUCTION

A Distributed Parallel Computing System (DPCS) is constructed composed of two clusters. Each cluster consists of eight (8) used IBM processors with 1.8 Ghz each and with Open Source LINUX Fedora 7 used as the operating system. Matlab Distributed Computing Engine (MDCE) is incorporated and used as a platform for the mathematical computations necessary to detect the edges of the abnormalities in the mammograms.

A parallel processing algorithm implemented on the mammograms is constructed adopting cluster computing from Mylonas (2000) and Srivinasan (2005).

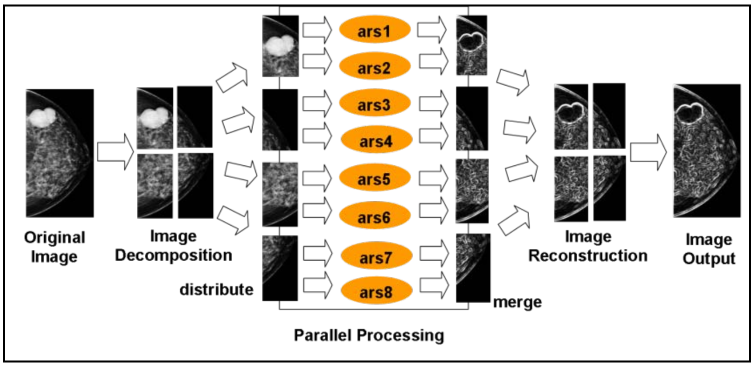


Fig 1. Parallel Computing Algorithm

In the algorithm an original image is divided into several subimages so that the master in the DPCS can distribute each subimage into the available processors. The subimages are then processed synchronously using wavelet transform to detect the edges of the abnormalities.

WAVELET AS EDGE DETECTOR

A wavelet transform is computed by convolving the image function $f(x, y)$ with a dilated wavelet. The first and second order derivatives of the Gaussian function are wavelets (Mallat and Zhong,1992). In this work we consider the wavelets from the first derivatives of the two-dimensional Gaussian function $\theta(x, y)$ written as

$$\psi^1(x, y) = \frac{\partial\theta(x, y)}{\partial x} \quad \text{and} \quad \psi^2(x, y) = \frac{\partial\theta(x, y)}{\partial y} \tag{1}$$

The wavelet transform of $f(x, y)$ at the scale *s* and position (x, y) with respect to the wavelets above are defined respectively as

$$W_s^1 f(x, y) = f(x, y) * \psi_s^1(x, y), \quad W_s^2 f(x, y) = f(x, y) * \psi_s^2(x, y) \tag{2}$$

The dilation by a scaling factor *s* for the two-dimensional function $\psi(x, y)$ results in

$$\psi_s^1(x, y) = \frac{1}{s^2}\psi^1\left(\frac{x}{s}, \frac{y}{s}\right), \quad \psi_s^2(x, y) = \frac{1}{s^2}\psi^2\left(\frac{x}{s}, \frac{y}{s}\right) \tag{3}$$

The edge points are located from $W_s^1 f(x, y)$ and $W_s^2 f(x, y)$. Sharp variation points are determined by the local maxima of $\left|W_s^1 f(x, y)\right|$ (Mallet and Zhong, 1992). Hence at any scale *s*, the wavelet transform modulus of $f(x, y)$ is

$$M_s f(x, y) = \sqrt{\left|W_s^1 f(x, y)\right|^2 + \left|W_s^2 f(x, y)\right|^2} \tag{4}$$

where the angle of the gradient vector with the horizontal direction is governed by

$$A_s f(x, y) = \tan^{-1}\left(\frac{W_s^2 f(x, y)}{W_s^1 f(x, y)}\right) \tag{5}$$

The sharp variation points are where the modulus $M_s f(x, y)$ has local maxima in the direction of the gradient given by $A_s f(x, y)$. Hence the line formed by the (x, y) points along this direction forms the edge.

IMPLEMENTATION

20 digital mammograms obtained from the National Cancer Society Malaysia (NCSM) are used as datasets. The edges of the abnormalities in each mammogram are computed using the three scale iterations based on the wavelet method. The performance of the ARS DPCS is evaluated in terms of speedup and efficiency.

Definition 1 (Amdhal 1967). The speedup of a PCS with p number of processors is the ratio of the execution time on one processor and the execution time on p number of processors

$$S_p = \frac{T_1}{T_p} \quad (6)$$

Definition 2 (Amdhal 1967). The efficiency of a PCS with p number of processors is the ratio of the speedup time on p processors and p number of processors

$$E_p = \frac{S_p}{p} \quad (7)$$

RESULTS

Results obtained show that the ARS DPCS is successful in improving the computing performance in terms of speedup and efficiency.

CONCLUSIONS

Results obtained imply that the DPCS is prospective for image processing and visualization. Hence the DPCS can be extended so that more jobs can be processed efficiently. Efficient and accurate detection of edges for abnormalities in mammograms is beneficial as the edge form the outline of the abnormalities which is very useful in surgery.

Acknowledgement: This research has been supported by the Universiti Teknologi MARA and the Ministry of Higher Education of Malaysia (MOHE) under the grant code 600-RMI/ST/FRGS 5/3/Fst (25/2008)

REFERENCES

Amdahl, G.M. 1967. Validity of the single processor approach to achieving large scale computing capabilities. AFIPS spring joint computer conference.

Mylonas, S.A, Trancoso, P. and Trimikliniotis, M. 2000. Adaptive Noise Canceling and Edge Detection in Images using PVM on a NOW. 10th Mediterranen Electrotechnical Conference.

S.Mallat, S.Zhong. 1992. Characterization of Signals from Multiscale Edges. IEEE Trans. Pattern Analysis and Machine Intelligence, Volume. 14, no. 7, pp.710-732.

S.Mallat. 1998. A Wavelet Tour of Signal Processing. Academic Press, Second Edition.

Srinivasan, N. and Vaudehi, V. 2005. Application of Cluster Computing in Medical Image Processing. IEEE.

Xiaodong Zhang and Deren Li. 2001. A Trous Wavelet decomposition Applied to Image Edge Detection. Geographic Information Sciences. Volume 7, no. 2, pp. 119-123.

SOFTWARE REQUIREMENT ANALYSIS TEMPLATE WITH AUTOMATION AIDED SYSTEM

Asyraf Azizan, Marzanah A. Jabar, and Fatimah Sidi
Faculty of Computer Science and Information Technology,
Universiti Putra Malaysia, 43400 Serdang, Selangor.

INTRODUCTION

Software engineering typically refers to a regimented and procedural methodology for developing software (Kerry 2009). The requirement state is the major process in software development life cycle (SDLC) which has a major effect in the quality of the software product. Errors made in this process are extremely expensive to correct when they are detected during implementing or testing period (Kayed 2009). In requirement elicitation and elaboration using natural language is still the choice for software developer to obtain the software requirement process from the users (Chih-Wei Lu Chih-Hung Chang Chu 2008). But most of the time the users do not know the technicality of computer or what the software developer really need, it is difficult to tell on how the system will function (Chao 2009). To overcome this problem, a requirement analysis template with an automation aided system is proposed. The main function is to close the understanding gap between stakeholder and the developer in order to construct a better architecture to achieve the usability of software. In exploring this problem, existing techniques need to be explored and answered with regard to the problem in eliciting requirement. For this purpose, four approaches for software requirement process have been studied, their strengths and limitation identified and problems to be understood.

RESULTS AND DISCUSSION

Table 1 is the summary of the literature review. It is tabulated based on the model and for each model the benefits and limitation are highlighted.

Table 1: Summary of Software Requirement Process Model

Model/framework	**Strengths**	**Limitation**
Software requirement analysis technology method based on event-driven	1. Suitable for structured analysis method and object oriented method. 2. Easy and feasible method.	1. Suitable for functional requirement only. 2. Users need to have basic skill in software engineering to use this model. 3. No automation aided system.
Model-driven Object-oriented Requirement Editor (MOR Editor)	1. Support requirement document modeling. 2. Model-driven document editing. 3. Support requirement	1. Still in prototype version.

	engineering objectize requirement artifact. 4. Link related requirement artifact. 5. Provide reusable requirement template. 6. Create a requirement document model which can integrate with artifact in analysis, design and implementation phases	
Integrated framework for semantic requirement engineering	1. Combines domain ontology, enterprise ontology and user ontology to enable semantic representation and reasoning of software requirement.	1. No tool was produced yet. Only a framework on how the system will function.
Process framework for requirement analysis and specification	1. Can be used as a reference for driving concrete Requirement Engineering processes. 2. Separation of the elicitation, analysis and refinement of user requirement from the construction of a system or software specification. 3. Supported by a tool and has been tested in several projects from different organization.	1. Has a repeated problem that is the effort required to build the Problem Domain models and to link the user requirements to them. 2. Automation system still in research stage. 3. Not explicitly differentiate between functional and non-functional requirements.

A software requirement analysis template with automation aided system is our new approach to gather user requirement. It uses the method of “5 W 1 H” that is what, who, when, where, why and how. The first “W” is what. This is the basic question when getting the user requirement. The question must be directly on what are the user or stakeholder want for their system. From the “what” question, the main requirement can be gathered that will involve in the system. It also can detail out information needed. For the second “W” is who. The source of information must be known. From that we can know who is the person or actor that involve with requirement. The third “W” is when. This is the matter of time. The fourth “W” is where. This will discover the place involve in the requirement. This is important for the developer to know when it involves the data transferring. The last

"W" is why. This question is to give more understanding for the developer. Using "why" question we can discover out their policies or rules that should be follow. This will explain more on the flow of the system later. It also will help the stakeholder to elaborate more to make sure that the developer completely understands on what they needs. Lastly the "H" is how. This question is to detail out the requirement. The requirement gathered then will be stored in repository for further use. With the automation system, it will help the user by suggesting the similar requirement that has been used before from the system repository.

CONCLUSION

A requirement analysis template with an automation aided system is proposed. The main function is to close the understanding gap between stakeholder and the developer in producing a clear requirement specification and to reduce the ambiguity.

In order to construct a better architecture to achieve the usability of software. to overcome the problems, a new tool has been proposed that has an automation function. It is using the inquisitive technique or also known probing technique "5 W 1 H" method that leads to deeply understanding on the requirement.

REFERENCES

Chao, Z. Z. L. Y. C. (2009). "Software Requirement Analysis Research Based on Event-Driven." International Forum on Computer Science-Technology and Applications, 2009. IFCSTA '09. **1**: 247 - 250

Chih-Wei Lu Chih-Hung Chang Chu, W. C. Y.-W. C. H.-C. C. (2008). "A Requirement Tool to Support Model-based Requirment Engineering." Computer Software and Applications, 2008. COMPSAC '08. 32nd Annual IEEE International (July 28 2008-Aug. 1 2008): 712 - 717

Kayed, A. H., N. Samhan, A.A. Alfayoumi, M. (2009). "Towards an Ontology for Software Product Quality Attributes." Fourth International Conference on Internet and Web Applications and Services, 2009. ICIW '09.: 200 - 204

Kerry, E. D., S. (2009). "Applying software engineering practices to produce reliable, high-quality and accurate automated test systems." AUTOTESTCON, 2009 IEEE

NORMALIZATION OF WAVE FUNCTION OF BAND STATES

Abdurahim A. Okhunov, Hasan Abu Kassim
Quantum Science Center
Department of Physics, Faculty of Science, Universiti Malaya
Email: abdurahim@um.edu.my

INTRODUCTION

A discription of rotational states is one of the oldest, yet not fully solved, problem in nuclear structure physics. There have been intensive interests in the studies of structure of deformed nucleus in experimentally (Leander 1985) and theoretically (Mikhaylov 1992, Gromov 1993). It is possible to note some general structure peculiarity of even–even deformed nuclei which have defined of theoretical direction to the description of experimental data. The stateproperty of ground (gr), $\beta_\nu^+ -$ and $\gamma_\nu^+ -$ rotational bands in even–even deformed rare–earth nuclei are influenced by the Coriolis admixtures of the states of $K^\pi = 1_\nu^+$ bands (Bohr 1971). The cases of existence of several rotational bands with negative and positive parity are located in very narrow interval of excited energy. That creates r prerequisite for mixing adiabatic states with the fixed values of projections of angular momentum K, to the symmetry axis of nuclei. With growth of the angular moment I performance of the law $E(I) \sim I(I+1)$ is violet. It can be connected with the change of the moment of inertia at change of rotational frequency, and with forming of the internal excited angular momentum with the different nature.

WAVE FUNCTIONS

From the (Usmanov 1997) we have for wave function of conditions taking into account the influence Coriolis interactions is described by expression (1).

$$\Psi^I_{M,K} = \sum_{K'} \Psi^I_{K,K'} \left| IMK \right\rangle = \sqrt{\frac{2I+1}{16\pi^2}} \left\{ \sqrt{2}\Psi^I_{gr,K} D^I_{M,0}(\theta) + \sum_{K'} \frac{\Psi^I_{K',K}}{\sqrt{1+\delta_{K',0}}} \left[D^I_{K,K'}(\theta) b^+_{K'} + (-1)^{I+K} D^I_{M,-K}(\theta) b^+_{-K} \right] \right. \tag{1}$$

By the theory of indignations it is possible to receive analytical expressions for the amplitude mixture $\Psi^I_{K,K'}$ in (1). Matrix elements from the operator H_{cor} in (2) is not diagonal by basis of wave functions, and others part of full Hamiltonian (2) is a diagonally. In the small values of angular momentum I represents the small amendment to not indignant operator.

$$\begin{aligned} H &= H_{rot}\left(I^2\right) + H^\sigma_{K,K'}(I) \\ H^\sigma_{K,K'}(I) &= \omega_K \delta_{K,K'} - \omega_{rot}(I)\left(\hat{j}_x\right)_{K,K'} \chi(I,K)\delta_{K,K'\pm 1} \end{aligned} \tag{2}$$

Including in basis state of a Hamiltonian (2) of bands with quantum numbers $K^\pi = 0_\nu^+,\ 2_\nu^+$ and 1_ν^+ , we define amendments of the first and second order and full wave functions systems it takes the following formula:

$$\Psi^I_{M,K} = \varphi_{\tau,gr}\left|IMgr\right\rangle + \varphi_{\tau,\beta_\nu}\left|IM\beta_\nu^+\right\rangle + \varphi_{\tau,1_\nu}\left|IM1_\nu^+\right\rangle + \varphi_{\tau,\gamma_\nu}\left|IM\gamma_\nu^+\right\rangle \tag{3}$$

Where $\varphi_{\tau,K}$ is a amendment for states ν^{th} bands. This amends have following type:

For the ground bands $(\nu = gr)$

$$
\begin{aligned}
&\varphi_{gr,gr}=1\\
&\varphi_{gr,\beta_1}=\omega_{rot}^2(I)\frac{\langle 1|j_x|gr\rangle\langle 1|j_x|\beta_1\rangle}{\omega_1\varepsilon_{\beta_1,gr}}\\
&\varphi_{gr,\beta_2}=\omega_{rot}^2(I)\frac{\langle 1|j_x|gr\rangle\langle 1|j_x|\beta_2\rangle}{\omega_1\varepsilon_{\beta_2,gr}}\\
&\varphi_{gr,1^+}=\omega_{rot}(I)\frac{\langle 1|j_x|gr\rangle}{\omega_1}\\
&\varphi_{gr,\gamma}=\omega_{rot}^2(I)\frac{\langle 1|j_x|gr\rangle\langle 1|j_x|\gamma\rangle}{\omega_1\varepsilon_{\gamma,gr}}.
\end{aligned}
\tag{4}
$$

For the β_1 bands $(\nu=\beta_1)$

$$
\begin{aligned}
&\varphi_{\beta_1,gr}=-\omega_{rot}^2(I)\frac{\langle 1|j_x|\beta_1\rangle\langle 1|j_x|gr\rangle}{\omega_{\beta_1}\varepsilon_{1,\beta_1}}\\
&\varphi_{\beta_1,\beta_1}=1\\
&\varphi_{\beta_1,\beta_2}=\omega_{rot}^2(I)\frac{\langle 1|j_x|\beta_1\rangle\langle 1|j_x|\beta_2\rangle}{\varepsilon_{1,\beta_1}\varepsilon_{\beta_2,\beta_1}}\\
&\varphi_{\beta_1,1^+}=\omega_{rot}(I)\frac{\langle 1|j_x|\beta_1\rangle}{\varepsilon_{1,\beta_1}}\\
&\varphi_{\beta_1,\gamma}=\omega_{rot}^2(I)\frac{\langle 1|j_x|\beta_1\rangle\langle 1|j_x|\gamma\rangle}{\varepsilon_{1,\beta_1}\varepsilon_{\gamma,\beta_1}}.
\end{aligned}
\tag{5}
$$

For the β_2 bands $(\nu=\beta_2)$

$$
\begin{aligned}
&\varphi_{\beta_2,gr}=-\omega_{rot}^2(I)\frac{\langle 1|j_x|\beta_2\rangle\langle 1|j_x|gr\rangle}{\omega_{\beta_2}\varepsilon_{1,\beta_2}}\\
&\varphi_{\beta_2,\beta_1}=\omega_{rot}^2(I)\frac{\langle 1|j_x|\beta_2\rangle\langle 1|j_x|\beta_1\rangle}{\varepsilon_{1,\beta_2}\varepsilon_{\beta_1,\beta_2}}\\
&\varphi_{\beta_2,\beta_2}=1\\
&\varphi_{\beta_2,1^+}=\omega_{rot}(I)\frac{\langle 1|j_x|\beta_2\rangle}{\varepsilon_{1,\beta_2}}\\
&\varphi_{\beta_2,\gamma}=\omega_{rot}^2(I)\frac{\langle 1|j_x|\beta_2\rangle\langle 1|j_x|\gamma\rangle}{\varepsilon_{1,\beta_2}\varepsilon_{\gamma,\beta_2}}.
\end{aligned}
\tag{6}
$$

For the γ bands $(\nu=\gamma)$

$$
\begin{aligned}
&\varphi_{\gamma,gr} = -\omega_{rot}^2(I)\frac{\langle 1|j_x|\gamma\rangle\langle 1|j_x|gr\rangle}{\omega_\gamma \varepsilon_{1,\gamma}} \\
&\varphi_{\gamma,\beta_\nu} = -\omega_{rot}^2(I)\frac{\langle 1|j_x|\gamma\rangle\langle 1|j_x|\beta_\nu\rangle}{\varepsilon_{1,\gamma}\varepsilon_{\gamma,\beta_\nu}} \\
&\varphi_{\gamma,1^+} = \omega_{rot}(I)\frac{\langle 1|j_x|\gamma\rangle}{\varepsilon_{1,\gamma}} \\
&\varphi_{\gamma,\gamma} = 1.
\end{aligned}
\tag{7}
$$

where $\varepsilon_{\nu,\alpha} = \omega_\nu - \omega_\alpha$ and $\langle 1|\hat{j}_x|K\rangle$ – matrix elements of carioles mixture of states $K^\pi = 0^+_\nu,\ 2^+_\nu$ and 1^+_ν band.

With the account normalizations for Carioles interactions we have

$$
\Psi^I_{\tau,\alpha} = \frac{\varphi^I_{\tau,K}}{N^I_\tau}, \quad N^I_\tau = \sqrt{\sum_\ell \left(\varphi^I_{\tau,\ell}\right)^2}
\tag{8}
$$

We can see from the formulas (4)-(7), if a value of matrix elements is large (big) and headband levels are located close, there is a hybridization of a band, which is strongly shown in values of probabilities of transitions.

RESULTS

The Coriolis mixture coefficients $\Psi^I_{K,K'}$ for the states of the ground band state, $\beta^+_\nu -$, $\gamma^+_\nu -$ and 1^+_ν bands for isotope ^{174}Yb is illustrated in Figure 1, taking by the description energy spectra states. We see that even in low value of spin β^+_1 and 1^+ mixture components in γ^+_1 band states is a strongly, from the closely bandhead energy its bands, that it is saying in the value of probability of electromagnetic transitions.

CONCLUSIONS

In the paper, we introduced a theoretical framework. To understand structure properties of states, we use the phenomenological model (Usmanov 1997), this takes into account the Coriolis mixture of states. We have shown from our calculations that in high values of angular momentum of nuclei $E(I) \sim I(I+1)$ law is violated with Coriolis coupling between $gr,\ \beta^+_\nu -,\ \gamma^+_\nu -$ and $K^\pi = 1^+$ rotational bands. The ground band state is more clearly than $\beta^+_\nu -$ and $\gamma^+_\nu -$ bands, and mixture effects are distinct.

Acknowledgement: The work here is fully supported by UMRG Grant RG120/10AFR, University of Malaya. The author also would like to thank the Malaysian Mathematical Sciences Society for organizing such seminar for the scientists.

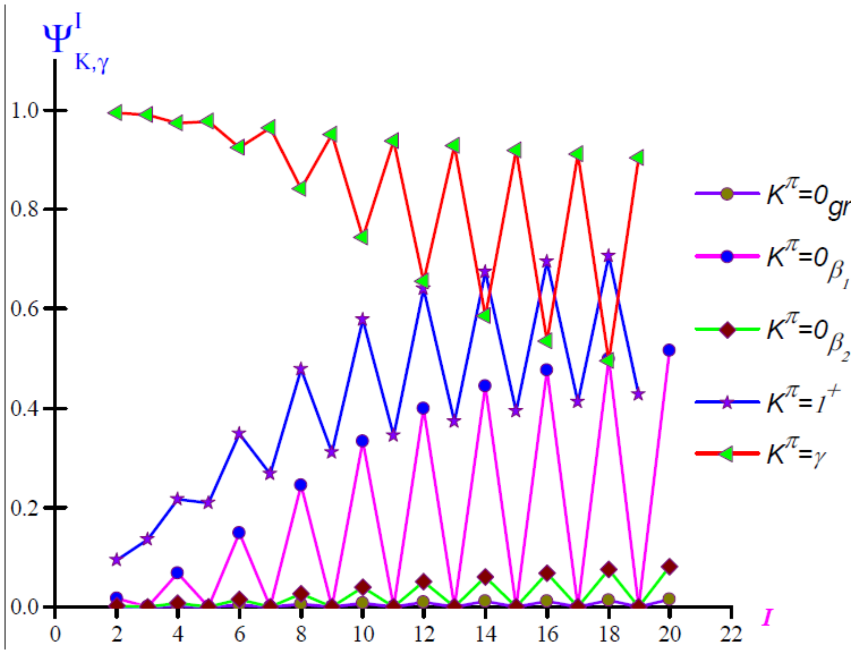


Figure 1. Wave function of $\gamma-$ band states

REFERENCES

Leander G.A. in Nuclear Structure, eds. Broglia R.A., Hagemann G.B. and Herskind B. 1985. *(North–Holland, Amsterdam): 249.*

Mikhaylov I.N., Usmanov Ph.N., Okhunov A.A., Briancon Sh. 1992. *Izv AS USSR, ser. fiz. Vol.56(1): 121.*

Gromov K.Ya., Usmanov Ph.N., Xolmatov A.X. et al. 1993. *Yadernay fizika Vol.56(12): 28.*

Bohr A, Mottelson B. 1971. *Nuclear structure. Vol.2, Mir, Moskwa*.

Usmanov Ph.N., Mikhailov I.N. 1997. *Phys. Part. Nucl. Vol.28: 348.*

GOODNESS OF FIT TESTS FOR RANKED SET SAMPLING

F. Azna A.Shahabuddin, Kamarulzaman Ibrahim and Abdul Aziz Jemain
School of Mathematical Sciences, Faculty of Science and Technology
Universiti Kebangsaan Malaysia
Email: azna@ukm.my

INTRODUCTION

Goodness of fit tests have been applied in many areas of research. Goodness of fit tests (GOF) measure the degree of agreement between the distribution of an observed sample data and a theoretical statistical distribution. The problems involve a comparison of the empirical distribution function (EDF) for a set of ordered observations of size *n*, say $F_n(x_{(i)})$, with a particular theoretical distribution with unknown parameters, denoted as

$F_0\left(x_{(i)}\right)$. The problem can be formulated under the test of hypothesis involving $H_0 : F(x_{(i)}) = F_0\left(x_{(i)}\right)$ where F_0 is the hypothesized continuous cumulative distribution function (cdf) with unknown parameters against $H_1 : F(x_{(i)}) \neq F_0(x_{(i)})$. The standard practice of GOF test is that the observations are sampled based on a simple random sampling (SRS) procedure. Another good and efficient sampling procedure which has received numerous attention in the current statistics literature is ranked set sampling. This sampling procedure was first introduced by McIntyre in 1952, (McIntyre, 1952). RSS has received a lot of interest from various researchers and recently there have been many development in the theories and methodologies of RSS, see for example Chen (2004), Stokes & Sager (1988), Patil, (1995), Patil et.al (1999), Bai & Chen (2004) and Jemain et.al (2008).

In this paper, the performance of several goodness of fit tests such as Kolmogorov Smirnov (KS), Anderson-Darling(AD), Cramer-von-Mises (CV) are investigated. In addition our proposed test FKS and FKS2 which incorporates a variance stabilizing transformation will be studied. The performances of these selected tests are studied under two sampling techniques which are Simple Random Sampling (SRS) and Ranked Set Sampling (RSS).

RANKED SET SAMPLING

The RSS procedure as suggested by McIntyre, (1952) is as follows: To obtain a RSS sample of size *k, k* sets of SRS samples each of size *k* are selected from the target population. The units within each set are then rank with respect to a variable of interest by visual judgment or any other inexpensive ranking mechanism but not involving actual measurements of the variable. From the first sample, the smallest ranked unit is selected and the actual measurement is made on the variable of interest denoted as $X_{(1)}$. Then the second SRS of size *k* is selected from the population and ranked without actual measurement as before. From this sample, the second smallest rank unit is selected and actual measurement is made on the variable of interest denoted as $X_{(2)}$. The process is continued until from the *k*-th sample, the *k*-th ranked unit is selected and measurement is taken, denoted as $X_{(k)}$. In a cycle of getting a ranked set sample of size *k*, sample units of size k^2 have actually been considered. The cycle can be repeated many times, and if the cycle is repeated *m* times, the final data sets for RSS would be of size *n=mk.*

The resulting ranked set sampled is denoted by $X_{(r)j}, r = 1, 2, ..., k; j = 1, 2, ..., m$ where $X_{(r)j}$ is the unit of *r*-th lowest ranked from the r-th sample in the *j*-th cycle. The random variables in each row are the order statistics associated with the SRS observations. The data obtain are independent but not identically distributed. From the data, observations in *r*-th column follows the probability density function (pdf) of the *r*- th order statistic $X_{(r)}$ for a SRS of size *k* given by :

$$f_{(r)}(x) = \frac{k!}{(r-1)!(k-r)!}\left[F(x)\right]^{r-1}[1-F(x)]^{k-r} f(x) \tag{1}$$

where $f(x)$ and $F(x)$ are the probability density function and cumulative distribution function (cdf) for a random sample $X_1, X_2, ..., X_k$ respectively.

We can easily verify the relationship between $f_{(r)}(x)$ and $f(x)$, see for example Bai and Chen (2004), as follows:

$$f(x) = \frac{1}{k}\sum_{r=1}^{k} f_{(r)}(x) \tag{2}$$

Also the following fundamental equality holds for all *x*:

$$F(x) = \frac{1}{k}\sum_{r=1}^{k} F_{(r)}(x) \tag{3}$$

where

$$F_{(r)}(x) = \int_0^{F(x)} \frac{\Gamma(k+1)}{\Gamma(r)\Gamma(k-r+1)} t^{r-1}(1-t)^{k-r}\, dt \tag{4}$$

and can be written in the form of Incomplete Beta function $F_r(x) = I_{F(x)}(r, n-r+1)$.

RESULTS

In this paper two situations are investigated for Normality test. The first part is testing for normality when the alternative hypotheses are from Normal distributions with various means and variances. The second part testing for normality when the alternative hypotheses are from other distributions beside N(0,1). To compare the performance under RSS and SRS we calculate the value of efficiency, where efficiency = power (RSS)/ power(SRS).

CONCLUSION

Our proposed statistics FKS2 perform better than the traditional GOF test KS, CV but AD perform better than FKS2 in some cases under the ranked set samples. RSS performs better under the normality test when the data are sampled from symmetric distributions. We found that the RSS procedure performed as efficient as SRS when the data are sampled from skewed distributions. RSS is more efficient for smaller n, in particular when n$\leq$ 50 in the two examples studied.

Acknowledgement: The work here is fully supported by MOHE: UKM-ST-06-FRGS0100-2010.

REFERENCES

McIntyre, G.A.1952. A method for unbiased selective sampling using ranked sets, *Australian Journal of Agricultural Research,* No.3, pp.385-390.

Bai, Z, Chen.Z & Sinha, B.K. 2004. *Ranked Set Sampling: Theory and Applications.* Springer.

Chen, S.H. 1983, Ranked set sampling theory with selective probability vector, *Journal of Statistical Planning and Inference*, No. 8, pp. 161-174.

Stokes, S.L. and Sager,T.W. 1988. Characterization of a ranked set sample with application to estimating distribution functions, *Journal of the American Statistical Association*, No. 83, pp. 374-381.

Patil, G.P.1995. Editorial: Ranked Set Sampling, *Environmental and Ecological Statistics*, No.2, pp.271-285.

Patil,G.P., Sinha, A.K., and Tallie,C. 1999. Ranked set sampling: a bibliography, *Environmental and Ecological Statistics*, No.6, pp.91-98.

Bai, Z.D. and Chen, Z. 2003. On the theory of ranked set sampling and its ramifications. *Journal of Statistical Planning and Inference*. No. 109, pp. 81-99.

Jemain, A.A., Al-Omari, A., and Ibrahim, K. 2008. Modified ratio estimator for the population mean using double median ranked set sampling, *Pakistan Journal of Science*, No. 24(3), pp. 217-226.

DATA QUALITY DIMENSION IN DATA WAREHOUSE

Fatimah Sidi, Marzanah A. Jabar
Faculty of computer sciences and information technology
Universiti Putra Malaysia
fatimahcd@fsktm.upm.edu.my , *marzanah@fsktm.upm.edu.my*

INTRODUCTION

Data quality (DQ) has become the critical concern in organization. The growth of daily data and complexity in data warehouse with enhancing the access from various sources become a challenge in for information user (Lee, 2002). The demand for quality data has increase the awareness of quality of information in making fast and reliable decision making. The dirty data will cause major problem in data warehouse as more people to gain the wrong information from it, despite of gaining competitive advantage for an organization. Relatively it inexpensive to clean up the data set, seldom been used and expensive to improve the DQ frequently used (Ballou and Tayi,1999).

The DQ dimension in our study was based on four of this dimension: accuracy, completeness, consistency and timeline whereas it was widely cited in literature as most important data quality dimension in utilization of data warehouse (Blake, 2011; Parssion, 2006). Here we extend previous research enhancing data quality dimension in data warehouse and we propose an integrated framework with product and service performance of information quality (PSP/IQ) model.

RESULT / DISCUSSION

We expect to achieve the data quality dimension in accuracy, completeness, consistency and timeline by adopting PSP/IQ model (Table 1).

Table 1. The PSP/IQ Model from Lee et al. (2002)

	Conforms to specifications		**Meets or exceeds consumer expectations**	
Product quality	• Free-of-error • Concise representation	• Completeness • Consistent representation	• Appropriate amount • Relevancy • Understandability	• Interpretability • Objectivity
Service quality	• Timeliness	• Completeness	• Believability • Accessibility	• Ease of operation • Reputation

From the PSP/IQ model, the product quality must meet the requirement: where the data warehouse is free of error, or 99% accuracy and completeness. The service quality indicate the process acquire to complete product information to use, in meeting the consumer expectation. The relevant and useful product information is to achieve the service quality expected, increase reliability, and easy maintenance.

Data quality through PSP/IQ model is expected to contribution in the following ways:

1. To determine the activities in data warehouse.
2. To identify the frequently used data.
3. To estimate the quality of data quality dimension.

CONCLUSION

We will study the product service performance information quality (PSP/IQ) model worked by Lee et al (2002). The data quality assessment will reveal the data quality dimension which useful in managing data warehouse. The four dimensions: accuracy, completeness, consistency and timeline have become the benchmark in this study. The accessing to data warehouse by user with varying need and quality of data should be supported by the organization activities. Enhancing data quality in data warehouse will result competitive advantages and appropriate decision making to the organization.

REFERENCES

Ballou, D.P., Tayi, G.K. (1999). Enhancing Data Quality in Data Warehouse Environments. Communications of the ACM. 42 (1): 73-78.

Blake, R., Mangiameli, P. (2011). The Effects and Interactions of Data Quality and Problem Complexity on Classification. Journal of Data and Information Quality. 2(2).

Khan, B.K., Strong, D.M., Wang, R. Y. (2002). Information Quality Benchmark: Product and service performance. Communications of the ACM. 45(4): 184-192.

Lee, Y. W., Strong, D. M., Kahn, B. K. & Wang, R. Y. (2002). AIMQ: A methodology for information quality assessment. Information Manager. 40, 133–146.

Parssian, A. (2006). Managerial decision support with knowledge of accuracy and completeness of the relational aggregate functions. Decision Support System. 42, 1494–1502.

Sen, A., Ramamurthy, K., Sinha, A. P. (2011). A Model of Data Warehousing Process Maturity. IEEE Transaction On Software Engineering : On Press.

SEVENTH ORDER EXPLICIT HYBRID METHODS FOR SOLVING SECOND ORDER ORDINARY DIFFERENTIAL EQUATIONS

Fudziah Ismail, Faieza Samat, Mohamed Suleiman and Norihan Md. Arifin
Department of Mathematics, Faculty of Science
Universiti Putra Malaysia
Email: fudziah@math.upm.edu.my

INTRODUCTION

There has been a great interest in the research of new methods for numerically solving the special second-order ordinary differential equations of the form

$$y'' = f(x, y), y(x_0) = y_0, y'(x_0) = y'_0 \tag{1}$$

The second-order equation can be directly solved by using Runge-Kutta-Nystrom (RKN) methods or multistep methods. In the development of numerical methods for solving (1), it is important to pay attention to the algebraic order as well as the phase-lag and dissipation order of the method. Phase-lag is the angle between the analytical solution and the numerical solution while dissipation is the distance from a standard cyclic solution. A pioneering work on phase-lag property has been done by Brusa and Nigro (1980). Some of the current developments of hybrid methods are the work of Tsitouras (2002, 2003, 2006) and Franco (2006). Here we derived explicit hybrid methods of algebraic order seven with higher order of phase-lag and dispersion using five stages per step. These methods will be compared with the existing methods in the literature.

EXPLICIT HYBRID METHOD

We consider the class of explicit hybrid methods established by Franco (2006) :

$$\left.\begin{aligned} &Y_1 = y_{n-1}, Y_2 = y_n \\ &Y_i = (1+c_i)y_n - c_i y_{n-1} + h^2 \sum_{j=1}^{i-1} a_{ij} f(x_n + c_j h, Y_j), i = 3,\ldots,s \\ &y_{n+1} = 2y_n - y_{n-1} + h^2\left[b_1 f_{n-1} + b_2 f_n + \sum_{i=3}^{s} b_i f(x_n + c_i h, Y_i)\right] \end{aligned}\right\} \tag{2}$$

The order conditions for this class of methods are given by Coleman (2003).

Let $H = \lambda h$ and $\mathbf{e} = (1 \ 1 \ \cdots \ 1)^T$. When the hybrid method is applied to equation $y'' = -\lambda^2 y$, $\lambda > 0$ we have

$$y_{n+1} - S(H^2) y_n + P(H^2) y_{n-1} = 0 \tag{3}$$

where $S(H^2) = 2 - H^2 \mathbf{b}^T (\mathbf{I} + H^2 \mathbf{A})^{-1} (\mathbf{e} + \mathbf{c})$, $P(H^2) = 1 - H^2 \mathbf{b}^T (\mathbf{I} + H^2 \mathbf{A})^{-1} \mathbf{c}$

The characteristic equation associated with (3) is

$$\xi^2 - S(H^2)\xi + P(H^2) = 0 \tag{4}$$

According to Houwen and Sommeijer (1987), phase-lag is defined as the difference $t = H - \theta(H)$ where H is the phase (or argument) of the exact solution of $y'' = -\lambda^2 y$ and

$\theta(H)$ is the phase of the principal root of (4). Corresponding to the characteristic equation (4) is the quantity $\varphi(H) = H - \arccos\left(\frac{S(H^2)}{2\sqrt{P(H^2)}}\right)$

Which is called the phase-lag (or dispersion error) while the quantity

$$d(H) = 1 - \sqrt{P(H^2)} \qquad (5)$$

is called the dissipation (or amplification) error.

DERIVATION OF THE NEW METHODS

In this section, we derive five-stage seventh order explicit hybrid methods algebraically. The new methods must satisfy the order conditions as given by Coleman (2003) with *s* = 6. There are 33 order conditions involved. By applying the simplifying condition

$$\sum_{j=1}^{6} a_{ij} c_j^{\alpha} = \frac{c_i^{\alpha+2} + (-1)^{\alpha} c_i}{(\alpha+1)(\alpha+2)}, \alpha = 0,1 \qquad (6)$$

some order conditions which are combinations of the other order conditions are eliminated leaving 15 order conditions. After some algebraic manipulations the only free parameter left is c_5. We obtain two methods depending on the value of c_5.

Strategies employed in choosing the free parameter of the methods are:

1) Minimize the function *P* which is given by $P = \sqrt{\sum_{i=1}^{8} s_i^2}$. Here, s_i's represent the order conditions of the eighth order method.
2) Increase the dissipation order.

For the first method, we select the parameter c_5 so that *P* is as small as possible which gives the value $c_5 = -\frac{28521}{50000}, E = 6.755534178017401 \times 10^{-4}$. This method which will be denoted as EHM7(8, 7) has the phase-lag of order 8 and dissipative of order 7.

For the second method, c_5 is selected so that the dissipation order is increased. After much algebraic manipulation and solving the resulting equations for c_5 yields $c_5 = \frac{406}{16347} - \frac{50993}{179817}\sqrt{5}$ and the error norm is $E = 6.156459599162350 \times 10^{-3}$. The phase-lag and dissipation error for this method are given by

$$\varphi(H) = -\frac{13}{7257600} H^9 + O(H^{11}) \quad \text{and} \quad d(H) = -2.631856949 \times 10^{-7} H^{10} + O(H^{20})$$

The method which will be denoted as EHM7(8, 9) has phase-lag of order 8 and dissipative of order 9.

NUMERICAL RESULTS

The following are some second-order problems taken from the literature that have been used to evaluate the performance of the methods.

Problem 1 : $y'' = -100y + 99\sin(x), y(0) = 1, y'(0) = 11, x \in [0,5]$

Solution: $y(x)=\cos(10x)+\sin(10x)+\sin(x)$.

Problem 2 : $y''=-y, y(0)=0, y'(0)=1, x\in[0,10]$, Solution: $y(x)=\sin(x)$.

Problem 3 :

$$y_1''=-y_1+\frac{1}{1000}\cos(x), y_1(0)=1, y_1'(0)=0, y_2''=-y_2+\frac{1}{1000}\sin(x), y_2(0)=0, y_2'(0)=0.9995$$

solution: $y_1(x)=\cos(x)+0.0005x\sin(x), y_2(x)=\sin(x)-0.0005x\cos(x)$.

Problem 4 :

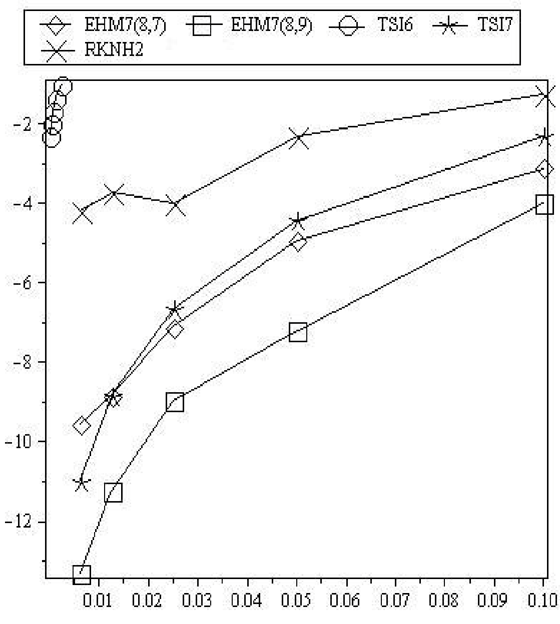
$$y''=-y-y^3+(\cos(x)+\varepsilon\sin(10x))^3-99\varepsilon\sin(10x), y(0)=1, y'(0)=10\varepsilon,\ x\in[0,20].$$

We choose $\varepsilon=10^{-3}$. Solution: $y(x)=\cos(x)+\varepsilon\sin(10x)$.

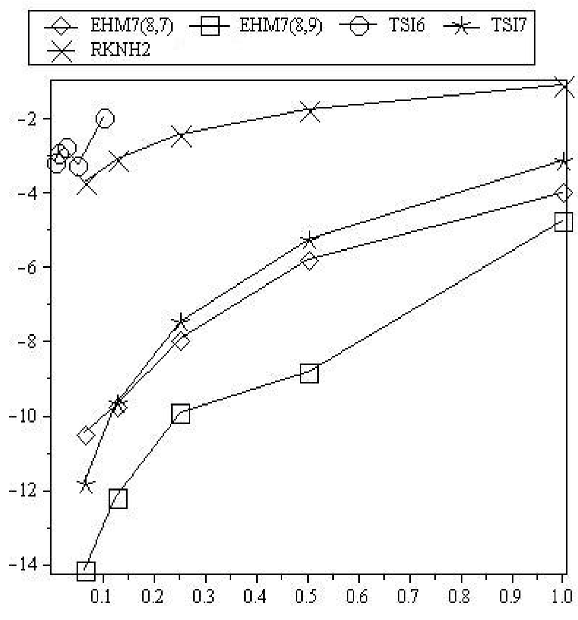
The methods that have been used in the comparisons are denoted by:

- TSI6: The sixth-order explicit hybrid method by Tsitouras (2003)
- TSI7: The seventh-order explicit hybrid method by Tsitouras (2002)
- RKNH2: The second-order RKN method by van Der Houwen and Sommeijer (1987)
- EHM7(8, 7): The first seventh-order explicit hybrid method with five stages derived in this paper.
- EHM7(8, 9): The second seventh-order explicit hybrid method with five stages derived in this paper.

Graphs of log(endpoint error) versus stepsize

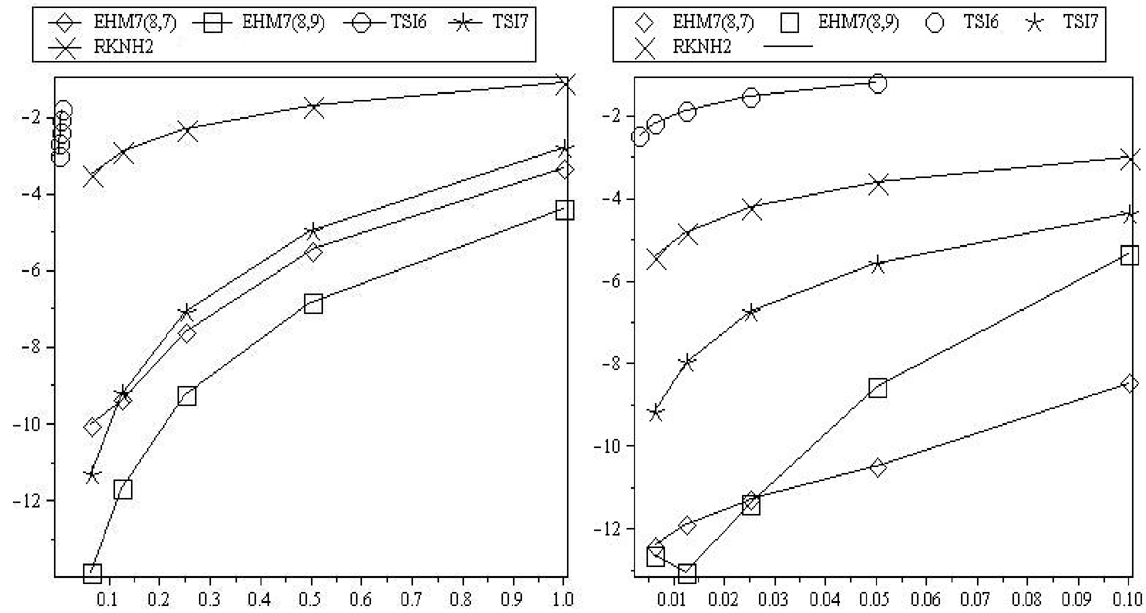


Problem 1



Problem 2

Graphs of log(endpoint error) versus stepsize



Problem 3 Problem 4

DISCUSSION AND CONCLUSION

From the numerical results it is observed that the new EHM7(8, 9) method is the most efficient for solving all the problems and method TSI6 is the least efficient method. For problem 4, method EHM7(8, 7) performed better for larger stepsizes and EHM7(8, 9) performed better for smaller stepsizes. As a conclusion we can say that the new methods are more efficient in terms of accuracy compared to the existing methods.

REFERENCES

Brusa, L. and Nigro, L. 1980. A one-step method for direct integration of structural dynamic equations. *International Journal for Numerical Methods in Engineering* 15: 685-699.

Coleman, J.P. 2003. Order conditions for a class of two-step methods for $y'' = f(x, y)$. *Journal of Numerical Analysis.* 23: 197-220.

Franco, J. M. 2006. A class of explicit two-step hybrid methods for second-order IVPs. *Journal of Computational .and Applied Mathematics*. 187: 41-57.

Tsitouras, Ch. 2002. Explicit two-step methods for second-order linear IVPs. *Computers and Mathematics with Applications*. 43: 943-949.

Tsitouras, Ch. 2003. Families of explicit two-step methods for integration of problems with oscillating solutions. *Applied Mathematics and Computation* 135: 169-178.

Tsitouras, Ch. 2006. Stage reduction on P-stable Numerov type methods of eighth order. *Journal of Computational and Applied.Mathematics*. 191: 297-305.

Van Der Houwen, P.J & Sommeijer, B.P. 1987. Explicit Runge-Kutta (-Nystrom) methods with reduced phase errors for computing oscillating solutions. *SIAM Journal of Numerical Analysis*. 24: 595-617.

SOLVING DELAY DIFFERENTIAL EQUATIONS BY USING PARALLEL TECHNIQUES

Fuziyah Ishak
Faculty of Computer and Mathematical Sciences, Universiti Teknologi MARA, 40450 Shah Alam, Selangor, Malaysia.
fuziyah@tmsk.uitm.edu.my

Mohamed B. Suleiman
Mathematics Department, Faculty of Science, Universiti Putra Malaysia, 43400 Serdang, Selangor, Malaysia

INTRODUCTION

Delay differential equations (DDEs) arise naturally in modeling real life phenomena. The application areas involving DDEs as stated in Driver (1977) include but are not limited to population growth, predator-prey population models, control systems and two-body problem of electrodynamics. Analytical solutions for DDEs are hard and at times are impossible to find. As an alternative, scientists resort to numerical solutions that can be made as accurately as possible. Earlier work can be referred to Suleiman and Ismail (2001), Jackiewicz and Lo (2006) and Suleiman and Ishak (2010).

Numerical solutions for DDEs require a lot of computational effort. Today's computer technology provides a better solution since greater computing power can easily be achieved by exploiting the usage of super computers equipped with high speed multiprocessors. This work focuses on the parallel implementation of a predictor-corrector multistep method for solving first order systems of DDEs of the form:

$$\begin{aligned} &y'(x) = f(x, y(x), y(x-\tau_i)), && x \in [a,b], \tau_i > 0, i = 1, 2, \ldots, n, \\ &y(x) = \varphi(x), && x \in [\bar{a}, a], \end{aligned} \tag{1}$$

where $\varphi(x)$ is the initial function, τ_i is either constant, time dependent or state dependent lag function and $\bar{a} = \min\limits_{x\in[a,b]} (x-\tau_i)$. The numerical solution is based on multistep method in variable stepsize variable order (VSVO) scheme where two approximations are obtained simultaneously in a single integration step.

PARALLEL IMPLEMENTATION

The multistep formulae are implemented in PECE mode where P stands for an application of a predictor, E stands for a function evaluation, and C stands for an application of a corrector. Two approximation formulae are obtained by integrating equation (1) and replacing the function f by interpolating polynomials in divided difference form. Details discussion on the derivation and the VSVO implementation can be referred to Ishak et al. (2008).

The computational cost in solving DDEs arises mainly in function evaluations. In DDEs, as opposed to ordinary differential equations, the derivatives of the unknown functions depend on the unknown functions at the present as well as the past states. The extra cost in function evaluations is reduced by assigning two processors to work independently on each approximation. These tasks are performed concurrently between two processors of a High Performance Computer with distributed-shared memory architecture. The processors communicate via a message passing protocol known as Message Passing Interface. The parallel algorithm is described in Figure 1.

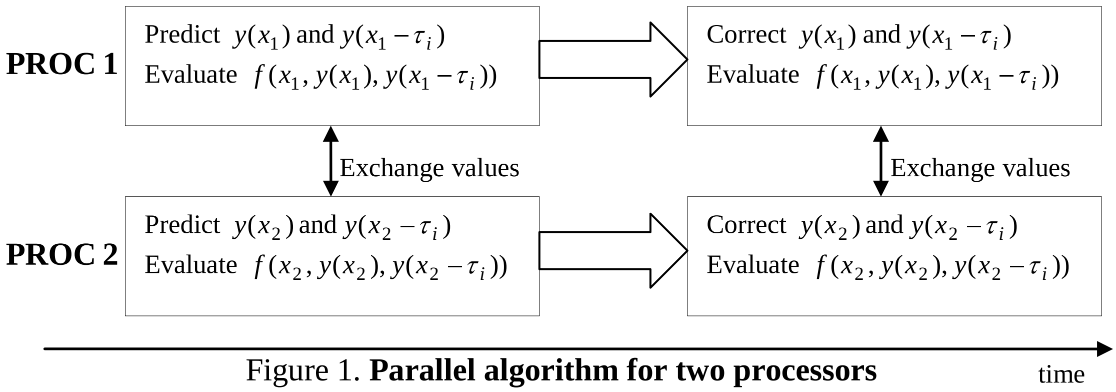


Figure 1. **Parallel algorithm for two processors**

RESULTS AND DISCUSSION

The effectiveness of the parallel implementation is demonstrated by applying the multistep method to systems of DDEs for various numbers of equations. The accuracy of the method is validated by the average and maximum errors. For comparison, the experiment is conducted using sequential and parallel versions of the algorithm. The performance of the parallel program is assessed in terms of speedup and efficiency. It is shown that the speedup and efficiency increase as the number of equations in the system increases.

CONCLUSION AND FUTURE RESEARCH

Numerical solution for DDEs requires a lot of computational effort. The workload can be reduced by applying concurrent computation on parallel machines. Thus, parallelizing the multistep method increases the overall performance of the method.

For future research, the multistep formulae will be derived to increase the number of approximations in every integration step. Thus, more processors can be assigned for computing in each step. While this work focuses on parallelism across time approach which is method dependent, we can also consider parallelism across space approach.

REFERENCES

Driver, R. D. 1997. *Ordinary and Delay Differential Equations*, Springer Verlag New York Inc., New York.

Ishak, F., Suleiman, M. & Omar, Z. 2008. Two-point predictor-corrector block method for solving delay differential equations, *MATEMATIKA*, 24(2): 131-140.

Jackiewicz, Z. & Lo, E. 2009. Numerical solution of neutral functional differential equations by Adams methods in divided difference form, *J. of Computational and Appl. Math*., 189: 592-605.

Suleiman, M. B. & Ishak, F.2010. Numerical solution and stability of multistep method for solving delay differential equations, *Japan J. of Industrial and Appl. Math*., 27(3): 395-410.

Suleiman, M. B. & Ismail, F. 2001. Solving delay differential equations using componentwise partitioning by Runge-Kutta method. *Applied Math. And Computation*, 122: 301-323.

A DIFFERENTIAL GAME DESCRIBED BY AN INFINITE SYSTEM OF DIFFERENTIAL EQUATIONS

Gafurjan Ibragimov, Risman Mat Hasim
INSPEM & Department of Mathematics, Faculty of Science,
University Putra Malaysia,
43400, Serdang, Selangor, Malaysia.
gafur@science.upm.edu.my, risman@math.upm.edu.my

INTRODUCTION

Let $\lambda_1, \lambda_2, \cdots$ be any sequence of positive numbers and r be a fixed number,

$$l_r^2 = \{\alpha = (\alpha_1, \alpha_2, \cdots): \sum_{i=1}^{\infty} \lambda_i^r \alpha_i^2 < \infty\}$$

be the space with the inner product and the norm defined by

$$(\alpha, \beta)_r = \sum_{i=1}^{\infty} \lambda_i^r \alpha_i \beta_i, \quad \alpha, \beta \in l_r^2, \quad \| \alpha \| = \left(\sum_{i=1}^{\infty} \lambda_i^r \alpha_i^2 \right)^{1/2}.$$

Let $L_2(t_0, T, l_r^2)$ be the space of the functions $f(t) = (f_1(t), f_2(t), \cdots), t_0 \le t \le T$, with measurable coordinates $f_k(t), t_0 \le t \le T$, satisfying the inequality

$$\sum_{k=1}^{\infty} \lambda_k^r \int_{t_0}^{T} \| f_k(t) \|^2 \, dt < \infty,$$

where $T > 0$ is a fixed number, and let $C(t_0, T, l_{r+1}^2)$ be the space of functions $z(t) = (z_1(t), z_2(t), \cdots)$, $t_0 \le t \le T$, such that $z(t) \in l_{r+1}^2$ for each $t \in [t_0, T]$ and the function $z(t), t_0 \le t \le T$, is continuous in the norm of the space l_{r+1}^2. Note that the number T can be chosen greater enough.

We consider differential game described by the infinite system of differential equations

$$\dot{z}_k = -\lambda_k z_k - u_k + v_k,\ \ z_k(0) = z_{k0},\ \ k = 1, 2, \cdots, \tag{1}$$

where $z_k, u_k, v_k, z_{k0} \in R^1$, $z_0 \in l^2_{r+1}$, $z_0 = (z_{10}, z_{20}, \cdots) \neq 0$, $u = (u_1, u_2, \cdots)$ is the control parameter of the pursuer and $v = (v_1, v_2, \cdots)$ is that of the evader. Suppose that $u(\cdot), v(\cdot) \in L_2(0, T, l^2_r)$.

Definition 1. *A function* $u(\cdot) = (u_1(\cdot), u_2(\cdot), \cdots)$ *(respectively* $v(\cdot) = (v_1(\cdot), v_2(\cdot), \cdots)$*) that satisfy the condition*

$$\int_0^T \| u(t) \|^2 \, dt \leq \rho^2 \left(\int_0^T \| v(t) \|^2 \, dt \leq \sigma^2 \right), \| u(t) \| = \left(\sum_{k=1}^{\infty} \lambda_k^r u_k^2(t) \right)^{1/2},$$

where ρ and σ ($\rho > \sigma$) are given positive numbers, is called a control of the pursuer (evader).

Definition 2. *A function of the form* $u(t, v) = w_0 + v, 0 \leq t \leq T,$ *is called a strategy of the pursuer, where* $w_0(\cdot) = (w_{10}(\cdot), w_{20}(\cdot), \cdots) \in L_2(0, T, l^2_r)$ *is an arbitrary function subject to*

$$\sum_{k=1}^{\infty} \lambda_k^r \int_0^T w_{k0}^2(t) dt \leq (\rho - \sigma)^2.$$

Definition 3. *We say that pursuit starting from the initial position* $z_0 \in l^2_{r+1}$ *can be completed for the time* ϑ *if there exists a strategy* $u(t, v)$ *of the pursuer such that for any evader's control* $v(t), 0 \leq t \leq \vartheta,$ *equality* $z(\tau) = 0$ *occurs at some* τ, $0 \leq \tau \leq \vartheta$.

We can extend the system (1) by introducing additional state variables p, q by equations

$$\dot{p} = -\| u(t) \|^2,\ p(0) = \rho^2,\ \ \dot{q} = -\| v(t) \|^2,\ q(0) = \sigma^2. \tag{2}$$

Note that $p(t)$ and $q(t)$ are the amounts of control resources of the players remained at the time t.

Definition 4. *A function* $V(z, p, q), V : l^2_{r+1} \times [0, \rho^2] \times [0, \sigma^2] \to l^2_r,$ *is referred to as the strategy of the evader if*

1) for any control $u = u(t), 0 \leq t \leq T,$ of the pursuer the system (1), (2) has a unique solution at $v = V(z(t), p(t), q(t)), 0 \leq t \leq T;$

2) the inequality $\int_0^T \| V(z(t), p(t), q(t)) \|^2 \, dt \leq \sigma^2$ holds.

Definition 5. If there exists a strategy V_0 of the evader such that $z(t) \neq 0, t \in [0, T]$ for any control $u(\cdot)$ of the pursuer, then we say that evasion is possible in the game (1), (2).

Problem. Find all initial positions $z_0 \in l^2_{r+1}$ from that evasion is possible in the game (1), (2).

MAIN RESULTS

The following theorem is true.

Theorem. *Let* $\rho > \sigma$ *and* $z_0 \in l_{r+1}^2$. *If* $z_0 \notin l_r^2$, *then evasion is possible on* $[0, T]$ *in the game (1), (2).*

CONCLUSION

We have studied a pursuit-evasion differential game with integral constraints. In the works Tukhtasinov (1995) and Ibragimov (2003) the numbers $\lambda_k, k = 1, 2, \cdots,$ satisfy the condition $0 < \lambda_1 \le \lambda_2 \le \cdots \to \infty$. In contrast to the previous works, in the present paper the numbers $\lambda_k, k = 1, 2, \cdots,$ are any positive numbers. It is assumed throughout the paper that $z_0^2 \in l_{r+1}$ since the existance-uniqueness theorem (see Ibragimov 2005) was proved under this condition. We have divided the space l_{r+1}^2 into two parts. We earlier proved that if $\rho > \sigma$ and $z_0 \in l_{r+1}^2 \cap l_r^2$, then pursuit can be completed. In this work, in the case $z_0 \in l_{r+1}^2 \setminus l_r^2$ and $\rho > \sigma$, we have proved that evasion is possible on $[0, T]$.

REFERENCES

Butkovskiy A.G. (1975). *Control Methods in Systems with Distributed Parameters*. Nauka, Moscow.

Chernous'ko F.L. (1992). *Bounded Controls in Systems with Distributed Parameters*. Prikl. Mat. Mekh., Vol 56. No 5. pp.810-826.

Egorov A.I. (2004). *Principles of the control theory*. Nauka, Moscow, 504 p.

Tukhtasinov, M. (1995). *On Some Problems in the Theory of Differential Pursuit Games in Systems with Distributed Parameters.* Prikl. Mat. Mekh., Vol. 59, No. 6, pp. 979–984.

Satimov N.Yu., and Tukhtasinov M. (2006). *Game problems on a fixed interval in controlled first-order evolution equations*. Mathematical notes, V.80, No 3-4, pp.578-589.

Satimov N.Yu. and Tukhtasinov M. (2007). *On Game Problems for Second-Order Evolution Equations*. Russian Mathematics (Iz. VUZ), Vol. 51, No.1, pp. 49–57.

Ibragimov G.I. (2003). *A problem of optimal pursuit in systems with distributed parameters*. J. Appl. Math. Mech. Vol. 66, No 5, pp.719-724.

Ibragimov G.I. (2004). *On possibility of evasion in a differential game described by infinite system of differential equations*. Uzbek Math. Journal, Tashkent, No.1, pp. 50-55

PLATO'S MATHEMATICAL FORMS: AN ISLAMIC CRITIQUE

Habsah Ismail,
Faculty of Educational Studies, Universiti Putra Malaysia
habsah@educ.upm.edu.my

Mat Rofa Ismail
Faculty of Science, Universiti Putra Malaysia
mrofa@yahoo.com

ABSTRACT

Plato's theory of Forms rests on the belief that these forms are representatives of eternal concepts that belong to a transcendent realm separated from the physical or the material realm. The Theory of Forms which can also be understood in terms of mathematics, held mathematical knowledge in high esteem as these forms collectively merged, depict the Highest Form of Goodness or God. However, Plato's mathematical forms, since the time of Aristotle, and, to date, had been refuted and amongst the point of attack, is the inconclusive justification by Plato in addressing and linking the concept of God and the true existence of these forms. Hence, this paper seeks to address this issue by providing an Islamic critique in an overview of Plato's mathematical forms.

REFERENCES

Stumpf,S.E. (1999). *Socrates to Sartre: A history of philosophy.* New York. McGraw-Hill college

Zietlin, I.M. 1993). *Plato's Vision: The classical origins of social and political thought.* New Jersey. Prentice Hall.

Sarwar, G. (1973). *Philosophy of the Quran.* Lahore. Ashraf Press.

Abdul Latiff Samian. (1997). *Falsafah Matematik.* Kuala Lumpur, Dewan Bahasa dan Pustaka.

Ernest, P. (2003). *The philosophy of Mathematics education.* London. Routledge Falmer.

Wan Mohd Nor Wan Daud. (1998). *The educational philosophy and practice of Syed Muhammad Naquib Al-Attas.: An exposition of the original concept of Islamization.* Kuala Lumpur. International Institute of Islamic Thought and Civilization (ISTAC).

ON RANKING FUZZY NUMBERS METHOD USING AREA DOMINANCE APPROACH

Harliza Mohd Hanif[1], Daud Mohamad[2], Nor Hashimah Sulaiman[3]
[1,2,3] *Faculty of Computer and Mathematical Sciences,*
Universiti Teknologi MARA, 40450 Shah Alam, Selangor, MALAYSIA.
harlizaaj86@yahoo.com, daud@tmsk.uitm.edu.my,
nhashima@tmsk.uitm.edu.my

INTRODUCTION

To rank fuzzy numbers correctly is crucial particularly in determining the right choice of alternatives in decision-making process.Study of ranking fuzzy numbers has begun since 1970 when fuzzy set theory began to see its practical applications in many areas such as decision making, optimization, control and others. Since then, many different approaches had been proposed including the area-based ranking of fuzzy numbers. Each method of ranking fuzzy numbers is bounded to certain limitations. Most proposed ranking methods merely focus on ranking normal and linear form of fuzzy numbers. When it comes to ranking fuzzy numbers that have the same mean and symmetrical spread, many methods can not rank them correctly. In some situations, the ranking order produced by certain method is inconsistent with the others. A ranking fuzzy number method based on area dominance has been introduced by Tseng and Klein in 1989. However with the latest development in the ranking methods that also consider non normal and non linear forms of fuzzy numbers, this method cannot rank these generalized form of fuzzy number. In this paper we improvised the method of Tseng and Klein by introducing the concept of upper dominance.

AREA DOMINANCE METHOD

Let *A* and *B* be two fuzzy numbers. Indifferent area of *A* and *B* is defined as the overlapping area between *A* and *B*. For non-overlapping areas between *A* and *B*, *A* dominates *B* if *A* is in the right-hand side of *B* and *B* is in the left-hand side of the fuzzy number *A*.

The second category is for the overlapping case between fuzzy numbers *A* and *B*. *A* dominates *B* if the area belongs to *A* and is on the right-hand side of the overlapping area, or the area belongs to *B* and is on the left-hand side of the overlapping area.

The notation of $R(A, B)$ and $R(B, A)$ are used to represent two fuzzy preference relations between *A* and *B* where

$$R(A,B) = \frac{(\text{areas where } A \text{ dominates } B) + (\text{area where } A \text{ and } B \text{ are indifferent})}{(\text{area of } A) + (\text{area of } B)}$$

$$R(B,A) = \frac{(\text{areas where } B \text{ dominates } A) + (\text{area where } A \text{ and } B \text{ are indifferent})}{(\text{area of } A) + (\text{area of } B)}$$

$$R(A,B) + R(B,A) = 1 \qquad (1)$$

If the $R(A, B)$ is greater than 0.5, then the ordering of *A* and *B* is defined as '*A* is preferred to *B*' ($A \succ B$). If $R(A, B)$ is equal to 0.5, then the ordering of *A* and *B* is defined as '*A* is indifferent to *B*' ($A \sim B$) and if $R(A, B)$ is less than 0.5, then the ordering of *A* and *B* is defined as *B* is preferred to *A* ($B \succ A$).

SPREAD OF FUZZY NUMBERS

The concept of spread has been applied in fuzzy ranking by some researchers. Among them are Lee and Li (1988), Chen and Lu (2002) and Chen and Chen (2003). Chen and Chen (2003) calculated the spread of a fuzzy number as

$$S = \sqrt{\frac{\sum_{j=1}^{4}\left(a_j - x^*\right)^2}{4-1}}$$

where

$$x^* = \frac{y^*(a_2+a_3)+(a_1+a_4)\left(w-y^*\right)}{2w}, \qquad y^* = \frac{\left(w\left(\frac{a_3-a_2}{a_4-a_1}+2\right)\right)}{6}$$

and (x^*, y^*) is the center of gravity (COG) of the fuzzy number.

AN IMPROVED ALGORITHM

Let *A*, *B* be fuzzy numbers and α be membership value such that $\alpha \in [0,1]$. Without lost of generality, let the first situation be as in Figure 1 where there is an area above the overlapping area between *A* and *B*.

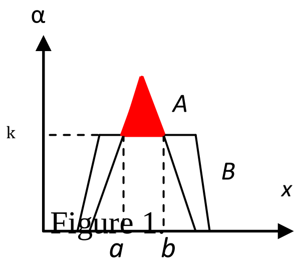


Figure 1:

A non-overlapping area (shaded region) is considered an upper dominance if

i. the lowest membership value of this area at $\alpha=\mu_k$ is non-zero, and

ii. $\mu_A(c) \geq \mu_k$ where $c \in [a,b]$ and $[a,b]$ is an overlapping interval at $\alpha=\mu_k$.

For the second situation, let, again without lost of generality, the fuzzy numbers *A* and *B* be as in Figure 2a or Figure 2b where the non-overlapping area has its left value *a* and its right value *b* at α=0 where the membership values of *a* and *b* in both A and B are 0 and is not classified as left and right dominance (as defined by Tseng & Klein's).

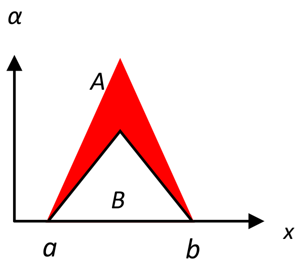


Figure 2a.

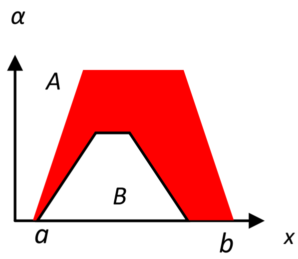


Figure 2b.

The non-overlapping area is considered as upper dominance if

i. $\mu_A(a) = \mu_B(a) = 0$ and $\mu_A(b) = \mu_B(b) = 0$

ii. $\mu_A(c) \geq \mu_B(c)$ where $c \in [a, b]$.

A is said to dominate *B* if

a) the non-overlapping area belongs to *A* and is on the right-hand side of the overlapping area, or

b) the non-overlapping area belongs to *B* and is on the left-hand side of the overlapping area, or

c) the non-overlapping area belongs to *A* and is at the upper dominance of the overlapping area.

We then define $I_{AB} = R(A,B)^{S(A)}$ and $I_{BA} = R(B,A)^{S(B)}$ to represent two fuzzy preference index between *A* and *B* where *R*(*A*, *B*) and *R*(*B*,*A*) are as in (1) and *S*(*A*) and *S*(*B*) denote the spread of fuzzy numbers *A* and *B* respectively using Chen and Chen (2003).

The fuzzy numbers A *is* ordered higher than B if I_{AB} is greater than I_{BA} and B is ordered higher than A if I_{BA} is greater than I_{AB}.

CONCLUSION

An improvement to Tseng and Klein (1989) is given in this paper. The proposed method is also been investigated for its consistency in ranking order as compared to some existing methods. It can rank non-normal fuzzy numbers as well as the non-linear form of fuzzy numbers. Furthermore, it also can rank two embedded symmetrical fuzzy numbers of different spread for which many other methods usually are not capable to rank. The consideration of spread factor to the method has made the ranking better and consistent with human intuition.

Acknowledgment. This research has been supported by the Universiti Teknologi MARA and the Minstry of Higher Education of Malaysia (MOHE) under the grant code 600-RMI/ST/FRGS 5/3/Fst (22/2008)

REFERENCES

L. H. Chen, H. W. Lu, An approximate approach for ranking fuzzy numbers based on left and right dominance, *Computers and Mathematics with Applications*, 41,1589-1602, 2001.

S. J. Chen and S. M. Chen, Fuzzy Risk Analysis Based on Similarity Measures of Generalized Fuzzy Numbers, *IEEE Transaction on Fuzzy System*, 11, 45-56, 2003.

E. S. Lee and R. L. Li, Comparison of fuzzy numbers based on the probability measure of fuzzy events, *Computer and Mathematics with Applications.*,10, 887-896, 1988.

T. Y. Tseng and C. M. Klein, New Algorithm for the Ranking Fuzzy Procedure in Fuzzy Decision-Making, *IEEE Transaction, System, Man and Cybernetic* 19, 1289-1296, 1989.

FUNCTIONAL FORM FOR THE GENERALIZED POISSON REGRESSION MODEL AND ITS APPLICATION IN INSURANCE

Hossein Zamani and Noriszura Ismail
School of Mathematical Sciences, Faculty of Science and Technology
Universiti Kebangsaan Malaysia
ni@ukm.my

INTRODUCTION

In actuarial literature as well as insurance practice, the Poisson regression model has been widely used for modeling claim count data. However, it is also recognized that the claim count data often display overdispersion and hence, inappropriate imposition of the Poisson may underestimate the standard errors and overstate the significance of the regression parameters, giving misleading inference about the regression parameters. The mixture of Poisson regression models such as the Poisson-inverse Gaussian, the Poisson-lognormal and the Poisson-gamma (negative binomial), have been suggested and applied for accommodating overdispersion in claim count data. Several parameterizations were performed for the negative binomial regression models, and the NB-1 and the NB-2 were developed and applied on count data (Cameron and Trivedi, 1986; Lawless, 1987). The generalized Poisson (GP) is an increasingly popular approach for modeling overdispersed as well as underdispersed count data. The GP-1 was applied by several researchers for dealing with overdispersion and underdispersion in count data (Consul and Famoye, 1992), whereas the GP-2 was used by Wang and Famoye (1997) and Ismail and Jemain (2007).

This paper develops a functional form of the GP regression model, which is referred as the GP-P model, that parametrically nests the Poisson and the two well known GP regression models (GP-1 and GP-2). Greene (2008) implemented the same approach for developing a functional form of the negative binomial regression model, which is referred as the NB-P, where the NB-1 and NB-2 are the special cases of the NB-P when $P=1$ and $P=2$ respectively.

GP REGRESSION MODELS

Let $(Y_1, Y_2, ..., Y_n)^T$ denotes the vector of count random variables where Y_i and Y_j are independent and identically distributed for any $i \neq j$, and n be the sample size. The p.m.f of the GP regression is,

$$\Pr(Y_i = y_i \mid \theta_i, v) = \frac{\theta_i(\theta_i + vy_i)^{y_i-1}\exp(-\theta_i - vy_i)}{y_i!}, \quad y_i = 0,1,2,... \qquad (1)$$

where $\theta_i > 0$ and $\max(-1, -\frac{\theta_i}{4}) < v < 1$. The mean and the variance are $E(Y_i) = \mu_i = (1-v)^{-1}\theta_i$ and $Var(Y_i) = (1-v)^{-3}\theta_i = (1-v)^{-2}\mu_i$, where $(1-v)^{-2}$ is the dispersion factor. The covariates can be included via a log link function, $\log(\mu_i) = \log((1-v)^{-1}\theta_i) = \mathbf{x}_i^T\boldsymbol{\beta}$.

The GP-1 has the following p.m.f.,

$$\Pr(Y_i = y_i \mid \mu_i, v) = \frac{((1-v)\mu_i + vy_i)^{y_i-1}(1-v)\mu_i \exp(-(1-v)\mu_i - vy_i)}{y_i!}, \quad y_i=0,1,2,..., \qquad (2)$$

with mean, $E(Y_i) = \mu_i$, and variance, $Var(Y_i) = (1-v)^{-2}\mu_i$. In this paper, we propose a parameterization for deriving a new form of the GP-1 model which has the same properties but different form of p.m.f., by rewriting $v = \varphi(1+\varphi)^{-1}$, so that $\theta_i = \mu_i(1+\varphi)^{-1}$. The p.m.f. of the new form of GP-1 model is,

$$\Pr(Y_i = y_i \mid \mu_i, \varphi) = \frac{\mu_i(\mu_i + \varphi y_i)^{y_i-1}\exp\left(-\frac{\mu_i + \varphi y_i}{1+\varphi}\right)}{(1+\varphi)^{y_i}\, y_i!}, \quad y_i = 0,1,2,..., \qquad (3)$$

with mean, $E(Y_i) = \mu_i$, and variance, $Var(Y_i) = (1+\varphi)^2\mu_i$, where φ denotes the dispersion parameter. The GP-1 model shown in equation (4) reduces to the Poisson when $\varphi = 0$, allows for overdispersion when $\varphi > 0$, and allows for underdispersion when $\varphi < 0$.

The parameterization for the GP-2 model is given by $v = \varphi\mu_i(1+\mu_i\varphi)^{-1}$, so that $\theta_i = \mu_i(1+\mu_i\varphi)^{-1}$, and the p.m.f. is,

$$\Pr(Y_i = y_i \mid \mu_i, \phi) = \frac{\mu_i(\mu_i + \phi\mu_i y_i)^{y_i-1}\exp\left(-\frac{\mu_i + \phi\mu_i y_i}{1+\phi\mu_i}\right)}{(1+\phi\mu_i)^{y_i}\, y_i!}, \quad y_i = 0,1,2,..., \quad (4)$$

with mean, $E(Y_i) = \mu_i$, and variance, $Var(Y_i) = (1+\varphi\mu_i)^2 \mu_i$. The GP-2 model shown in equation (5) reduces to the Poisson when $\varphi = 0$, allows for overdispersion when $\varphi > 0$, and allows for underdispersion when $\varphi < 0$.

In this section, a functional form of the GP regression model, which is referred as the GP-P model, is developed. The GP-1 and the GP-2 described in p.m.f.s (3)-(4) are the special cases of the GP-P. The GP-P is given by the parameterization of $\theta_i = \mu_i(1+\phi\mu_i^{P-1})^{-1}$, so that $v = \phi\mu_i^{P-1}(1+\phi\mu_i^{P-1})^{-1}$. Hence, the p.m.f. of the GP-P regression model is,

$$\Pr(Y_i = y_i \mid \mu_i, \phi, P) = \frac{\mu_i(\mu_i + \phi\mu_i^{P-1} y_i)^{y_i-1}\exp\left(-\frac{\mu_i + \phi\mu_i^{P-1} y_i}{1+\phi\mu_i^{P-1}}\right)}{(1+\phi\mu_i^{P-1})^{y_i}\, y_i!}, \quad y_i = 0,1,2,..., \quad (5)$$

with mean, $E(Y_i) = \mu_i$, and variance, $Var(Y_i) = (1+\varphi\mu_i^{P-1})^2 \mu_i$. When $\varphi = 0$, the GP-P reduces to the Poisson, when $\varphi > 0$, we have overdispersion and when $\varphi < 0$, we have underdispersion. In addition, the GP-P reduces to the GP-1 when $P = 1$, and reduces to the GP-2 when $P = 2$.

APPLICATION

The data for private car Own Damage (OD) claim counts obtained and compiled from ten insurance companies in Malaysia will be considered as an example. The data, which was based on 1.01 million private car policies for a three-year period of 2001-2003, was supplied by Insurance Services Malaysia (ISM). The exposure was expressed in a car-year unit and the incurred claims consist of claims already paid as well as outstanding. By excluding zero exposures, we have a total of 547 count data to be fitted. Table 1 shows the parameter estimates and the t-ratios for the fitted models of the Malaysian motor claims. The results show that the regression parameters for all models have similar estimates. As expected, the GP-1, the GP-2 and the GP-P models provide similar inferences for the regression parameters, i.e. their t-ratios are smaller than the Poisson. The likelihood ratio may be employed to assess the adequacy of the GP-1 or the GP-2 over the Poisson since both models reduce to the Poisson when φ equals zero. The likelihood ratios for testing the Poisson against the GP-1 and the GP-2 indicate that the GP-1 and the GP-2 are better models compared to the Poisson. The likelihood ratio may also be implemented for testing the adequacy of the GP-P over the GP-1 or the GP-2, since the GP-P reduces to the GP-1 and the GP-2 when $P = 1$ and $P = 2$ respectively. The likelihood ratios for testing the GP-1 against the GP-P and the GP-2 against the GP-P indicate that the GP-P is a better model.

Table 1: Parameter estimates and *t*-ratios

Parameters	Poisson	GP-1	GP-2	GP-P

	Est.	*t*-ratio	Est.	*t*-ratio	Est.	*t*-ratio	Est.	*t*-ratio
Intercept	-3.04	-192.72	-3.07	-66.87	-3.09	-43.84	-3.09	-53.18
2-3 year	0.51	41.10	0.53	14.53	0.57	8.99	0.55	11.47
4-5 year	0.52	39.94	0.50	13.13	0.51	8.02	0.52	10.55
6-7 year	0.43	33.62	0.45	12.02	0.37	5.90	0.45	9.34
8+ year	0.24	18.97	0.23	6.18	0.29	4.47	0.26	5.44
1001-1300 cc	-0.31	-24.05	-0.26	-6.96	-0.10	-1.54	-0.23	-5.00
1301-1500 cc	-0.16	-14.37	-0.14	-4.06	0.19	2.90	-0.13	-3.03
1501-1800 cc	0.14	12.73	0.13	4.18	0.29	5.36	0.14	3.64
1801+ cc	0.12	10.55	0.10	2.98	0.35	6.26	0.16	3.90
Local type 2	-0.46	-31.90	-0.43	-10.39	-0.43	-6.22	-0.42	-8.13
Foreign type 1	-0.21	-18.06	-0.16	-4.54	-0.42	-7.36	-0.20	-4.93
Foreign type 2	0.18	10.85	0.24	5.07	0.23	3.93	0.24	4.77
Foreign type 3	-0.02	-0.80	0.07	1.25	-0.25	-3.39	-0.03	-0.36
East	0.35	19.91	0.40	8.32	0.27	4.58	0.37	6.56
Central	0.32	29.43	0.31	9.45	0.28	4.80	0.31	7.40
South	0.26	20.41	0.26	6.85	0.39	6.77	0.30	6.43
East Malaysia	0.13	8.88	0.12	2.74	0.09	1.54	0.11	2.16
φ	-	-	2.02	19.08	0.02	12.35	0.64	7.86
P	-	-	1.00	-	2.00	-	1.27	45.78
$\log L$		-3,613.39		-2,164.45		-2,262.40		-2,111.85

CONCLUSION

This paper develops a functional form of the GP regression model, which is referred as the GP-P, that parametrically nests the Poisson and the two well known GP models (GP-1 and GP-2). The estimation of the regression parameters, the dispersion parameter, φ, and the functional parameter, P, for the GP-P is implemented using a maximum likelihood procedure.

It is worth to note that even though the p.m.f. of the GP-1, the GP-2 and the GP-P models are written in mathematically complex formulas, the mean and the variance are conceptually simpler to interpret, i.e. the mean for the GP-1, the GP-2 and the GP-P are equal to the Poisson, but the variance may be equal, or larger, or smaller than the Poisson and hence, allowing these models to handle either equi- or over- or underdispersion. In particular, the mean-variance relationship of the GP-1 is in a linear form, the GP-2 in a cubic form, and the GP-P in a functional form.

ACKNOWLEDGMENT

The authors gratefully acknowledge the financial support received in the form of a fundamental research grant (UKM-ST-06-FRGS0108-2009) from the Ministry of Higher Education (MOHE), Malaysia. The authors are pleasured to thank Insurance Services Malaysia Berhad (ISM) for supplying the data.

REFERENCES

Cameron, A.C. and Trivedi, P.K. (1986). Econometric models based on count data: comparisons and applications of some estimators and tests. *Journal of Applied Econometrics* 1: 29-53.

Consul, P.C. and Famoye, F. (1992). Generalized Poisson regression model. *Communication Statistics (Theory and Methodology)* 2(1): 89-09.

Greene, W. (2008). Functional forms for the negative binomial model for count data. *Economics Letters* 99: 585-590.

Ismail, N. and Jemain, A.A. (2007). Handling overdispersion with negative binomial and generalized Poisson regression models. *Casualty Actuarial Society Forum* Winter: 103-158.

Lawless, J.F. (1987). Negative binomial and mixed Poisson regression. *The Canadian Journal of Statistics* 15(3): 209-225.

Wang, W. and Famoye, F. (1997). Modeling household fertility decisions with generalized Poisson regression. *Journal of Population Economics* 10: 273-283.

MUSYARAKAH MODELS OF JOINT VENTURE INVESTMENTS BETWEEN TWO PARTIES

[1]**Maheran Mohd Jaffar** (maheran@tmsk.uitm.edu.my)
[2]**Rashidah Ismail** (shidah@tmsk.uitm.edu.my)
[3]**Hamdan Abdul Maad** (hamdan@tmsk.uitm.edu.my)
[4]**Abd Aziz Samson** (aziz@tmsk.uitm.edu.my)
[1,2,3,4]*Pusat Pengajian Matematik, Fakulti Sains Komputer dan Matematik,*
Universiti Teknologi MARA Malaysia
40450 Shah Alam, Selangor, Malaysia

INTRODUCTION

Currently, the Islamic financial products have become an alternative to the conventional financial products. The difference between them is that *syariah* contracts are involved in managing the process from the beginning of packaging the Islamic products until the end of the contract in order to avoid any involvement of *riba*.

Riba is an Arabic words which from *syariah* point of view means the *premium* along with the principle amount that must be paid by the borrower to the lender as the conditition for the loan or for the extension of maturity period [1]. In order to avoid *riba*, the management of the Islamic financial products can use contract like *musyarakah* which are also known as the equity-financing or the profit and loss contracts.

Musyarakah contract is a joint venture contract between two or more parties and the profit is shared according to the agreed profit sharing ratio. All partners have the right to participate actively in the joint venture of *musyarakah* and it is only natural for the partner with entrepreneurship expertise to take charge. The loss shall be borne by the partners according to their capitals or an agreed proportion ([2]; [3]; [4]).

According to [4], even though the Islamic scholars encourage the use of *musyarakah* principles, none of the Islamic banks uses these concepts in more than 10% of their financial portfolio. In Malaysia, the use of musyarakah concept in financial activities has shown to increase slowly from 0.5% in 2002, 1.4% in 2008 and 1.7% in 2009 [5].

This paper discusses the mathematical models of some *musyarakah* products of a joint venture between two parties and the weakness of each model.

MUSYARAKAH MODELS

Musyarakah 1. The capital provider and entrepreneur invest E_0 and Q_0 respectively at the initial time $t=0$. The profit gained at time *t* on the initial investments of E_0 and Q_0 are r_tE_0 and r_tQ_0 respectively. At the end of contract, only the profit r_tE_0 is shared between the capital provider and the entrepreneur with the profit sharing rates of $k:(1-k)$. The investment of the capital provider and the entrepreneur at time *t* are E_t and Q_t respectively as follows:

$$E_t = E_{t-1} + r_t k E_0 \tag{1}$$

$$Q_t = Q_{t-1} + r_t Q_0 + r_t(1-k)E_0 \tag{2}$$

for t = 1, 2, 3....*n*. Equations (1) and (2) can be written in matrix form as follows:

$$\begin{pmatrix} E_t \\ Q_t \end{pmatrix} = \begin{pmatrix} E_{t-1} \\ Q_{t-1} \end{pmatrix} + \begin{pmatrix} r_t k & 0 \\ r_t(1-k) & r_t \end{pmatrix} \begin{pmatrix} E_0 \\ Q_0 \end{pmatrix}$$

for t = 1, 2, 3...*n* that can be solved for $t=n$ as

$$E_n = E_0(1 + k\sum_{i=1}^{n} r_i)$$

$$Q_n = Q_0(1+\sum_{i=1}^{n} r_i) + E_0(1-k)\sum_{i=1}^{n} r_i$$

This model denies and contradicts the justification of internalizing *musyarakah* between the parties involved because of the existence of tenor in the contract. In fact, it is unfair and doing injustice to the capital provider. This model is not suitable for managing *musyarakah* products.

Musyarakah 2. The capital provider and entrepreneur invests E_0 and Q_0 respectively at the initial time $t = 0$ which means the total investment is $E_0 + Q_0$. Let us consider the profit gained at time *t* on the initial investments of E_0 and Q_0 is $r_t(E_0 + Q_0)$. At the end of contract, only the profit $r_t(E_0 + Q_0)$ is shared between the capital provider and the entrepreneur, with the profit sharing rates of $k:(1-k)$. The investment models of capital provider E_t and the entrepreneur Q_t at time *t* are as follows:

$$E_t = E_{t-1} + r_t k(E_0 + Q_0) \tag{3}$$

$$Q_t = Q_{t-1} + r_t(1-k)(E_0 + Q_0) \tag{4}$$

for t = 1, 2, 3....*n*. Equations (3) and (4) can be put in matrix form as follows:

$$\begin{pmatrix} E_t \\ Q_t \end{pmatrix} = \begin{pmatrix} E_{t-1} \\ Q_{t-1} \end{pmatrix} + \begin{pmatrix} r_t k & r_t k \\ r_t(1-k) & r_t(1-k) \end{pmatrix} \begin{pmatrix} E_0 \\ Q_0 \end{pmatrix}$$

for t = 1, 2, 3...*n* that can be solved for $t = n$ as

$$E_n = E_0(1+k\sum_{i=1}^{n} r_i) + Q_0 k\sum_{i=1}^{n} r_i$$

$$Q_t = E_0(1-k)\sum_{j=1}^{t} r_t + Q_0(1+(1-k)\sum_{i=1}^{t} r_i)$$

The weakness is still related to the existence of tenor in *musyarakah* contract that both parties can only take out their profit at the end of the tenor. Logically, it must use compound interest model. The next case discusses the management of joint venture that overcomes the weaknesses of the above two cases.

Musyarakah 3. It is assumed the profit rate at time *t*, is r_t and the tenor contract is *n*. At initial stage of the joint venture $t = 0$, capital provider and entrepreneur invest E_0 and Q_0 respectively. The profit gained from the investment of the capital provider is shared between the capital provider and the entrepreneur with a ratio of $k:(1-k)$. The profit gained from the investment of the entrepreneur is shared between the capital provider and the entrepreneur with a ratio of $(1-j):j$. If the profit rate at time *t* is r_t, then the investment models of capital provider E_t and the entrepreneur Q_t at time *t* are as follows:

$$E_t = E_{t-1} + r_t k E_{t-1} + r_t(1-j)Q_{t-1} \tag{5}$$

$$Q_t = Q_{t-1} + r_t(1-k)E_{t-1} + r_t jQ_{t-1} \tag{6}$$

for t = 1, 2, 3...n. Equations (5) and (6) can futher be simplified as

$$E_t = (1 + r_t k)E_{t-1} + r_t(1 - j)Q_{t-1} \tag{7}$$

$$Q_t = r_t(1 - k)E_{t-1} + (1 + r_t j)Q_{t-1}, \tag{8}$$

for t = 1, 2, 3...n. The scalar $(1 + r_t k)$ can be interpreted as the growth rate of the capital provider investment E_{t-1} while $r_t(1 - j)$ is the growth rate of the entepreneur investment Q_{t-1}. The scalar $r_t(1-k)$ can be interpreted as the growth rate of the capital provider investment E_{t-1} while $(1 + r_t j)$ is the growth rate of the entepreneur investment Q_{t-1}. Equations (7) and (8) can be put in matrix form as follows:

$$\begin{pmatrix} E_t \\ Q_t \end{pmatrix} = \begin{pmatrix} 1 + r_t k & r_t(1 - j) \\ r_t(1 - k) & 1 + r_t j \end{pmatrix} \begin{pmatrix} E_{t-1} \\ Q_{t-1} \end{pmatrix}$$

for t = 1, 2, 3...n. The equation shows the interaction takes place between the investment of both parties mathematically. The coefficient of

$$\begin{pmatrix} E_{t-1} \\ Q_{t-1} \end{pmatrix} \text{ is } \begin{pmatrix} 1 + r_t k & r_t(1 - j) \\ r_t(1 - k) & 1 + r_t j \end{pmatrix}$$

which contains non zero value. Besides that, with the existence of another profit sharing rate j, it shares the profit accordingly to ensure fairness and justice for all parties involved. Thus the model of this joint venture governs the principle of *musyarakah* which internalizes its true spirit of an Islamic investment.

CONCLUSION

Any profit and loss sharing contract needs to use compound interest model in the calculation of profit or investment due to the existence of tenor in the contract. Hence any practice of simple interest model will violate the true management of the joint venture. The new *musyarakah* model takes into account two profit sharing rates in order to perform a fair and justified investment of joint venture between the capital provider and the entrepreneur. It certainly encourages the opportunity for the entrepreneur to invest and provides initial capital which indirectly secures the inefficiency and mismanagement on his part as his own capital will be at stake. With the existence of two profit sharing rates, the risks are also shared accordingly.

ACKNOWLEDGMENT

This research is funded by the Fundamental Research Grant Scheme (FRGS), Ministry of Higher Education Malaysia, that is managed by the Research Management Institute, Universiti Teknologi MARA (600-RMI/FRGS 5/31/Fst (26/2008).

REFERENCES

[1] M.Arif, “Islamic banking in Malaysia: Framework, performance, and lessons”. Journal of Islamic Economics, vol. 2, issue 2, pp.67-78, 1989.

[2] S.A.S. Al-Harran, Islamic Finance. Partnership Financing. Subang Jaya, Malaysia: Pelanduk Pub. (M) Sdn. Bhd, 1996.

[3] Sudin Harun and B. Shamugam, Islamic banking system. Concept & applications, Subang Jaya: Pelanduk Pub. (M) Sdn. Bhd, 2001.

[4] Maheran Mohd Jaffar. Model matematik pelaburan musyarakah dan musyarakah mutanaqisah. Doctoral dissertation, Pusat Pengajian Sains Matematik, Fakulti Sains dan Teknologi, Universiti Kebangsaan Malaysia, 2006.

[5] Bank Negara Malaysia, retrived online 29 January 2010, http://www.bnm.gov.my.

A SERIES OF DERIVATIVE OPERATORS FOR ANALYTIC FUNCTIONS

Maslina Darus
School of Mathematical Sciences
Faculty of Science and Technology
Universiti Kebangsaan Malaysia
Email: maslina@ukm.my

INTRODUCTION

Derivative operators have been the trend of research in the area of geometric function theory. In this particular work, we will present a series of derivative operators studied by various authors. Perhaps the first one was initiated by Ruscheweyh in 1975. He stated that

Definition 1 (Ruscheweyh 1975). For $f \in A$, the operator $D^{\delta} f : A \to A$, defined by

$$D^{\delta} f(z) = \frac{z}{(1-z)^{\delta+1}} * f(z) \qquad (\delta > -1\ z \in U),$$

where is the convolution and

$$D^{m} f(z) = \frac{z(z^{m-1} f(z))^{(m)}}{m!} \qquad (m \in N_0 = N \cup \{0\}; z \in U).$$

If $f \in A$, then

$$D^{m} f(z) = z + \sum_{n=2}^{\infty} C(m,n) a_n z^n,$$

where

$$C(m,n) = n+m-1 \quad = \frac{\prod_{j=1}^{n-1}(j+m)}{(n-1)!}, \qquad (m \in N_0, n \geq 2).$$
$$m$$

Remark For $z \in U$, we can write

$$D^0 f(z) = f(z)$$
$$D^1 f(z) = zf'(z)$$
$$2D^2 f(z) = z.(D^1 f(z))' + D^1 f(z)$$
$$\vdots$$
$$z(D^m f(z))' = (m+1)D^{m+1} f(z) - mD^m f(z).$$

Since 1975, many mathematicians studied the properties analytically and geometrically (see Ahuja (1985), Owa *et.al* (1986) dan Ahuja & Silverman (1989)).

Next, we give another operator stated as follows:

Definition 2 (Salagean 1983). For $f \in \mathcal{A}$ and $j \in N$, the operator $D^j f : A \to A$, defined by

$$D^j f(z) = f(z) * (z + \sum_{n=2}^{\infty} n^j a_n)$$

and

$$D^0 f(z) = f(z), D^1 f(z) = Df(z) = zf'(z)$$
$$D^j f(z) = D(D^{j-1} f(z)).$$

If $f \in A$, then

$$D^j f(z) = z + \sum_{n=2}^{\infty} n^j a_n z^n, (j \in N_0).$$

In 2006, Darus and Al-Shaqsi gives the following:

Definition 3 For $f \in A$, the generalized ruscheweyh derivative operator $D_\lambda^\delta : A \to A$ defined by

$$D_\lambda^\delta f(z) = \frac{z}{(1-z)^{\delta+1}} * D_\lambda f(z) \qquad (\delta > -1, z \in U),$$

where is the convolution and $D_\lambda f(z) = (1-\lambda) f(z) + \lambda z f'(z)$. Thus we can write

$$D_\lambda^m f(z) = \frac{z(z^{m-1} D_\lambda f(z))^{(m)}}{m!} \qquad (m \in N_0; z \in U; \lambda \geq 0).$$

If $f \in A$, then

$$D_\lambda^m f(z) = z + \sum_{n=2}^{\infty} [1 + \lambda(n-1)] C(m,n) a_n z^n,$$

where $m \in N_0, \lambda \geq 0, z \in U$ and $C(m,n)$ given in Definition 1.

Remark For $z \in U$, we have

$$D^0 f(z) = D_\lambda f(z)$$
$$D^1 f(z) = z(D_\lambda f(z))'$$
$$2D_\lambda^2 f(z) = z.\,(D_\lambda^1 f(z))' + D_\lambda^1 f(z)$$
$$\vdots$$
$$z(D_\lambda^m f(z))' = (m+1)D_\lambda^{m+1} f(z) - mD_\lambda^m f(z).$$

For $\lambda = 0$, the operator reduces to Definition 1.

RESULTS

We will study the work given by Al-Oboudi (2004), who gave the generalized derivative operator of Salagean. New operator regarding the generalized Salagean derivative operator will be introduced and many new properties will be presented here. Those properties include the coefficient ectimates and the growth and distortion theorems.

CONCLUSIONS

Many new results can be obtained from every single operators we introduced. But certain cautions should be taken care since not all operators are defined to be univalent. In other words, not all of them can be described as starlike or convex to guarantee the univalency of new operators.

Acknowledgement: The work here is fully supported by MOHE: UKM-ST-06-FRGS0244-2010. The author also would like to thank the Malaysian Mathematical Sciences Society for organizing such seminar and meetings among the scientists.

REFERENCES

Ahuja, O. 1985. Integral operators of certain univalent functions. *Int. J. Math. Math. Sci*. 8: 653–662.

Ahuja, O. & Silverman, H. 1989. Function classes related to Ruscheweyh derivatives. *J. Aust. Math. Soc. Ser. A* 47(3): 438–444.

Al-Oboudi, F. M. 2004. On univalent functions defined by a generalized Salagean operator. Int. J. Math. Math. Sci. 2004 (27): 25–28.

Darus, M. & Al-Shagsi, K. 2006. On Harmonic univalent functions defined by a generalized Ruscheweyh derivatives operator. Lobachevskii J. Math.22: 19–26.

Owa, S., Fukui, S., Sakaguchi, K. & Ogawa, S. 1986. An application of the Ruscheweyh derivatives. Int. J. Math. Math. Sci. 9: 721–-30.

Ruscheweyh, S. 1975. New criteria for univalent functions. Proc. Amer. Math. Soc. 49: 109–115.

Salagean, G. S. 1983. Subclass of univalent function. Lecture Notes in Mathematics 1013,Springer- Verlag, Berlin pp. 362–372.

ISLAMIC EPISTEMOLOGY OF EDUCATION:

Integration of Intellect and Values in Mathematical Science

Mat Rofa Ismail
Department of Mathematics, Faculty of Science, Universiti Putra Malaysia
Email: rofa@math.upm.edu.my

ABSTRACT

The central idea underlying Islamic epistemology of education is the integration between the intellectual science and the transmitted revealed science. The former is dealing with the capacity of human's intellect to comprehend the very nature of his ability to think about objects, problems, arguments, methodology and conclusion, whilst the later serves as the paradigm based on the fundamental tenet of Islamic faith. The integration of both elements forms a unique Islamic epistemological interpretation of nature and values. Rather unfortunate, this very foundation of value-laden approach is seen to be an antithesis to positivism or mechanism definition of science which is considered to be value-free activities. The discussion of mathematics in this paper is viewed from the National philosophy of Education and its relevance to the epistemological approach described above. As an illustration we would like to discuss a model of the integration of both elements in our following slides based on a selected topic in mathematics.

DISCUSSION

As a model for discussion, the Fibonacci series is considered here to show the integration of intellect and values in mathematical thinking. The unique ratio, which is to be defined by the limit of the ratio of two consecutive terms of the series as n tends to infinity, known as the golden ratio. It is highlighted in this discussion to represent the harmonious balance of the universe. The ratio that to be found in the natural world shows the existence of a unique law that governs the universe as a whole. The universe is being created in the best proportional values. This is the symbol of harmonious universe described by certain equations of dynamic phenomena. Philosophy relates mathematical and natural sciences to the real goal of the epistemological virtue, that is metaphysical ultimate. In this case, the golden ratio reflects the epistemological interpretation of the unique ultimate of the Absolute One, the Creator of the universe.

The rotating nature of galaxies in the macro world is to be compared to the micro world of sub-atom structure which revolves in the same way, the same orientation and proportional order. The golden ratio lies between the two worlds, connecting them together especially when the curvature oscillating expansion of the objects is considered. This is not merely part of so-called law of nature but indeed, it is to my humble interpretation, it is part of Law of God. It shows the existence of ONE AND ONLY God, the Creator of the universe, great and small creatures, macro and micro worlds. The golden ratio witnesses the case.

God, The One and Only, The Absolute, Who creates the galaxies, atomic structures or even beautiful flowers in golden proportional ratio, is also The One who commands Prophet Abraham as to build Kaabah in Makkah, Prophet David as to build al-Aqsa Mosque in Jerusalem and Prophet Noah as to build his ark using the same number of reference. The ratio can be interpreted as the "finger print" of divine origin.That is my humble interpretation. What's yours?

TCP WITH ADAPTIVE DELAYED ACK STRATEGY IN MULTI-HOP WIRELESS NETWORKS

Mohamed Othman and Ammar Mohammed AI-Jubari
Department of Communication Technology and Network
University Putra Malaysia, 43400 UPM Serdang, Selangor D.E., Malaysia
mothman@fsktm.upm.edu.my; ammar_aljobari@yahoo.com;

INTRODUCTION

In recent years, emergence of new technologies such as IEEE 802.11, Bluetooth is making possible the deploying of multi-hops wireless networks for research and development, even for commercialization purposes. As the technologies become more and more advances, supporting transport control protocol (TCP) seems a challenges issue (Postel, J., 1981, Allman, M., et al., 1999). The TCP provides reliable end to end connection over unreliable network and it is the most widely used in Internet applications especially in wired network. In practice, TCP suffers a significant degradation in it performance over the ad hoc wireless network. Several improvements have been proposed to make TCP more reliable in the wireless network. In this research, we proposed an adaptive delayed ACK strategy for dynamically adjusting TCP receiver delay window.

PROPOSED ADAPTIVE DELAY ACK STRATEGY

The main goal of this research is to have a better throughput by lowering the number of ACK, which guarantees the TCP reliability. Till now, no strategy achieves the required TCP performance particularly in wireless network, as compared in wired network. The proposed strategy is to adjust the delay window dynamically based on several conditions such as transmission window, number of hops, and packet lost event. To reduce the number of ACK packets appropriately, the proposed strategy makes the ACK generated by reaching the delay window. In the strategy, unless the sender's retransmission timer expires, the receiver always increases the delay window based on the increase in the congestion window (*cwnd*) size. Out-of-order packets cause immediately ACK generation to inform the sender of the packet loss/recovery in a timely manner, as introduced in the recommendation of RFC 2581 (Allman, M., et al., 1999). The proposed strategy is presented in the form of algorithm as shown in algorithm I below.

Algorithm 1 TCP-ADW PRINCIPAL OPERATION

```
ackcount ← 0, dwin ← 2, δ ← 1/h
if consecutive DATA segment received then
    if ackcount > 0 then
        record inter-arrival time
        Currentcwnd ← cwnd
    end if
    ackcount ← ackcount + 1
    if ackcount = dwin then
        produce ACK
        ackcount ← 0
    end if
    if dwin < Maxdwin then
        if Current_cwnd ≥ previous_cwnd then
            dwin ← δ ∗ cwnd
        else
            dwin ← 2
        end if
    else
        dwin ← Maxdwin
    end if
else
    produce ACK
    ackcount ← 0
    dwin ← 2
end if
```

PERFORMANCE EVALUATION

The main focus is to keep the TCP throughput at acceptable level by adjusting the delayed ACK window to the optimal size. Several simulations are carried on the chain topology with seven hops wireless networks. NS2 is used in this simulation. To show the performance of the proposed strategy known as TCP-ADW, and comparing it against three strategies such as TCP-DCA, TCP with standard delayed ACK (TCP-SDA), and the regular TCP without delayed ACK (Braden, R., 1989, Chen, et al., 2008). Figures 1, 2, 3, and 4 show the respective throughput results with difference number of hops, while Table 1 shows the simulation results obtained, the number of ACKs versus data packet sent in long path (thirteen hops).

Throughput results with single TCP flow

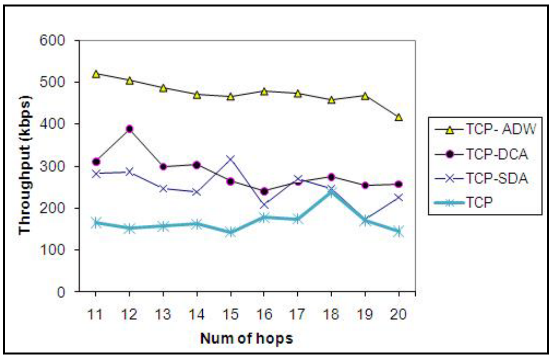


Figure 1. TCP throughput over chain topology with hop length 11 to 20

The performance gain of TCP-ADW over TCP-DCA is in the range of 30% ~ 99%, with an average of 66%.

The performance gain of TCP-ADW over TCP-SDA is from 48% up to 170%, with an average of 101%.

93% over TCP, up to 233%, with an average of 183%.

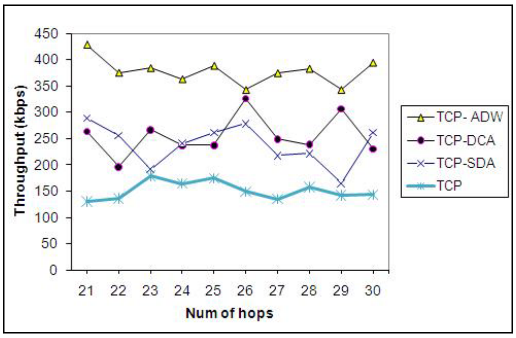


Figure 2. TCP throughput over chain topology with hop length 21 to 30

The performance gain of TCP-ADW over TCP-DCA is in the range of 5% ~ 91%, with an average of 51%

TCP-ADW over TCP-SDA is 23%, up to 108%, with an average of 62%.

121% over TCP, up to 228%, with an average of 152%.

Throughput results with single TCP flow

Ratio of ACKs vs. data packets

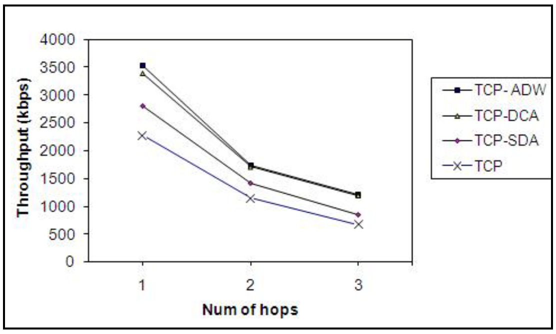


Figure 3. TCP throughput over chain topology with hop length 1 to 3

3% over TCP-DCA

30% over TCP-DA

62% over TCP

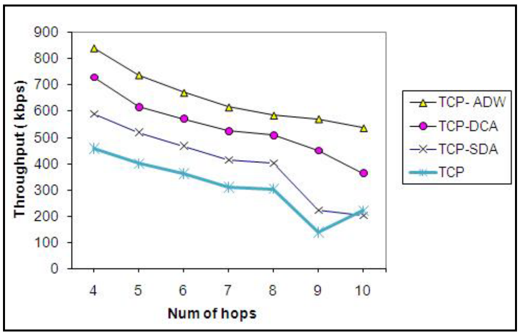


Figure 4. TCP throughput over chain topology with hop length 4 to 10

The performance gain of TCP-ADW over TCP-DCA is in the range of 15% ~ 27%, with an average of 19%.

The performance gain of TCP-ADW over TCP-SDA is in the range of 42% ~ 156% with an average of 63%.

For the regular TCP, the improvement is 84% ~ 209% with an average of 127%.

Table 1 shows a typical example of number of ACKs and data packets sent in case of 13 hops. Since TCP-ADW generates as few ACKs as possible, the advantage of TCP-ADW is obvious.

TABLE 1: NUMBER OF ACKS VS. DATA PACKETS SENT IN LONG PATH (13 HOPS)

TCP Strategy	Throughput	Num of Packets	Num of ACKs	Ratio
TCP-ADW	487.207	8619	944	0.109
TCP-DCA	298.783	5286	1861	0.352
TCP-SDA	246.602	4363	2261	0.518
TCP	156.883	2776	2713	0.977

CONCLUSION

An adaptive delayed ACK strategy is proposed to improve TCP performance over multi-hop wireless networks. The key idea of our dynamic adaptive acknowledgment strategy is to provide capability to a TCP receiver to adjust itself in terms of the data to ACK ratio. The outcome of the simulation evaluations showed that our strategy outperforms over the three TCPs which have been proposed by previous researchers. The adaptive strategy improves throughput, which is the key issues. The simulation results show that proposed strategy improve TCP throughput approximately 233% compared to the regular TCP.

ACKNOWLEDGEMENTS

This research was supported by the Research University Grant Scheme (RUGS), Universiti Putra Malaysia (RUGS Number: 05/01/07/0180RU). The initial research paper was presented in ITSim 2010 and at the time authors also would like to thank the Malaysian Mathematical Sciences Society for organizing such seminar and meetings among the scientists.

REFERENCES

Braden, R., 1989, Requirement for internet hosts communication layers, RFC 1122, IETF Network Working Group.

Chen, J., Gerla, M., Lee, Y.Z., and Sanadidi, M.Y., 2008, TCP with delayed ACK for wireless networks, Ad Hoc Networks, 6(7): 1098-1116.

Allman, M., Paxson, V., and Stevens, W., 1999, Transmission Control Protocol, RFC 2581, IETF Network Working Group.

Postel, J., 1981, Transmission Control Protocol," RFC 793, IETF Network Working Group.

NS2 network simulator , available from: http:// www.isi.edu/nsnamlns/.

Bluetooth Special Interest Group, available from http://www.bluetooth.com.

IEEE 802.11 WLAN standard, available from http://standards.ieee.org/ getieee802.

IMPLEMENTATION OF A PARALLEL MODIFIED EXPLICIT GROUP ACCELERATED OVER-RELAXATION ALGORITHM ON DISTRIBUTED MEMORY ARCHITECTURE

Mohamed Othman and Shukhrat Rakhimov
Department of Communication Technology and Network
University Putra Malaysia, 43400 UPM Serdang, Selangor D.E., Malaysia
mothman@fsktm.upm.edu.my; sh.rakhimov@gmail.com

INTRODUCTION

The modified explicit group (MEG) method and their parallel implementation on shared memory multi processors for solving two dimensional (2D) Poisson equation were developed and discussed in Othman 2000 and Othman 2004, respectively. Both sequential and parallel implementations were shown to be the most superior as compared to the EDG and EG methods, see Evans, et al., 1982, Evans et al., 1990, Abdullah 1991, Yousif et al., 1995. Recently, the MEG Accelerated Over-Relaxation (AOR) iterative method was introduced by Othman et al., 2009 and the results were shown that the method was superior as compared to the EG- and EDG- AOR methods. Again, the methods were found to be suitable for parallel implementation, see Evans et al., 1990, Yousif et al., 1995, Martins, et al., 2002, Othman et al., 2004, Ali et al., 2007, Foo et al., 2010.

PROPOSED PARALLEL AOR MEG ITERATIVE METHOD

The implementation of the parallel MEG AOR iterative algorithm on distributed memory architecture, specifically on the clustered of Sun Fire V240 and V1280 is discussed. In the implementation, two main processes are illustrated; they are the domain decomposition and interprocess communication techniques. In domain decomposition technique, the initial domain is decomposed into several subdomains according to certain strategy. For instance, initial domain Ω is decomposed into two subdomains Ω_1 and Ω_2 and Figure 1 shows both subdomains for domain size n=18, and number of processors p=2.

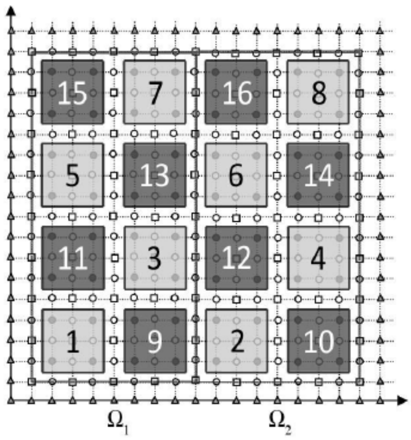


Figure 1. Domain decomposition for parallel MEG AOR method with n=18, p=2.

In the second process, the interprocess communication and it described of how the data at the boundary or boundaries are used by another process at the adjacent subdomain and vice versa. In the case of chessboard strategy, two stages per iteration were considered. In each stage, the computation of each data near to the boundary of each subdomin requires the values at the adjacent sudomain, which have been updated during the previous stage. These processes of iterative will continue until the local and global convergence criterion is achieved. With reference to both techniques, a parallel MEG AOR iterative algorithm is illustrated as in algorithm I below

Algorithm 1 Parallel MEG AOR Iterative Algorithm.

1: Discretize the domain into four type points
2: Combine the iterative points • into blocks of four and arrange in red black strategy
3: Decompose initial domain Ω_1 into p subdomains, in Fig 1, 2-subdomain, Ω_1 and Ω_2
4: Initialization
5: **repeat**
6:   Initialization the right Ω_1 and left blocks Ω_2
7:   **for** all red blocks of points **do**
8:     Apply the MEG AOR equation
9:     **if** current block is on the left boundary of the subdomain **then**
10:       copy current block to the right block
11:     **end if**
12:     **if** current block is on the right boundary of the subdomain **then**
13:       copy current block to the left block
14:     **end if**
15:   **end for**
16:   Perform the interprocess communication
17:   **for** all black blocks **do**
18:     repeat steps 6–16.
19:   **end for**
20:   **if** local convergence is not achieved **then**
21:     Set the flag to false and perform step 6
22:   **end if**
23: **until** global convergence is achieved
24: Evaluate all points of type □ using

$$v_{i,j} \leftarrow 0.25\left(v_{i-1,j-1} + v_{i+1,j-1} + v_{i+1,j+1} + v_{i-1,j+1} - 2h^2 \cdot f_{i,j}\right)$$

25: Evaluate all points of type ○ using

$$v_{i,j} \leftarrow 0.25\left(v_{i-1,j} + v_{i,j-1} + v_{i+1,j} + v_{i,j+1} - 4h^2 \cdot f_{i,j}\right)$$

PERFORMANCE EVALUATION

In order to evaluate the performance of all the parallel EG-, EDG- and MEG- AOR algorithms, several experiments were carried out for solving the 2D Poisson equation which subject to the Dirichlet boundary conditions and satisfying the exact solution $u(x,y)=e^{xy}$, $(x,y) \in \delta\Omega$. Throughout the experiments, a tolerance $\varepsilon=10^{-10}$ and boolean logical flag were used as the local and global convergence test criterions. The experimental values of ω were obtained within ±0.01 by running the program for different values of ω and choosing the one(s) that gave the minimum number of iterations.

From the results obtained, the parallel MEG- AOR algorithm is the most superior among the three parallel algorithms as the number of processors increased. While the speedup and efficiency of the parallel MEG- AOR algorithm with difference size are shown in Figures 3 and 4, respectively.

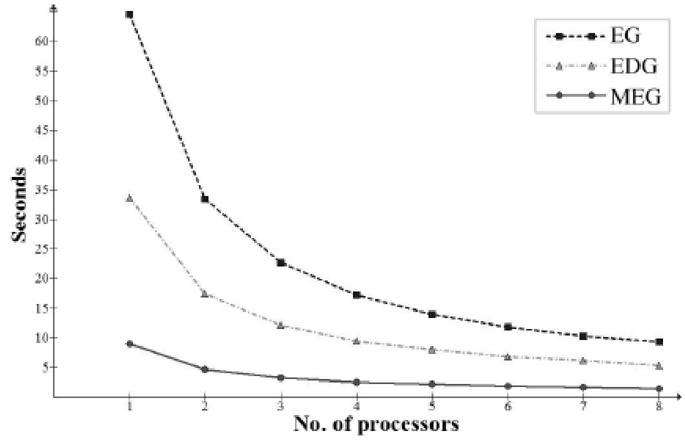


Figure 2. Execution time (seconds) vs. number of processors for parallel EG-, EDG- and MEG- AOR methods (size n = 450).

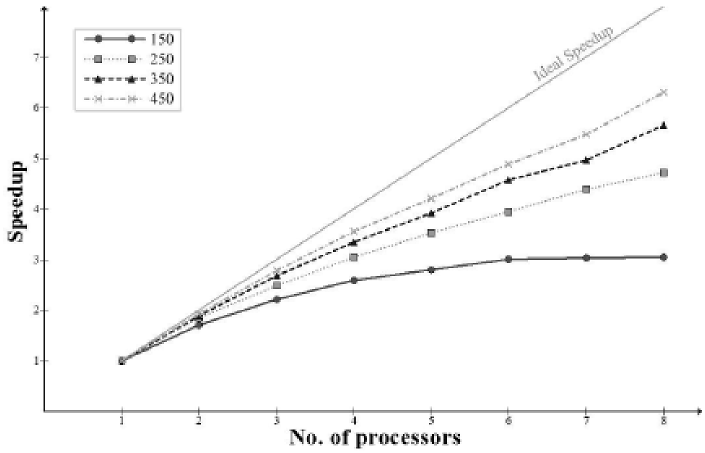


Figure 3. Speedup vs. number of processors for the parallel MEG- AOR methods with n=150, 250, 350, 450.

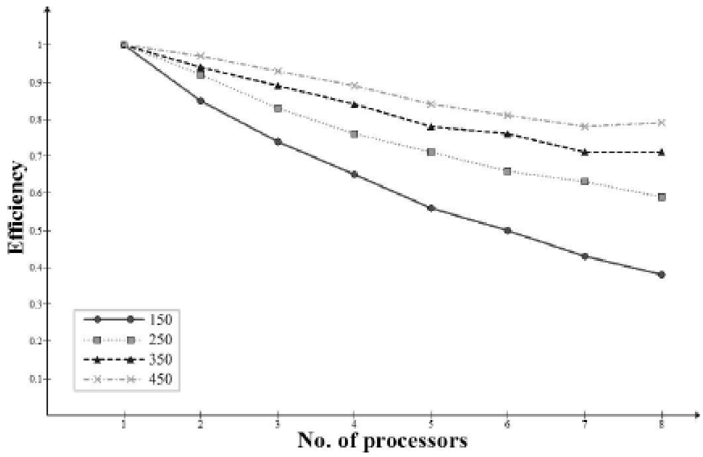


Figure 4. Efficiency vs. number of processors for the parallel MEG- AOR method with n=150, 250, 350, 450.

CONCLUSION

It can be concluded that the proposed parallel MEG AOR method is implemented successfully on distributed parallel machine architecture. The proposed method has shown their superior performance as compared with the previous parallel EG AOR and EDG AOR methods. The same happen for speedup and efficiency, especially for huge size of grid. As for huge size of grid, the communication latency is not influential the performance results.

ACKNOWLEDGEMENTS

The research was supported by Fundamental Research Grant Scheme No. 02-10-07-321FR under the Ministry of High Education of Malaysia.

REFERENCES

Evans, D.J., Biggins, M.J., 1982, The Solution of Elliptic Partial Differential Equations by A New Block Over-Relaxation Technique. Intern. Journal of Computers Mathematics, 10: 269-282

Evans, D.J., Yousif, W.S., 1990, The Implementation of the Explicit Block Iterative Methods on the Balance 8000 Parallel Computer, Parallel Computing, 16: 81-97

Abdullah, A. R., 1991, The Four Explicit Decoupled Group (EDG) Method: A Fast Poisson Solver. Intern. Journal of Computers Mathematics, 38: 61-70

Yousif, W.S., and Evans, D.J., 1995, Explicit De-coupled Group Iterative Methods and Their Parallel Implementations. International Journal of Parallel, Emergent and Distributed Systems, 7: 53-71

Martins, M.M., Yousif, W.S., and Evans, D.J.:, 2002, Explicit group AOR method for solving elliptic partial differential equations, Neural, Parallel and Science Computation, 10: 411-422

Othman, M., and Abdullah, A. R., 2000, An Efficient Four Points Modified Explicit Group Poisson Solver. Intern. Journal of Computers Mathematics, 76: 203-217

Othman, M., Abdullah, A. R. and Evans, D.J., 2004, A Parallel four points modified explicit group algorithm on shared memory multiprocessors. International Journal of Parallel, Emergent and Distributed Systems, 19: 1-9

Ali, N.H.M., and Chong, L.S., 2007, Group Accelerated OverRelaxation Methods on Rotated Grid. Applied Mathematics and Computation, 191(2): 533-542

Othman, M., Rakhimov, Sh., Jumat, S., and Suleiman, M., 2009, A Red Black Accelerated Over Relaxation Four Points Modified Explicit Group Iterative Solver. The 2nd International Conference on Mathematical Modeling and Computation and The 5th East Asia SIAM Conference, Brunei Darussalam, June 8-11

Foo K.P., and Ali, N.H.M., 2010, Parallel Group Explicit Accelerated OverRelaxation Methods on Distributed Memory Multicomputer. Journal of Communication and Computer, 7(9): 45-54

SOME RESULTS ON CLOSED-ORBIT COUNTING

Mohd Salmi Md Noorani
School of Mathematical Sciences,
Faculty of Science and Technology
Universiti Kebangsaan Malaysia

In this talk we give a survey of current results on the problem of closed orbit counting for shift systems. We progress from the one-dimensional case until the current findings for Z^n-actions.

AMS subject Classification 2000: Primary:22D40; Secondary: 37A15
Keyword and phrases: Shift systems, closed orbits, counting, group actions

INTRODUCTION

One of the classical problems in the field of dynamical systems is closed orbit counting. In the simplest situation, a closed (or also known as periodic) orbit of a dynamical system f (acting on some space X) is a set $\tau = \{x, f(x), \dots, f^{n-1}(x)\} \subset X$ where $f^n(x) = x$ for some $x \in X$ and $n \ge 1$. When $f^k(x) \ne x$ for $1 \le k < n$ then we say τ has (least) period $|\tau|$. It is then natural to ask questions such as: How many closed orbits are there for a given period?; How does the cardinality of such orbits grow with respect to the periods?; How are these orbits distributed in the space X ?, etc. In other words closed orbit counting are then problems related to the counting of such sets as the period increases.

In this talk we give a survey of current results on the problem of closed orbit counting for a particular class of dynamical systems called the shift system. We shall start the survey with the situation of Z-actions and provide counting results for the well-known situation in one-dimension. Next, we provide results for the notoriously difficult situation of Z^n-actions. In particular we discuss the very recent work of Miles and Ward on orbit counting problems for such systems. The situation for this more general situation is still very much open for investigations.

Main Results

The one-dimensional shift system is defined as follows:

Let A be a $0-1$ irreducible $n \times n$ matrix and let $\{1, \dots, n\}$ be given the discrete topology. Define

$$X_A = \left\{ x \in \prod_{i=-\infty}^{\infty} \{1, \dots, n\} : A(x_i, x_{i+1}) = 1 \textit{ forall } n \in Z \right\}$$

so that X_A is a compact zero dimensional space. Also define the map $\sigma : X_A \to X_A$ by $(\sigma x)_i = x_{i+1}$. Then the pair (X_A, σ), or simply σ, is called a *shift of finite type*. When the entries of A are all 1's then X_A is known as the (one-dimensional) full shift on n-symbols.

The higher-dimensional full shift S^{Z^n} -- called the full Z^n-shift -- is defined as follows:

Let S be a finite (alphabet) set. Let $Y \subseteq Z^n$ and call elements of Y as locations. Then following terminology from statistical mechanics we shall call maps $x: Y \subseteq Z^n \to S$ as configurations and read $x_n = x(n)$ as the value of x at location n where $n \in Y$. For each $m \in Z^n$, the corresponding Z^n-action is denoted by σ_m and is defined as

$$\sigma_m : S^{Z^n} \to S^{Z^n}$$

such that

$$(\sigma_m x)_n = x_{m+n}.$$

A potpourri of results to be discussed in this survey will include the following theorems:

Theorem 1[Parry & Pollicott [4], Noorani[3]] For Z-action such mixing shifts of finite type we have

$$\#\{\tau : |\tau| = x\} \sim \frac{e^{hx}}{x}$$

where τ denotes a typical closed orbit of the action, $|\tau|$ its corresponding period and h is the topological entropy.

For higher-dimensional shift systems, we have:

Theorem 2 [Alrefaei *et al* [1]] Let H and V be two $k \times k$ $0-1$-matrices. Define a subsystem of the full $\mathbb{Z}^2$-shift X given by

$$Y = \{x = (x_n) \in X : H(x_n, x_{n+e_1}) = V(x_n, x_{n+e_2}) = 1, \forall n \in \mathbb{Z}^2\}.$$

If we further assume that $HV = VH$ and HV also a $0-1$-matrix then the number of fixed points of $\beta^{(n_1, n_2)}$ is equal to $Trace\ H^{n_1} V^{n_2}$.

Theorem 3 [Miles & Ward [2]] For the full Z^n-shift with $n \geq 2$ and $|S| = b$ alphabets, we have the estimates

$$C_1 N^{n-2} \leq \frac{\#\{\tau : |\tau| \leq N\}}{b^N} \leq C_2 N^{n-2} (\log N)^{n-1}.$$

Acknowledgement

This work was financially supported by the Malaysian Ministry of Higher Education FRGS Grant UKM-ST-06-FRGS0146-2010.

REFERENCES

[1] A. Alrefaei, S. Dzulkipli, M. S. M. Noorani, Periodic Points and Zeta functions for Two-dimensional Shifts of Finite Type, *Far East Journal of Dynamical Systems*, **9**, 255 - 263, 2008.

[2] R. Miles & T. Ward, A dichotomy in orbit growth for commuting automorphisms, *J. London Math. Soc.*, **81**, 715--726, 2010.

[3] M. S. M. Noorani, Counting Closed Orbits of Hyperbolic Diffeomorphisms, *Results in Mathematics*, **50**, 241--257, 2007.

[4] W. Parry and M. Pollicott, Zeta functions and the periodic orbit structure of hyperbolic dynamics, *Astérisque*, **187--188**. 1990.

GENERALIZED LOG-RANK TEST FOR PARTLY INTERVAL-CENSORED FAILURE TIME DATA VIA MULTIPLE IMPUTATION

Noor Akma Ibrahim and Azzah Mohammad Alharpy
Laboratory of Computational Statistics and Operations Research and
Department of Mathematics, Faculty of Science
Universiti Putra Malaysia
Email: nakma@putra.upm.edu.my

INTRODUCTION

Survival comparison is one of the main objectives in most of survival studies. In this work we will discuss the comparison test for partly interval-censored failure time. By partly interval-censored data, we mean that the exact failure times are observed for some subjects but for the remaining subjects, the failure times are observed only to belong to an interval. Partly interval-censored data often occur in medical and health studies that entail periodic follow-ups.

Many test procedures have been proposed to solve the comparison problem when data are right-censored (e.g. Fleming and Harrington, 1991; Kalbfleisch and Prentice, 2002). Finkelstein (1986) derived the log-rank test as a score statistic of a proportional hazard model. Petroni and Wolve (1994) proposed a class of nonparametric two-sample test for discrete interval-censored data. These are just to name a few of many well-established researches on right-censored and interval-censored.

For the case of partly interval-censored data, the research is rather limited. Peto and Peto (1972) in his discussion of partly interval-censored data, adopted a method treating an exact observation as an interval-censored observation with a very short interval. Turnbull (1976) described a general scheme of incomplete failure time data and derived self-consistency equations for computing the maximum likelihood estimator of the survival functions.

In this paper, we develop nonparametric test procedures for partly interval-censored data using the idea based on the generalized log-rank tests for partly interval-censored failure time data (Zhao et al., 2008) and a generalized log-rank test for interval-censored failure time data via multiple imputation (Huang et al., 2008).

Consider a survival study that involves independent subjects from p different treatments. Let $T_i > 0$ be a random variable to denote the failure time of interest for the i^{th} subjects, $i = 1,...,n$. Let n_l be the number of subject from l treatment, with distribution function $F_l(t)$ and survival function $S_l(t) = 1 - F_l(t)$, where $l = 1,..., p$; $n = n_1 + ... + n_p$. The goal is to determine whether the p treatments could have arisen from an identical failure time distribution that is $H_0 : S_1(t) = S_2(t) = ... = S_p(t)$. In the case of exact data, suppose that the $\{t_k\}_{k=1}^{r}$ is the unique order of T_l, and let d_{kl} be the number of failures at time t_k for l treatment, with total failures

$d_k = \sum_{l=1}^{p} d_{kl}$, n_{kl} is the number of subject at risk just before t_k for l for treatment, with total at risk $n_k = \sum_{l=1}^{p} n_{kl}$. The log-rank statistic U^0 is defined by $U^0 = \left(U_1^0, ..., U_p^0\right)^T$ where

$$U_l^0 = \sum_{k=1}^{r}\left(d_{kl} - \frac{n_{kl}d_{kl}}{n_k}\right) \quad l = 1, ..., p$$

The covariance matrix of U^0 is defined by $V^0 = V_1^0 + ... + V_r^0$, where V_k^0 is a p x p matrix of rank $p-1$ with entries

$$\left(V_k^0\right)_{l_1 l_2} = \begin{matrix} n_{kl_1}\left(n_k - n_{kl_1}\right) d_k \left(n_k - d_k\right) n_k^{-2} \left(n_k - 1\right)^{-1} \ if \ l_1 = l_2 = 1, ..., p \\ -n_{kl_1} n_{kl_2} d_k \left(n_k - d_k\right) n_k^{-2} \left(n_k - 1\right)^{-1} \quad if \quad l_1 \neq l_2 = 1, ..., p \end{matrix}$$

The statistic

$$\left(U^0\right)^T \left(V^0\right)^{-1} \left(U^0\right)$$

is expected to have an approximate χ^2_{p-1} distribution under the null hypothesis .

For the case of partial interval-censored data, suppose that we observe the exact failure time for N_1 subjects by $0_1 = \left\{\{t_i\}_{i=1}^{N_1}; N_1 = \sum_{l=1}^{p} n_{l1}\right\}$ and the interval-censored failure time for the remaining subject is given by $0_2 = \left\{\left(L_i, R_i\right], i = N_1 + 1, ..., n; N_2 = n - N_1\right\}$ where L_i and R_i are (+ve) independent of T_i such that $L_i < R_i$ with probability one. Let $t_1 < t_2 < ... < t_M$ be the time point at which the probability function may have a mass and $q_k = P\left(T = t_k\right)$ denote the common probability function of the p treatment under the null hypothesis. The collection $\{t_k\}_{k=1}^{M}$ is the unique order element of $\{0, t_i, L_i, R_i, i = 1, ..., n\}$. Define $\beta_{ik} = I\left(t_k = t_i\right)$ where $i = 1, ..., N_1$ and $\alpha_{ik} = I\left(t_k \in \left(L_i, R_i\right]\right)$ where $i = N_1 + 1, ..., n$. Under the condition that the mechanism generating interval-censored is independent of $T_i^{'}s$, the likelihood function for partly interval-censored data is proportional to

$$L(q) = \prod_{i=1}^{N_1} dF\left(T_i\right) \prod_{i=N_1+1} \left[F\left(R_i\right) - F\left(L_i\right)\right]$$

where $dF\left(T_i\right) = F\left(T_i\right) - F\left(T_i^-\right)$.

Based on the previous assumption, we have modified the Turnbull estimator (1976) in terms of partly interval-censored data and estimation can be done by using EM algorithm.

Our imputation procedure is to impute an exact failure time data from partly interval-censored observations. The multiple imputation scheme that we have used follows Rubin's multiple imputation. The test statistic for comparing p treatments for partly interval-censored data is $\bar{U}^T\left(\hat{V}\right)^{-1}\bar{U}$, where $\bar{U}$ is the sum of the average within imputation covariance associated with U and the between imputation variance of U. $\hat{V}=\frac{\sum_{h=1}^{H}V^h}{H}+\left(1+\frac{1}{H}\right)\frac{\sum_{h=1}^{H}\left[U^h-\bar{U}\right]\left[U^h-\bar{U}\right]^T}{H-1}$, H is a positive integer. We conducted a simulation study to investigate the performance of the proposed test in which we considered a two sample comparison problem.

RESULTS

The results are based on 10000 replications and 10 multiple imputations. Various values of probability, q, and initial values of β were used with three levels of sample size. The power of the test decreases as q increases. However the power increases as the sample size increases and the power also increases as the absolute vaues of β increases. When there exist more exact data, the power of the test increases.

CONCLUSIONS

From the simulation results the size of the test obtained is reasonable at both significance levels of 5% and 1%. The generalized log-rank test proposed for the partly interval-censored failure time data via multiple imputation works well under the situations considered.

Acknowledgement: The work here is supported by MOHE: FRGS/1/10/ST/UPM/02/23:01-04-10-892FR. The author also would like to thank the Malaysian Mathematical Sciences Society for organizing such seminar and meetings among the scientists.

REFERENCES

Finkelstein, D. M. 1986. A proportional hazards model for interval-censored failure time data. *Biometrics 42: 845-854*.

Fleming, T. R. & Harrington, D. P. 1991. *Counting Process and Survival Analysis*. John Wiley, New York.

Huang, J., Lee, C. & Yu, Q. 2008. A generalized log-rank test for interval-censored failure time data via multiple imputation. *Statistics in Medicine* 27: 3217-3226.

Kalbfleisch, J. D. & Prentice, R. L. 2002. *The Statistical Analysis of Failure Time Data* , Wiley, New York.

Peto, R. & Peto, J. 1972. Asymptotically efficient rank invariant test procedures. *Journal of the Royal Statistic Society, Series A*, 135(2): 185-207.

Petroni, G. R. & Wolfe, R. A. 1994. A two sample test for stochastic ordering with interval-censored data. *Biometrics* 50: 77-87**.**

Turnbull, B. W. 1976. The empirical distribution with arbitrarily grouped censored and truncated data. *Journal of the Royal Statistic Society, Series B,* 38(3): 290-295.

Zhao, X., Zhao, Q., Sun, J. & Kim, J. S. 2008. Generalized log-rank tests for partly interval-censored failure time data. *Biometrical Journal 50(3): 375-385.*

NUMERICAL SOLUTION OF STAGNATION-POINT FLOW AND HEAT TRANSFER TOWARDS A SHRINKING SHEET IN A NANOFLUID

Roslinda Nazar & Rokiah Ahmad
School of Mathematical Sciences
Faculty of Science and Technology
Universiti Kebangsaan Malaysia
Email: rmn@ukm.my

INTRODUCTION

The recent discovery of nanofluid, which is a new kind of fluid suspension consisting of uniformly dispersed and suspended nanometer-sized (10–50 nm) particles and fibers in the base fluid, marks the next approach as a cooling technology (Das et al. 2007). Nanofluids usually contain the nanoparticles such as metals, oxides, or carbon nanotubes. It is known that nanofluids can tremendously enhance the heat transfer characteristics of the base fluid. In this study, the steady stagnation-point flow and heat transfer towards a shrinking sheet in a nanofluid is theoretically studied. Three different types of nanoparticles, namely copper (Cu), alumina (Al_2O_3) and titania (TiO_2) are considered. The nanofluid equations model as proposed by Tiwari and Das (2007) has been used. The governing equations are solved numerically using the shooting method. Numerical results are obtained for the skin friction coefficient, the local Nusselt number as well as the velocity and temperature profiles for some values of the governing parameters, namely the nanoparticle volume fraction parameter φ, the shrinking parameter λ and the Prandtl number Pr. It should be mentioned to this end that for a regular Newtonian fluid (φ = 0), the present problem reduces to that first studied by Wang (2008).

BASIC EQUATIONS AND PROBLEM FORMULATION

Consider the steady two-dimensional stagnation-point flow of a nanofluid past a shrinking sheet with the linear velocity $u_w(x) = c\,x$, and the velocity of the far (inviscid) flow is $u_e(x) = a\,x$, where a and c are constant, and x is the coordinate measured along the surface. The flow takes place at $y \geq 0$, where y is the coordinate measured normal to the shrinking surface. It is assumed that the constant temperature of the shrinking sheet is T_w and that of the ambient nanofluid is T_∞, where $T_w > T_\infty$ (heated shrinking sheet). The basic steady conservation equations for a nanofluid in Cartesian coordinates x and y are (see Tiwari and Das (2007)),

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho_{nf}}\frac{\partial p}{\partial x} + \frac{\mu_{nf}}{\rho_{nf}}\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) \tag{2}$$

$$u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = -\frac{1}{\rho_{nf}}\frac{\partial p}{\partial y} + \frac{\mu_{nf}}{\rho_{nf}}\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) \tag{3}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha_{nf}\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right) \tag{4}$$

subject to the boundary conditions

$$\begin{aligned} &v=0,\quad u=u_w(x)=c\,x,\quad T=T_w \quad \text{at } y=0 \\ &u\to u_e(x)=a\,x,\quad v\to 0,\quad T\to T_\infty \quad \text{as} \quad y\to\infty \end{aligned} \tag{5}$$

Here u and v are the velocity components along the x and y axes, respectively, T, p, α_{nf}, ρ_{nf} and μ_{nf} are the temperature, fluid pressure, thermal diffusivity, effective density, and effective viscosity of the nanofluid, respectively, which are defined as in Oztop and Abu-Nada (2008). We look for a similarity solution of Eqs. (1) - (4) with the boundary conditions (5) of the following form:

$$\psi=(a\nu_f)^{1/2}\,x\,f(\eta),\quad \theta(\eta)=(T-T_\infty)/(T_w-T_\infty),\quad \eta=(a/\nu_f)^{1/2}\,y \tag{6}$$

where ν_f is the kinematic viscosity of the fluid and the stream function ψ is defined in the usual way as $u=\partial\psi/\partial y$ and $v=-\partial\psi/\partial x$, which identically satisfy Eq. (1). On substituting (6) into Eqs. (2)-(4), we obtain the following ordinary differential equations:

$$\frac{1}{(1-\varphi)^{2.5}\left(1-\varphi+\varphi\,(\rho_s/\rho_f)\right)}f'''+f\,f''+1-f'^2=0 \tag{7}$$

$$\frac{1}{\Pr}\frac{(k_{nf}/k_f)}{\left[1-\varphi+\varphi\left((\rho C_p)_s/(\rho C_p)_f\right)\right]}\theta''+f\,\theta'-f'\theta=0 \tag{8}$$

subject to the boundary conditions

$$f(0)=0,\quad f'(0)=\lambda,\quad \theta(0)=1,\quad f'(\infty)=1,\quad \theta(\infty)=0 \tag{9}$$

where primes denote differentiation with respect to η, φ is the nanoparticle volume fraction parameter, Pr is the Prandtl number and $\lambda=c/a\ (>0)$ is the stretching parameter and $\lambda<0$ is a shrinking parameter. It is worth mentioning that Eqs. (7) and (8) reduce to those first derived by Wang (2008) when $\varphi=0$ (regular Newtonian fluid), and Pr = 1.

The physical quantities of practical interest in this study are the skin friction coefficient at the surface of the shrinking sheet C_f and the local Nusselt number Nu, which can easily shown to be given by

$$\mathrm{Re}_x^{1/2}\,C_f=\frac{1}{(1-\varphi)^{2.5}}f''(0),\quad \mathrm{Re}_x^{-1/2}\,Nu=-\frac{k_{nf}}{k_f}\theta'(0) \tag{10}$$

where $\mathrm{Re}_x=u_e(x)\,x/\nu_f$ is the local Reynolds number.

RESULTS AND DISCUSSION

The nonlinear ordinary differential equations (7) and (8) subject to the boundary conditions (9) are solved numerically using the shooting method. Following Oztop and Abu-Nada (2008), we have considered the range of nanoparticle volume fraction to be $0\le\varphi\le 0.2$.

The Prandtl number Pr considered in this study is Pr = 1. Further, it should also be pointed out that the thermophysical properties of fluid and nanoparticles (Cu, Al_2O_3, TiO_2) used in this study are also as in Oztop and Abu-Nada (2008). In order to validate the present numerical method used, we have compared our results with those obtained by Wang (2008) for various values of the shrinking parameter ($\lambda < 0$) when $\varphi = 0$ (regular fluid) and Pr = 1 as shown in Tables 1 and 2. The comparisons with Wang (2008) are found to be in excellent agreement. Tables 1 and 2 also present the numerical values for the present problem in nanofluid with different nanoparticles (Cu, Al_2O_3, TiO_2) for various φ. It is found that dual solutions exist for $\lambda < -1$, up to a certain critical value of λ, say λ_c, beyond which the boundary layer separates from the surface, thus no solution is obtained. It is also shown in Tables 1 and 2 that for a particular nanoparticle, say Cu, as φ increases, the skin friction coefficient and the local Nusselt number also increase. On the other hand, for a fixed value of φ, say $\varphi = 0.1$, the skin friction coefficient is highest for Cu (nanoparticles with high density), followed by TiO_2 and Al_2O_3, while the local Nusselt number is largest for Cu, followed by Al_2O_3 and TiO_2 (nanoparticles with low thermal conductivity).

Table 1. Values of $\mathrm{Re}_x^{1/2} C_f$ for the shrinking sheet ($\lambda < 0$) with Pr = 1 for Cu, Al_2O_3, TiO_2 and φ = 0 (regular fluid), 0.1, 0.2. Results in parenthesis ( ) are the 2nd (lower branch) solutions

λ	$f''(0)$		Cu		Al_2O_3		TiO_2	
	φ = 0 (regular fluid)		φ = 0.1	φ = 0.2	φ = 0.1	φ = 0.2	φ = 0.1	φ = 0.2
	Present	Wang (2008)	Present		Present		Present	
-0.25	1.40224	1.40224	2.14368	2.98374	1.82256	2.34163	1.84217	2.38247
-0.5	1.49567	1.49567	2.28651	3.18254	1.94400	2.49765	1.96491	2.54121
-0.75	1.48930	1.48930	2.27677	3.16898	1.93572	2.48701	1.95654	2.53038
-1	1.32882	1.32882	2.03143	2.82750	1.72713	2.21902	1.74571	2.25772
-1.1	1.18668 (0.04920)	-	1.81414 (0.07526)	2.52506 (0.10475)	1.54239 (0.06399)	1.98166 (0.08221)	1.55898 (0.06467)	2.01622 (0.08364)
-1.15	1.08223 (0.116702)	1.08223 (0.116702)	1.65447 (0.17841)	2.30281 (0.24832)	1.40663 (0.15168)	1.80724 (0.19488)	1.42176 (0.15332)	1.83876 (0.19828)
-1.2	0.93247 (0.23363)	-	1.42552 (0.35719)	1.98415 (0.49717)	1.21198 (0.30369)	1.55716 (0.39018)	1.22502 (0.30695)	1.58431 (0.39698)
-1.2465	0.55430	0.55430	0.89323	1.24328	0.75942	0.97571	0.76759	0.99273

Table 2. Values of $\mathrm{Re}_x^{-1/2} Nu$ for the shrinking sheet ($\lambda < 0$) with Pr = 1 for Cu, Al_2O_3, TiO_2 and φ = 0 (regular fluid), 0.1, 0.2. Results in parenthesis ( ) are the 2nd (lower branch) solutions

λ	$-\theta'(0)$		Cu		Al_2O_3		TiO_2	
	φ = 0 (regular fluid)		φ = 0.1	φ = 0.2	φ = 0.1	φ = 0.2	φ = 0.1	φ = 0.2
	Present	Wang(2008)	Present		Present		Present	
-0.25	0.66857	0.66857	0.86202	1.03949	0.80339	0.93681	0.78502	0.89501
-0.5	0.50145	0.50145	0.70168	0.87555	0.62943	0.75219	0.61394	0.71593
-0.75	0.29378	0.29376	0.50504	0.67521	0.41488	0.52533	0.40287	0.49578
-1	0	0	0.23308	0.40000	0.11525	0.21058	0.10797	0.19002
-1.1	-0.17696 (-4.26577)	-	0.07358 (-2.78669)	0.23996 (-2.27462)	-0.06258 (-3.69342)	0.02522 (-3.46468)	-0.06716 (-3.66295)	0.00975 (-3.42772)
-1.15	-0.29789 (-2.76344)	-0.29799 (-2.76345)	0.03334 (-1.83645)	0.13343 (-1.51080)	-0.18285 (-2.41407)	-0.09941 (-2.27652)	-0.18567 (-2.39321)	-0.11156 (-2.25086)
-1.2	-0.47186 (-1.88316)	-	-0.18352 (-1.25320)	-0.01501 (-1.01270)	-0.35356 (-1.65139)	-0.27512 (-1.55706)	-0.35396 (-1.63698)	-0.28277 (-1.15162)
-1.247	-0.99004	-0.99004	-0.57251	-0.39105	-0.80755	-0.73450	-0.80200	-0.73153

Figure 1 displays the velocity profiles $f'(\eta)$ for nanoparticles Cu, Al_2O_3, TiO_2 when φ = 0.1 and λ = -1.2 (shrinking sheet), while Fig. 2 illustrates the corresponding temperature profiles when Pr = 1. These profiles show that dual solutions exist for the shrinking sheet. The solid and dashed lines in these figures represent the first (upper branch) and second (lower branch) solutions, respectively. The velocity and temperature profiles for Al_2O_3 and TiO_2 are almost identical while the velocity profile for Cu is higher and the temperature profile for Cu is lower. Therefore, it is found that nanoparticles with low thermal conductivity, TiO_2, have better enhancement on heat transfer compared to Al_2O_3 and Cu. These profiles satisfy the far field boundary conditions (9) asymptotically, which support the numerical results obtained and thus, cannot be neglected mathematically.

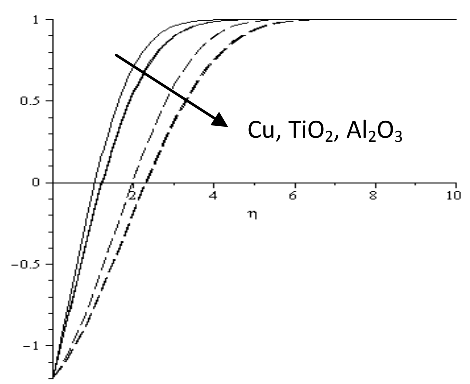


Figure 1. Velocity profiles $f'(\eta)$ when φ=0.1, λ = -1.2 (shrinking sheet) and Pr = 1

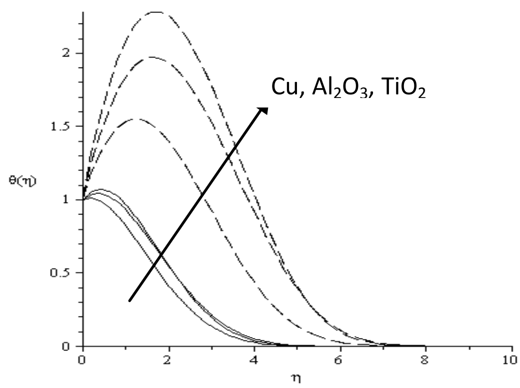


Figure 2. Temperature profiles $\theta(\eta)$ when φ=0.1, λ = -1.2 (shrinking sheet) and Pr = 1

Acknowledgement

The financial support received in the form of a fundamental research grant scheme (FRGS) from the Ministry of Higher Education, Malaysia is acknowledged.

REFERENCES

Das, S.K., Choi, S.U.S., Yu, W. & Pradeep, T. 2007. *Nanofluids: Science and Technology*. New Jersey: Wiley.

Oztop, H.F. & Abu-Nada, E. 2008. Numerical study of natural convection in partially heated rectangular enclosures filled with nanofluids *Int. J. Heat Fluid Flow* 29: 1326-1336.

Tiwari, R.J. & Das, M.K. 2007. Heat transfer augmentation in a two-sided lid-driven differentially heated square cavity utilizing nanofluids. *Int. J. Heat Mass Transfer* 50: 2002-2018.

Wang, C.Y. 2008. Stagnation flow towards a shrinking sheet. *Int. J. Non-Linear Mech*. 43: 377-382.

TECHNICAL ANALYSIS EMPLOYING FUZZY SYSTEM

Saiful Hafizah Jaaman, Abdul Razak Salleh
School of Mathematical Sciences, Faculty of Science and Technology
Universiti Kebangsaan Malaysia
Email: shj@ukm.my

INTRODUCTION

Technical analysis is the study of market behaviour, primarily through the use of charts by analyzing the past sequence of stock prices and volumes for the purpose of forecasting future price trends. Technical analysis is commonly used in financial markets to assist investors to make buying and selling decisions. The concern in technical analysis is the historical movement of prices and forces of supply and demand that affect those prices. There are three premises on which technical analysis is based: (1) market action discounts everything (2) prices move in trends (3) history repeats itself (Pring 1991). Thus, a technical analyst believes that everything is concealed in price and look for particular patterns on the charts that are supposed to have predictive value. Technical analysts focus on investor’s psychology and response to a certain price structure and price movements to identify the price at which an investor is willing is small or sell, depending on the investor’s expectation. If investor expects the price to rise, he or she will buy it, however if investor expects the price to fall, he or she will sell it.

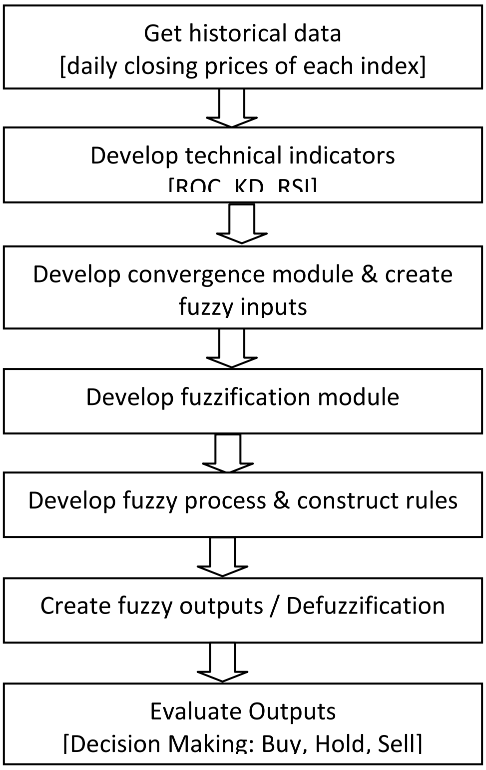
As is known in investment, market participants anticipate future development and take action now based on what they think the security is worth and their actions will drive the price movement. Since stock market processes are highly nonlinear, many researchers (among others Refenes *et. al* 1997, Benachenchou 1996, Simutis 2000) have been focusing on technical analysis to improve the investment return. This paper examines an investment strategy of buying and selling built using technical analysis and fuzzy system on four different indices, namely the Malaysian Stock Exchange, Shanghai Stock Exchange, Standard & Poor's 500 and Nikkei.

TECHNICAL ANALYSIS AND FUZZY SYSTEM

Fuzzy systems have been widely used in expert systems, machinery and home appliances. Only lately applications of fuzzy system in the finance field have been reported, exploiting the ability of fuzzy systems to model the vague and imprecise information. There are many technical analysis indicators and theories. The most difficult part is to choose which indicator to use for the success of technical analysis depends on how one interprets the available signals. Technical analysis deals with probability hence multiple indicators can be utilized to improve the result. In most cases, the answer by each indicator is not a definite yes or no (buy or sell) answer. Based on this, the opportunity to improve stock price evaluation by using fuzzy logic, neural network and artificial intelligence can be very useful. Fuzzy systems have been used with various technical indicators in previous studies. Zhou and Dong (2004) and Dourra and Siy (2002) model the cognitive uncertainty incorporated in technical analysis by using a fuzzy-logic approach. Chang and Liu (2008) make use of the fuzzy system and introduce the Takagi-Sugeno-Kang (TSK) to predict stock price. Shuhadah and Schneider (2010) employ a fuzzy inference system with technical indicators to create trading strategy of buying and selling in the stock market. The results combining technical analysis and fuzzy logic look very promising.

Each of technical analysis indicators has its limitation. The best result could be achieved by combining many indicators at the same time and evaluate their output collectively. In this study three technical indicators were employed, these were the Rate of Change (ROC), KD technical index and the Relative Strength Index (RSI). This study employs the fuzzy system process proposed by Dourra and Siy (2002) shown in figure 1.

Figure 1. Fuzzy Inference System



The rule base of the fuzzy inference system is initialised following the technical analysis guidelines for the three technical indicators used. The trading strategy classification rules are as follow:

Rule 1: IF ROC is large THEN C is to sell.

Rule 2: IF ROC is medium THEN C is to hold.

Rule 3: IF ROC is small THEN C is to buy.

Rule 4: IF RSI is large THEN C is to sell.

Rule 5: IF RSI is medium THEN C is to hold.

Rule 6: IF RSI is small THEN C is to buy.

Rule 7: IF %D is large THEN C is to sell.

Rule 8: IF %D is medium THEN C is to hold.

Rule 9: IF %D is small THEN C is to buy.

Combining the trading rules of the three technical indicators give the following sets of rule:

1. IF ROC is large and RSI is large and %D is large THEN C is to sell.
2. IF ROC is medium and RSI is medium and %D is medium THEN C is to hold.
3. IF ROC is small and RSI is small and %D is small THEN C is to buy.

RESULTS AND DISCUSSION

Daily stock index data from July 2008 to June 2009 of four different stock markets; Composite Index of the Malaysian Stock Exchange (KLCI), Nikkei, Standard & Poor's 500 (S&P 500) and Shanghai Stock Exchange (SSE) have been fed to this system for evaluation. The fuzzy system is designed for generating a buy, hold or sell signal. The results are presented in table 1.

Table 1. Trading Strategy

Market	Time to buy	Fuzzy Indicator	Time to sell	Fuzzy Indicator
KLCI	10-Okt-08	22.90793	05-Nov-08	81.55117
S&P 500	9-Okt-08	14.06088	28-Nov-08	75.56285
Nikkei	10-Okt-08	15.0145	04-Nov-08	79.75047
SSE	17-Sept-08 27-Okt-08	18.08065 14.85365	24-Sept-08	78.76714

The fuzzy inference system specify the rules that when the three technical indicators are small the strategy is to buy the shares, when the indicators are medium then it is suggested that investors hold the shares if they have already bought the shares and when the indicators are large the rule is to sell the shares. As shown in table 1, based on the index of each stock market the fuzzy indicator denotes that for all three stock markets; KLCI, S&P 500 and Nikkei, the time to buy shares occur in the month of October 2008 and with a holding period of about a month the time to sell occurs in the month of November 2008. Only for Shanghai Stock Exchange the fuzzy indicator reveals two buying signals; on 17th September 2008 and 27th October 2008 and one selling signal on 24th September 2008. The one month holding period produce investment returns between 10% - 15% which are acceptable.

The fuzzy system adds value as it provides excellent pattern recognition that fits well to the perception of the investors. The fuzzy system used information derived from technical analysis indices as input. It is designed to mimic human behaviour in interpreting technical indicators and by using different technical indicators as inputs, a flexible system is built. Nevertheless, humans usually use a variety of strategies, changing from one to another depending on the circumstances and information at hand. Thus, more research is needed in order to supply the system with such extended flexibility.

Acknowledgement
The financial support received in the form of a fundamental research grant scheme (FRGS) Code: UKM-ST-06-FRGS0103-2009 from the Ministry of Higher Education, Malaysia is acknowledged.

REFERENCES

Benachenhou, D. 1996. Smart trading with FRET. *Trading On The Edge*. Wiley. New York.

Chang, P.C. & C.H. Liu. 2008. A TSK Type Fuzzy Rule Based System for Stock Prediction. *Expert Systems with Applications*. 34: 135-144.

Dourra, H. & P. Siy. 2002. Investment using Technical Analysis and Fuzzy logic. *Fuzzy Sets and Systems*. 127: 221-240.

Pring, M.J. 1991. *Technical Analysis.* New York.

Refenes, A.N., N. Burgess & Y. Bentz. 1997. Neural networks in fnancial engineering: a study in methodology, *IEEE Trans. Neural Networks.* 8 (6): 1222–1267.

Simutis, R. 2000. Fuzzy logic based stock trading system. *Proc. IEEE-IAFE Conf. on Computational Intelligent of Financial Engineering*. New York.

Zhou, X.S. & M. Dong. 2004. Can fuzzy logic make technical analysis 20/20? *Financial Analyst Journal*, 60: 54–73.

SIMILARITY SOLUTIONS OF MHD MIXED CONVECTIVE SURFACE-TENSION-DRIVEN BOUNDARY LAYER FLOWS

Seripah Awang Kechil, Noraini Ahmad, Norma Mohd Basir
Faculty of Computer and Mathematical Sciences
Universiti Teknologi MARA
seripah@tmsk.uitm.edu.my, noraini.ahmad@rocketmail.com,
norma@tmsk.uitm.edu.my

INTRODUCTION

The gradients in temperature and surfactant concentration at the interface of two fluid layers give rise to surface tension variations that induce interfacial flows from region of low surface tension to region of high surface tension. The surface tension-driven convection or also known as Marangoni convection, is of central importance in industrial, biomedical and daily life applications such as coating flow technology, microfluidics, surfactant replacement therapy for neonatal infants, film drainage in emulsions and foams and drying of semi-conductor wafers in microelectronics. The development of the momentum, thermal and concentration of Marangoni mixed convection boundary layers have been considered by Magyari and Chamkha (2007, 2008) Al-Mudhaf and Chamkha (2005) and Pop et al. (2001). Magyari and Chamkha (2007, 2008) found the exact analytical solutions for the MHD thermosolutal Marangoni convection and thermosolutal Marangoni convection in the presence of heat and mass generation or consumption.

In this particular work, we will extend the work of Zueco and Beg (2011) on the hydromagnetic mixed Marangoni convection boundary layers by including the effects of surfactant concentration gradient and absorption on the interface

RESULTS/DISCUSSION

We will transform the dimensional governing equations to nondimensional boundary layer equations by introducing scaling and similarity variables. The boundary layer equations obtained will be solved numerically using the Runge-Kutta Felhberg with shooting technique. Comparative study with results of Chamkha et al. (2006) and Zueco and Beg (2011) will be carried out. Graphical representation of the boundary layer profiles will be analysed to assess the effects of magnetic field and absorption on the boundary layer thickness.

CONCLUSION

The effects of the magnetic field and suction or injection are known to either impose viscous drag forces or viscous driving forces on the velocity of the fluid motion (Magyari and Chamkha, 2007, 2008). We will show whether the effects of these physical parameters are significant on the velocity, temperature and concentration boundary layers.

REFERENCES

Al-Mudhaf, A., & Chamkha, A. J. (2005). Similarity solutions for MHD thermosolutal Marangoni convection over a flat surface in the presence of heat generation or absorption effects. Heat and Mass Transfer. 42: 112-121.

Chamkha, A.J., Pop, I., Takhar, H.S. (2006).Marangoni mixed convection boundary layer flow. Meccanica. 41: 219-232.

Magyari, E., & Chamkha, A. J. (2007). Exact analytical solutions for thermosolutal Marangoni convection in the presence of heat and mass generation or consumption. Heat and Mass Transfer. 43: 965-975.

Magyari, E., & Chamkha, A. J. (2008). Exact analytical results for the thermosolutal MHD Marangoni boundary layer. International Journal of Thermal Sciences. 47: 848-857.

Pop, I., Postelnicu, A., Grosan, T. (2001). Thermosolutal Marangoni forced convection boundary layers. Meccanica. 36, 555-571.

Zueco, J., Beg, O.A. (2011). Network numerical simulation of hydromagnetic Marangoni mixed convection boundary layers. Chemical Engineering Communications. 198: 552-571.

PETRI NETS OVER GROUPS AND THEIR LANGUAGES

Sherzod Turaev, Mohd Hasan Selamat

Faculty of Computer Science and Information Technology,
University Putra Malaysia, Malaysia
Email:{sherzod, hasan}@fsktm.upm.edu.my

Nor Haniza Sarmin

Faculty of Science, Universiti Teknologi Malaysia
81310 UTM Skudai, Johor, Malaysia
Email: nhs@utm.my

INTRODUCTION

The use of algebraic structures and techniques in the study of formal language and automata theory often leads to elegant and mathematically satisfying characterizations of languages and automata. For example, preimages of subsets of monoids (groupoids) under morphisms characterize regular languages [2] (context-free languages [1]). Algebraic structures are also used as regulated mechanisms in Chomsky grammars. Grammars over different monoids and groups are introduced in [3,5]. Finite automata over groups have been investigated in [4].

Petri nets, introduced by Carl Adam Petri in 1962, provide a powerful mathematical formalism for describing and analyzing the flow of information and control in concurrent systems. Since Petri nets are a generalization of finite automata, it is of interest to study *Petri nets over monoids and groups*. In this paper we introduce Petri nets over groups and investigate their languages, which have not been studied at all until now. Informally, a Petri nets over a group is a place/transition Petri net equipped with a counter. With each transition of the Petri net, an element of the group is associated. The occurrence of transitions of the Petri net starts from the initial marking with its counter containing the neutral element of the group. The occurrence of a transition changes the value of the counter by applying the group operation to the current value of the counter and the element of the group associated to the transition. An occurrence sequence of transitions is considered to be successful if and only if it finishes at a final marking with its counter containing the neutral element.

The families of context-free and matrix l*anguages are denoted by* ***CF*** *and* ***MAT****, respectively.* ***lPN*** *and* ***rPN*** *denote the families* of *L*- and *R*-type Petri net languages with the arbitrary labeling policy.

PETRI NETS OVER GROUPS

Let $N = (P, T, W, \mu_0, F)$ be a Petri net where P and T are disjoint finite sets of *places* and *transitions*, respectively, $W : (P\times T)\cup (T\times P) \to N$ is a *flow function*, μ_0 is the *initial marking*, and F is a set of *final markings*. Let $\boldsymbol{K} = (K, \circ, e)$ be a group under operation $\circ$ with the neutral element e.

Definition 1. A *Petri net over a group* $\boldsymbol{K}$ (GPN for short) is a construct

$$A = (P, T, W, \mu_0, F, K, \varphi)$$

where $N = (P, T, W, \mu_0, F)$ is a Petri net, and $\varphi : T \to K$ is a total function.

Definition 2. A transition $t \in T$ is enabled by marking μ if and only if $\mu(p) \geq W(p, t)$ for all $p \in P$.

In this case t can occur. Its occurrence transforms the marking μ into the marking μ' defined for each place $p \in P$ by $\mu'(p) = \mu(p) - W(p,t) + W(t,p)$, and writes kl (i.e., kl stands for $k \circ l$) in the counter where k is the old content of the counter and l is the element of K assigned to t, i.e., $\varphi(t) = l$. We write $(\mu,k)\ [t,l)\ (\mu,m)$ to indicate that the firing of t in μ leads to μ' changing the content of the counter from k to $m = kl$.

Definition 3. A finite sequence $t_1t_2 \cdots t_n$, $t_i \in T$, $1 \leq i \leq n$, is called an *occurrence sequence enabled at a marking* μ_0 *and finished at a marking* μ_n changing the content of the counter from k_0 to k_n if there are markings $\mu_1, \mu_2, \ldots, \mu_n$ and elements $l_1, l_2, \ldots, l_n$ of K such that

$$(\mu_0, k_0)\ [t_1,l_1)\ (\mu_1, k_1)\ [t_2,l_2)\ (\mu_2, k_2)\ \ [t_n,l_n)\ (\mu_n, k_n)$$

where $k_{i-1}l_i = k_i$, $1 \leq i \leq n$.

In short the occurrence sequence above can be written as

$$(\mu_0, k_0)\ [v,\sigma)\ (\mu_n, k_n)$$

where $v = t_1t_2 \cdots t_n$ and $\sigma = l_1l_2 \cdots l_n$.

Definition 4. An occurrence sequence of transitions $v \in T^*$ is called a *successful* if it starts at the initial marking μ_0 with the counter containing the neutral element and finishes at a final marking $\mu \in F$ with the counter containing the neutral element.

LANGUAGES OF PETRI NETS OVER GROUPS

Definition 5. A *labeled Petri net over group* $\boldsymbol{K}$ is a pair $D = (A, \ell)$ where $A = (P, T, W, \mu_0, F, K, \varphi)$ is a Petri net over a group $\boldsymbol{K}$, and $\ell : T \to \Sigma \cup \{\lambda\}$ is a transition labeling function.

The labeling function ℓ is extended to occurrence sequences in natural way, i.e., if $v \in T^*$ is an occurrence sequence then $\ell(vt) = \ell(v)\ell(t)$ and $\ell(\lambda) = \lambda$.

For an occurrence sequence $v \in T^*$, $\ell(v)$ is called a *label sequence*.

In general, a language generated (or accepted) by a Petri net over a group is a set of label sequences corresponding to successful occurrence sequences of the Petri net. Different labeling policies, definitions of final marking sets and types of groups result in different language families generated by Petri nets over groups. In this paper we restrict our investigation to the arbitrary labeling strategy, two types of final marking sets, namely, the set of all reachable markings from the initial marking μ0, denoted by $\boldsymbol{R}(D, \mu_0)$, and a finite set of final markings, and the group of integers under addition, i.e., $(\boldsymbol{Z},+,0)$. Then the following types of languages are defined.

Definition 6. An *l-type valence Petri net language* is a language generated by a labeled Petri net $D = (A, \ell)$ over $(\boldsymbol{Z},+,0)$ defined by

$$L(D) = \{\ell(v) \in \Sigma^* \mid v \in T^*, (\mu_0, e)\ [v,\pi)\ (\mu, e), \mu \in F, \pi \in \boldsymbol{K}\}.$$

Definition 7. An *r-type valence Petri net language* is a language generated by a labeled Petri net $D = (A, \ell)$ over $(\boldsymbol{Z},+,0)$ defined by

$$L(D) = \{\ell(v) \in \Sigma^* \mid v \in T^*, (\mu_0, e)\ [v,\pi)\ (\mu, e), \mu \in \boldsymbol{R}(D, \mu_0), \pi \in \boldsymbol{K}\}.$$

We denote by **lGPN** and **rGPN** the families of *l*- and *r*-type languages, respectively. Further, we cite the main results of our paper. The following inclusions follow from the definition of valence Petri net languages.

Theorem 1.

$$\mathbf{lPN} \subseteq \mathbf{lGPN} \text{ and } \mathbf{rPN} \subseteq \mathbf{rGPN}.$$

The next theorem shows the relationships of valence Petri net languages to matrix languages.

Theorem 2.

$$\mathbf{rGPN} \subseteq \mathbf{lGPN} \subseteq \mathbf{MAT}.$$

We also show that the families of valence languages and context-free languages are incomparable.

Theorem 3. For $x \in \{r, l\}$,

$$\mathbf{xGPN} - \mathbf{CF} \neq \varnothing.$$

Acknowledgement: This work was partially supported by University Putra Malaysia via RUGS 05-01-10-0896RU.

REFERENCES

[1] F. Bédard, F. Lémieux and P. McKenzie. Extensions to Barrington's M-program model. Theoretical Computer Science 107, 1993, 31–61.

[2] S. Eilenberg. Automata, languages, and machines, Volume B. Pure and Applied Mathematics. New York: Academic Press, 1976.

[3] H. Fernau and R. Stiebe. Sequential grammars and automata with valences. Theoretical Computer Science 276, 2002, 377–405.

[4] V. Mitrana and R. Stiebe. Extended finite automata over groups. Discrete Applied Mathematics 108, 2001, 287–300.

[5] Gh. Păun. A new generative device: valence grammars. Revue Roumaine de Mathématiques Pures et Appliquées 6, 1980, 911–924.

ON THE GRÜNEISEN PARAMETER AND ELASTIC MODEL FOR SUPERIONIC SILVER BORATE GLASS

H.A.A. Sidek, M.K. Halimah, K.A. Matori, Z.A. Wahab & W.M. Daud
Department of Physics, Faculty of Science
Universiti Putra Malaysia, Malaysia
sidek@science,upm.edu.my

INTRODUCTION

The Grüneisen parameter (γ) is of considerable importance to physicist because it sets limitations on the thermoelastic properties (directly related to the equation of state) and an also important tool for investigation of anharmonic effect in any solid. One way to derive the Grünisen parameter is by considering the entropy of the system in the quasiharmonic form where the individual vibrational mode i is introduced as a measure of the volume dependence of the mode frequency ω_i, and is defined as

$$\gamma_i = -\frac{d\ln\omega_i}{d\ln V}$$

The anharmonic properties of solids are usually described in terms of an average thermal Grüneisen parameter γ^{th} which can be evaluated experimentally using

$$\gamma^{th} = \left.\frac{\partial S}{\partial V}\right|_T \frac{V}{C_V} = \alpha V / \beta_V^S C_P = \alpha V / \beta_V^T C_V$$

where α is the coefficient of volume thermal expansion, V is the volume, β_V^T and β_V^S is isothermal and adiabatic compressibility respectively and, C_v and C_p are specific heats at constant volume and constant pressure respectively.

Superionic silver borate glass itself can be treated as elastically isotropic solid. By writing the longitudinal and shear velocities of ultrasonic waves as v_l and v_s respectively and the vibration frequency (qi v_i) where qi is the wave vector, the mode Grüneisen parameters of each modes can be defined as

$$\gamma_l = -\frac{d\ln q_i v_l}{d\ln V},\ \gamma_s = -\frac{d\ln q_i v_S}{d\ln V}$$

If we assume that the wave vectors q_i scale as $V^{-1/3}$, then the γ's are independent of the index i and can be written as

$$\gamma_l = \frac{1}{3} + \frac{1}{\beta^T}\frac{d\ln v_1}{dP} \text{ and } \gamma_s = \frac{1}{3} + \frac{1}{\beta^T}\frac{d\ln v_S}{dP}$$

where P is pressure. The value of each elastic Grüneisen parameter as well as the mean value $\overline{\gamma}^{el}$ for glass system can be rewritten in terms of the pressure derivatives of the effective second order elastic constant (SOEC):

$$\gamma_l = \frac{-1}{3} + \frac{B^s}{2C_{11}}\frac{2dC_{11}}{dP},\ \gamma_s = \frac{-1}{3} + \frac{B^s}{2C_{44}}\frac{2dC_{44}}{dP} \text{ and } \overline{\gamma}^{el} = \frac{(\gamma_1 + 2\gamma_s)}{3}$$

These quantities are used to describe the anharmonicity of the long wavelength acoustic phonons in a glass in the quasiharmonic approximation.

Makishima and Mackenzie have derived a semiempirical formula for theoretical calculation of elastic modulus (E), based on chemical compositions of the glass, the packing density of atoms and the bond energy (U_o) per unit volume. The authors assumed that for a pair of ions of opposite sign with spacing r_o, $U = -e^2 / r_o$, and for many interactions between ions within a crystal. They proposed the elastic modulus of multicomponent glasses (Xi molar fraction of each glass component) is given by the product of the dissociation energy per unit volume (G) and the packing density of ions (V_t).

$$E = 83.6 V_t \sum_i GiXi$$

This expression gives E in kilobars if units of G_i are in kilocalories per cubic centimeter.

The other calculated elastic properties such as shear modulus (G), bulk modulus (K) and Poisson's ratio (s) can be estimated readily by combining the E_m for each glass system as follows:

$$G_m = \frac{3EK}{9K - E}\,,\ K_m = 100V_t \sum_i GiXi \quad \text{and } \sigma_m = \frac{E_m}{2G_m} - 1$$

Meanwhile the bulk compression model was first proposed by Bridge and Higazy who computed a theoretical bulk modulus K (K_{bc}) for a glass, using available network bond-stretching force constants (f) on the assumption that an isotropic deformation in glass merely changes network bond lengths (l) without changing bond angles. They modelled the value of K_{bc} and n_b as

$$K_{bc} = \frac{n_b r^2 f}{9} \quad \text{and} \quad n_b = nn_f = \left(\frac{N_f N_A \rho}{M}\right)$$

where r is the bond length and f (newtons/meter) is the first-order f, n is the coordination bond number and n_f is the number of network bonds per unit formula. N_A is Avogadro's number, and ρ and M represent density and molar mass of glass syatem. The K for a polycomponent oxide glass on the basis of bond compression model is given by the equation

$$K_{bc} = \sum_i \left(X_i n_i l_i^2 f_i\right)\frac{\rho N_A}{9M}$$

where X_i is the mole fraction of oxide and $f = 1.7/r^3$. The bond compression model gives the calculated Poisson's ratio (σ) by an equation containing the average cross-link density per cation in the glass (n_c) in the form

$$\sigma = 0.28\left(n_c^{'}\right)^{-1/4} \ \text{where}\ n_c^{'} = \left(\frac{1}{\eta}\right)\sum_i \left(n_c\right)_i \left(N_c\right)_i \ \text{and}\ \ \eta = \sum_i \left(N_c\right)_i$$

and where $n_c^{'}$ is the average cross-link density per unit formula, n_c equals number of bonds less 2, N_c is the number of cations per glass formula unit, and η is the total number of cations per glass formula unit. The other calculated elastic modulus readily follows by combining the K_{bc} and σ for each glass system:

$$G = \left(\frac{3}{2}\right)K_{bc}\left(\frac{1-2\sigma}{1+\sigma}\right),\ L = K_{bc} + \left(\frac{4}{3}\right)G \ \text{and}\ E = 2G\left(1+\sigma\right)$$

This paper reports the Grüneisen parameter of superionic silver borate glasses and emphasize has also been given to the elastic models of Makishima and Mackenzie, and bond compression in order to understand the elastic behaviour of such glasses.

ACKNOWLEDGEMENT

We like to thanks the Ministry of Higher Education, Malaysia who funded this research project under the Fundamental Research Grant Scheme (FRGS) Program.

REFERENCES

[1]. G. Carini Jr, Carini G, D'Angelo G, Tripodo G, Bartolotta A and Salvato G (2005) *Phys. Rev.* B **72** 14201.

[2]. G. A. Saunders, H. A. A. Sidek, J. D. Comins, G. Carini, and M. Federico, (1987) Philos. Mag. B **56**, 1

[3]. E. P. J. Papadakis, (1967) Acoust. Soc. Am. 42, 1045

[4]. Mierzejewski, G. A. Saunders, H. A. A. Sidek, and B. Bridge (1988) J. Non-Cryst. Solids 104, 323

[5]. H.A.A. Sidek, M.K. Halimah, W.M. Daud, A.M. Khamirul, Z.A.Talib, A.W. Zaidan, A.S. Zainal, Z. Azmi, A .Zulkifly, S.A. Halim (2007) Journal of Wuhan University of Technology –Mater. Sci. Ed., **B12**, 863-867

ROMBERG METHOD FOR THE PRODUCT INTEGRAL ON THE INFINITE INTERVAL

[1,2]Z.K. Eshkuvatov, [1,2]N.M.A. Nik Long, [3]S. Bahramov
[1]*Department of Mathematics, Faculty of Science, UPM*
[2]*Institute for Mathematical Research, UPM.*
[3]*National University of Uzbekistan*
E-mail: ezaini@science.upm.edu.my

Introduction: Integration problems on infinite interval are not defined same as the finite intervals usually it is defined as improper integrals, which is

$$\int_a^{\infty} w(x)f(x)dx = \lim_{N\to\infty}\int_a^{N} w(x)f(x)dx, \qquad (1)$$

where *w(x)* is a weight function and infinite integral (1) exists whenever the latter limit exists. As we know the continuity of integrant function or boundedness of function *f(x)* is not enough the existence of the infinite boundary integral (1).

Quadrature formula of the type

$$I_n(f) = \sum_{j=0}^{n} w_{j,n} f\left(x_{j,n}\right), \qquad (2)$$

for the estimate of the product integral

$$I\left(fw\right) = \int_a^b w(x)f(x)dx\,, \qquad (3)$$

where *a* and *b* are finite numbers and *w(x)* is a weight function, have been extensively studied in the 1970-1990 years (see [1,4,6]) and literature cited therein.

Romberg integration for (3) with *w(x)=1* and its application for solving Volterra integral equation on the finite interval have been investigated in [2].

Unfortunately, few works can be found on the investigation of the product integral in the infinite interval [3, 5, 7].

There are many techniques on the reducing the infinite interval into the finite interval, but reduction techniques brings singularity of the integrand function and application of the Romberg rule is not suitable for those parts where singularity appears.

In this work we consider integral (1) and reduced it into the interval [0, 1] and use the mixed method: cubic Newton's divided difference formula on $[t_{n-3}, t_n]$ and Romberg method on $[t_0, t_{n-3}]$ with equal step size, $t_i=t_0+kh$, $h=1/n$ where $t_0=0$ and $t_n=1$. Error analysis is established for modified method.

Methodology: The trapezoidal rule is one of the simplest of the integration formulas, but it is seldom sufficiently accurate. Thus, the Romberg method uses the composite Trapezoidal rule to give preliminary approximations, and then applies Richardson extrapolation to obtain improved approximations.

Recall the ordinary integration problem

$$I(f)=\int_a^b f(x)dx \qquad (4)$$

Definition 1: Composite trapezoidal approximations for (4) of a function f on the interval $[a,b]$ is

$$R_{k,1}=\frac{1}{2}\left[R_{k-1,1}+h_{k-1}\sum_{i=1}^{2^{k-1}-1} f\left(a+2i-1\right)h_k\right]$$

where $h_k=\dfrac{b-a}{m_k}=\dfrac{b-a}{2^{k-1}}$, and $R_{1,1}=\dfrac{h_1}{2}\left[f\left(a\right)+f\left(b\right)\right]$ for each $k=2,3,4,\ldots,n$.

Definition 2: The Romberg integration rule for each $k=2,3,\ldots,n$ and $j=2,3,\ldots,k$, is defined as follows

$$R_{k,j}=R_{k,j-1}+\frac{R_{k,j-1}-R_{k-1,j-1}}{4^{j-1}-1}=\frac{1}{4^{j-1}-1}\left(4^{j-1}R_{k,j-1}-R_{k-1,j-1}\right) \qquad (5)$$

Theorem 1 ([6]): Let $f\in C^{2k+2}[a,b]$ be a real function to be integrated over $[a,b]$ and $R_{m,k}$ be defined in Romberg's method (5), then remainder term $R_{m,k}$ is zero for $f\in P_{2k}$, and truncation error of $R_{m,k}$ reads

$$E\left(f\right)=R_{m,k}-\int_a^b f(x)dx=r_k h^{2k}\left(b-a\right)f^{(2k)}\left(\xi_x\right),$$

where $r_k=\dfrac{2^{k(k-1)}\left|B_{2k}\right|}{\left(2k\right)!}$, with Bernoulli numbers B_k.

The Romberg technique has the desirable feature that it allows an entire new row in the table to be calculated by doing only one additional application of the Composite Trapezoidal rule. It then uses a simple averaging on the previously calculated values to obtain the remaining entries in the row. Moreover, the Romberg method has the advantage that all of the weights w_n are positive and the abscissas x_i are equally spaced. The development of Romberg integration relies on the theoretical assumption that $f(x)$ is smooth enough so that the error in the trapezoidal rule can be expanded in a series involving only even powers of h.

Change of variable: Due to the reducing technique, most of the infinite integral will be reduced to [0, 1] or [-1,1] interval. The substitution $t = \frac{x-a}{x}$ in the product integral (1) yields

$$\int_a^\infty w(x)\, f(x)\, dx = a\int_0^1 w\left(\frac{a}{1-t}\right) f\left(\frac{a}{1-t}\right)\frac{dt}{(1-t)^2}. \tag{6}$$

If density function f and weight function w in (6) is smooth enough around right bound 1 then product integral (6) is well defined. It is well known fact that Romberg method does not well work where in the presence of singularity. Fortunately, spline approximation is good for the singularity problem.

For mixed method, we will split the interval [0, 1] into [0, t_{n-3}] and [t_{n-3},1], where $t_k = kh,\ h = \frac{1}{n},\ k = 0,1,\ldots,n$.

$$I(wf) = a\int_0^1 \frac{w_1(t)\, f_1(t)}{(1-t)^2}dt = a\left(\int_0^{t_{n-3}} + \int_{t_{n-3}}^1\right)\frac{w_1(t)\, f_1(t)}{(1-t)^2}dt$$

The second integral will be approximated with cubic Newton polynomial $P_3(x)$, while the first integral is approximated by modified Romberg method. Hence, we will proof the following theorem.

Theorem 2: Let $f \in C^4[0,1]$, and $R_{m,3}$ be defined in Romberg's method, then remainder term $E(fw)$ is zero for all $f \in P_4$, and truncation error of $E(f)$ has the form

$$E(f) = \left|I(wf) - \left(R_{m,3}^w + P_3^w\right)\right| \le O\left(h^5\right).$$

CONCLUSIONS

Theorem 2 shows that mixed method is exact for the polynomial of degree 4 and order of convergence is at least 5. Many new results can be obtained from the approximation of mixed method.

Acknowledgement: The work here is fully supported by FRGS (01-12-10-989FR) -2011. The author also would like to thank the Malaysian Mathematical Sciences Society and Samarkand State University for organizing such seminar and meetings among the scientists.

REFERENCES

[1]. R Gonzailez-Vera, J.C. Santos-Leon (1996), Properties of certain piecewise polynomial product integration rules. Applied Numerical Mathematics 21 (1996) 241-264

[2]. Mladen Mestrovic´, Eva Ocvirk (2007). An application of Romberg extrapolation on quadrature method for solving linear Volterra integral equations of the second kind. Applied Mathematics and Computation 194, 389–393.

[3]. F. Jezequel and J.-M. Chesneaux (2004), Computation of an infinite integral using Romberg's method. *Numerical Algorithms* **36:** 265–283.

[4]. Philip Rabinowitz and Ian H. Sloa (1984), Product integration in the presence of a singularity. SIAM J. Numer. Anal. Vol. 21, No. 1. Pp. 149-166.

[5]. Philip Davis, Philip Rabinowitz, (1984), Methods of numerical integration. Academic press, INC, New York.

[6]. I.C. Simpson (1978), Numerical Integration over a Semi-Infinite Interval, Using the Lognormal Distribution. Numer. Math. 31, 71-76.

[7]. A. Yazıcı1, T. Ergenc, I. Altas (2003), Romberg Integration: A Symbolic Approach with Mathematica. P.M.A. Sloot et al. (Eds.): ICCS 2003, LNCS 2657, pp. 691–700, Springer-Verlag Berlin Heidelberg 2003.

ELASTOPLASTIC DEFORMATION OF RESERVOIR WALLS

Abdirashidov A. , Berdiyev Sh. ,Aminov B.
Faculty of Mechanics and Mathematics, Samarkand State University, Uzbekistan
abdira@samdu.uz

INTRODUCTION

Hydroelastic behavior of system two coaxial cylindrical shells, interacting through metastable liquid and subject to influence pulsed and hydrodynamics loads were researched in works (Abdirashidov A., Galiev Sh.U., 1988). From results stated in them follow that them are received disregarding plastic deformation of walls and their following fracture, as well as is not taken into account influence of the temperature surrounding ambiences on non stationary behavior of hydroelastic systems. Need of undertaking the more exact calculations, as well as longing to get the full belief about occurring phenomenal require using the models, adequately describing tense-deformed condition of container. In work (Galiev Sh.U., 1988) is brought one of the algorithms of calculation elastic-plastic deformation hard tel. This work is denoted the study of influence boiling metastable liquid on elastic-plastic deformation and fracture of two coaxial cylindrical shells, under external pulsed influences, as well as study an influence of temperature of shells on their tense-deformed condition. Moving the shells is described on base of Timoshenko equations, fracture of shells are taken into account with provision for plastic deformation of shell material, but behavior of liquid is described by equations of hydrodynamics with provision for boiling. The geometric features of elements of design assume the form $R_1 = 0.5$ m; $R_2 = 0.75$ m; $h_1 = h_2 = 0.005$ m. the amplitude of load is varied. The compared results of calculations springy and elastic-plastic deformation of internal and external shells, interacting with metastable liquid with features $P_0 = 0.1$ MPa ; 0.982 g/sm^3 – density ; $T = 293^0$ K . it is expected that temperature of surrounding ambiences changes the plastic features of material of shell (for instance, limit of plasticity toughness): 200; 300; 400 MPa . The amplitude of load form is 30 MPa. Also have conducted the calculations for liquid with the following initial features: $P_0 = 10$ MPa ; 0.7505 g/sm^3 – density ; $T = 555{,}22^0$ K .

RESULT AND DISCUSSION

Also have conducted the counted calculations plastic thinning thickness shells made from different material. Compared deflection internal (a) and external (b) of shell at moments of time 0,5; 1; 1,5; 2; 2,5; 3 ms with features: 400 MPa (limit of plasticity of material of shell), 600 MPa (limit of toughness of material of shell) for steel (E_1=500 MPa – module hardening; *a* - 0,72; 0,51; 0,52; 0,73; 0,62 sm; *b* - 0,20; 0,46; 0,50; 0,58; 0,49 sm), D16AT (100 MPa – limit of plasticity, 350 MPa – limit of toughness; E_1=1500 MPa – module hardening; *a* - 0,91; 1,2; 1,4; 1,7; 1,8 sm; *b* - 0,40; 1,15; 1,75; 2,35; 2,36 sm) and BrKMS (200 MPa – limit of plasticity, 400 MPa – limit of toughness; E_1=1000 MPa – module hardening; *a* - 0,78; 0,72; 0,95; 1,22; 1,15 sm; *b* - 0,25; 0,92; 0,98; 1,21; 1,24 sm), as well as for steels with provision for its temperature (200 MPa – limit of plasticity, *a* - 0,78;

0,79; 1,15; 1,30; 1,25 sm; *b* - 0,27; 0,98; 1,12; 1,37; 1,41 sm) at intensities of load 50 MPa. Plastic thinning thickness of shells begins in frontal point and hereinafter it spreads from this points. The reduction of intensity of load brings to progressive approximation of elastoplastic deformed form of shells.

CONCLUSION

From these results follows that increase of temperature surrounding ambiences and account of the temperature the most designs, as well as reduction of toughness features of material of shells brings to intensive growing elastic-plasticity thinning thickness walls of reservoir and the following fracture of design. Thereby, account of influence of the temperature of designs – mast condition of dynamic calculation thermohydroelastoplastic systems. Developed counted strategy and gotten results allow to study the wave processes in hydroelastoplastic systems.

REFERENCES

Abdirashidov A., Galiev Sh.U. (1988). Effect of bubbling-up of a liquid on pulsed deformation of the tank walls//Strength of Materials. №2, pp. 259-264.

U.Galiev Sh. Nonlinear waves in bounded continue. Naukova dumka. Kiev, 263 pp, 1988 (in Russian).

THE CAUCHY PROBLEM FOR THE HELMHOLTZ EQUATION

Abdukarimov A.
Samarkand branch of Tashkent University of information Technology
abdukarimov54@mail.ru
Islomov I.
Department of Mechanics and Mathematics, Samarkand State University

Let $x = (x_1, x_2)$ and $y = (y_1, y_2)$ of the point two-dimensional Euclidean space R^2. We shall consider the Helmholtz equation

$$\Delta U + \lambda^2 U = 0, \tag{1}$$

where Δ - is Laplace operator, $\lambda > 0$.

By D_ρ we denote bounded domain with boundary, consisting of rays pieces $|y_1| = \tau y_2, \tau = tg\frac{\pi}{2\rho},\ \rho > 1,\ 0 < y_2 \le y_0 < +\infty$, with origin in zero, and smooth curve *S*, lying inside angle $\frac{\pi}{\rho}$.By $H(D_\rho)$ we denote the space of the solution of equation (1) in D_ρ. If $U(y) \in H(D_\rho) \cap C^1(\overline{D}_\rho)$, where $\overline{D}_\rho = D_\rho \cup \partial D_\rho$, then true the Green formula

$$U(x) = \int_{\partial D_\rho} \left\{ \Phi \frac{\partial U}{\partial n} - U \frac{\partial \Phi}{\partial n} \right\} ds, \qquad x \in D_\rho, \tag{2}$$

where n – is unique external normal, Φ - is elementary solution of the equation (1).

In the formula (2) we substitute $\Phi(y,x)$ by

$$\Phi_\sigma(y - x, \lambda) = \frac{1}{2\pi} \int_0^\infty Im \left[\frac{E_\rho\,[\sigma(w - x_2)\exp\,[(w - x_2)]^2\,]}{w - x_2} \right] \frac{I_0(\lambda u)\,u}{\sqrt{u^2 + \alpha^2}} du,$$

$\sigma > 0, \rho > 1, w = i\sqrt{u^2 + \alpha^2} + y_2$, where $E_\rho(z)$- the whole function Mittag-Leffler [2].

Theorem 1. *Let* $U(y) \in H(D_\rho) \cap C^1(\bar{D}_\rho), U(y) = f(y), \frac{\partial U}{\partial n}(y) = g(y),$

$y \in S$, *where* $f, g \in C^1(S)$ *is given functions on S and*

$$|U(y)| + \left|\frac{\partial U}{\partial n}\right| \leq 1, \; y \in \partial D \backslash S, \tag{3}$$

$$U_\sigma(x) = \int_S \left[U \frac{\partial \varphi_\sigma}{\partial n} - \varphi_\sigma \frac{\partial U}{\partial n} \right] ds, \qquad x \in D_\rho. \tag{4}$$

Then the following inequality is true

$$|U(x_0) - U_\sigma(x_0)| \leq C_{\lambda,\rho}\, exp(-\sigma\gamma), x_0 = (0, x_2) \in D_\rho, \tag{5}$$

where $C_{\lambda,\rho}$ - constant dependens from λ and ρ.

Theorem 2. *Let* $U(y) \epsilon H(D_\rho) \cap C^1(\bar{D}_\rho)$ *satisfies to the boundary condition (3) on the* ∂D_ρ. *Then*

$$|U(x_0) - U_\sigma(x_0)| \leq C_{\lambda,\rho}\,(x_0) exp(-\sigma\gamma), \quad x_0 = (0, x_2) \epsilon D_\rho,$$

where

$$C_{\lambda,\rho}(x_0) = C_{\lambda,\rho} \cdot \int_S \frac{ds}{r}, \quad r = |y - x|, \gamma = \tau x_2 > 0, \; \sigma \geq \sigma_0 > 0. \tag{6}$$

Let $U(y) \in H(D_\rho) \cap C^1(\bar{D}_\rho)$ and instead of $U(y)$, $\frac{\partial U(y)}{\partial n}$ on S are given their approximation f_δ, g_δ :

$$\max_s |U - f_\delta| + \max_s \left| \frac{\partial U}{\partial n} - g_\delta \right| \leq \delta, \; 0 < \delta < 1.$$

We define σ *and* R by the following equalities

$$\sigma = \tau^{-\rho} R^{-\rho} \ln \delta^{-1}, \; R = \max_{s} \sqrt[\rho]{\mathrm{Re}\left(i|y_1| + y_2\right)^{\rho}} \; .$$

Theorem 3. Let $U \in H\left(D_\rho\right) \cap C^1\left(\bar{D}_\rho\right)$ *and* U *on* ∂D_ρ *satisfies the condition (3).Then*

$$\left|U\left(x_0\right) - U_{\sigma\delta}\left(x_0\right)\right| \le C_{\lambda,\rho}\left(x_0\right)\delta^{\frac{x_2}{R}}, \quad x_0 \in D_\rho,$$

where $C_{\lambda,\rho}\left(x_0\right)$ *is defined from (6),*

$$U_{\sigma\delta}\left(x\right) = \int_{s^*} \{\mathrm{f}_\delta \frac{\partial L_\sigma}{\partial \mathrm{n}} - \mathrm{g}_\delta L_\sigma\}\mathrm{ds}, \quad \mathrm{x} \in D_{\rho.}$$

$L_\sigma\left(x, y\right)$ - a elementary solution of the equation (1) with special characteristic.

REFERENCES

[1] Yarmuhamedov Sh. About continuation of the solution of the Helmholtz equation //DAN 1997, t. 357, 3, 320-323p.

[2] Dzjarbashyan M.M. The integral representation of functions in complex domain. M.: Science, 1966.

ASYMPTOTICS OF EIGENVALUES OF A SYSTEM OF TWO BOSONS ON A LATTICE

Abdullaev J. I.
jabdullaev@mail.ru

INTRODUCTION.

Spectral properties of the two-particle Hamiltonian $H(k) = H_0(k) - V, \quad k \in \mathbb{T}^3$ on a lattice are investigated in [1,2]. There is considered the two-particle Hamiltonian $H(k), \quad k \in \mathbb{T}^3$ with the contact potential $\hat{v}(n_1 - n_2) = \mu\delta_{n_1 n_2}$ and established the uniqueness of eigenvalue of the Hamiltonian $H(k)$. It is proved that the eigenvalue $z(k) = z(k_1, k_2, k_3)$ is symmetric function, even for each $k_i \in [-\pi, \pi], i \in \{1,2,3\}$ and increasing for any $k_i \in [0, \pi], i \in \{1,2,3\}$ ([2]). The existence positive eigenvalues below the band spectrum for nontrivial values of the quasi-momentum $k \in \mathbb{T}^3 \equiv (-\pi, \pi]^3$, provided that the Hamiltonian $H(0)$ has a zero energy resonance. It is shown that in [3], the Hamiltonian $H(k)$ has infinitely many eigenvalues lying below the band, if the width $\omega_j(k)$ of the band in the direction e_j vanishes for some $j \in \{1,2,3\}$. The Hamiltonian $H(k)$ has only finite number eigenvalues outside of the band spectrum if $k \in (-\pi, \pi)^3$ and the potential $\hat{v}$ satisfied the condition $|n|^{3+\kappa} \; \hat{v}(n) \to 0, \kappa > 1/2$ as $n \to \infty$ (see. [3]).

In this paper we will consider the fiber Hamiltonian $H(k)=H_0(k)-V, k\in\mathbb{T}^3$ (see. (2)) of a system of two bosons with a general potential $\hat{v}.$ We shall study the discrete spectrum of the Hamiltonian $H(k)$ and show infiniteness the number of eigenvalues. The problem on existence of a solution of the Schrödinger equation $H(k)f=zf$ is reduced to the existence of the motionless points of the compact operator $G(k,z)=V^{\frac{1}{2}}(H_0(k)-z)^{-1}V^{\frac{1}{2}}.$ We describe a set $M(j)\subset\mathbb{T}^3, j\in\{1,2,3\}$ (see. (3)) which is $\omega_j(k)\equiv 0$ and a class of potentials $W(j), j\in\{1,2,3\}$ (see. (4)) such that for any $(k,\hat{v})\in M(j)\times W(j)$ the Hamiltonian $H(k)$ has infinite number of eigenvalues $z_n(k), n\in\mathbb{Z}_+,$ and we obtain the asymptotic formula for the eigenvalues $z_n(k)$ as $n\to\infty$.

DESCRIPTION OF THE TWO-PARTICLE OPERATOR.

The total Hamiltonian $\hat{H}$ of a system of two bosons on a three-dimensional lattice $\mathbb{Z}^3$ acts on the Hilbert space $\ell_2^{sym}((\mathbb{Z}^3)^2)=\{f\in\ell_2((\mathbb{Z}^3)^2):\ f(n,m)=f(m,n)\}$ by $\hat{H}=\hat{H}_0-\hat{V}_2,$ where the free Hamiltonian $\hat{H}_0$ acts on $\ell_2^{sym}((\mathbb{Z}^3)^2)$

$$\hat{H}_0=-\frac{1}{2m}\Delta_{x_1}-\frac{1}{2m}\Delta_{x_2}.$$

Here m is a mass of bosons, which is assumed to be unity in what follows, $\Delta_{x_1}=\Delta\otimes I$ and $\Delta_{x_2}=I\otimes\Delta,$ the Laplacian Δ is a difference operator which describes the transport of a particle from a site to the nearest neighboring site, i.e.

$$(\Delta\hat{\psi})(x)=\sum_{j=1}^{3}[\hat{\psi}(x+e_j)-2\hat{\psi}(x)+\hat{\psi}(x-e_j)],\quad \hat{\psi}\in\ell_2(\mathsf{Z}^3),$$

where e_j being the unit vector along the j-th direction in $\mathbb{Z}^3.$ Interaction between of two bosons is described by the operator $\hat{V}_2$

$$(\hat{V}_2\hat{\psi})(x_1,x_2)=\hat{v}(x_1-x_2)\hat{\psi}(x_1,x_2),\quad \hat{\psi}\in\ell_2^{sym}((\mathsf{Z}^3)^2).$$

Throughout this paper we assume that the potential $\hat{v}(x)$ satisfies the condition

$$\hat{v}(x)=\hat{v}(-x)\geq 0, \forall x\in\mathsf{Z}^3,\quad \sum_{x\in\mathsf{Z}^3}\hat{v}(x)<\infty. \tag{1}$$

Under assumption (1) the operator $\hat{V}_2$ is a bounded self-adjoint operator on $\ell_2^{sym}((\mathbb{Z}^3)^2).$ Consequently the two-particle Hamiltonian $\hat{H}$ is also bounded self-adjoint operator.

The transition to the momentum representation is performed by the Fourier transform $\mathsf{F}: L_2((\mathsf{T}^3)^2)\to\ell_2((\mathsf{Z}^3)^2).$ The Hamiltonian $H=\mathsf{F}^{-1}\hat{H}\mathsf{F}$ of a system of two bosons in the momentum representation acts on the Hilbert space $L_2^{sym}((\mathbb{T}^3)^2)$ and commute with the group of unitary operators $U_s, s\in\mathbb{Z}^3$:

$$(U_s f)(k_1,k_2)=\exp(-i(s,k_1+k_2))f(k_1,k_2),\quad f\in L_2^{sym}((\mathbb{T}^3)^2).$$

Therefore the operators U_s and H are decomposed into the direct integrals

$$U_s = \int_{\mathbb{T}^3} \oplus U_s(k)dk, \quad H = \int_{\mathbb{T}^3} \oplus H(k)dk.$$

The fiber Hamiltonian $H(k) = H_0(K) - V$ acts on $L_2^e(\mathbb{T}^3) = \{f \in L_2(\mathbb{T}^3) : f(-q) = f(q)\}$ by

$$(H(k)f)(q) = \varepsilon_k(q) f(q) - (2\pi)^{-\frac{3}{2}} \int_{\mathbb{T}^3} v(q-s) f(s)ds. \tag{2}$$

The unperturbed fiber Hamiltonian $H_0(k)$ is operator multiplication of the function

$$\varepsilon_k(q) = \varepsilon(\frac{1}{2}k+q) + \varepsilon(\frac{1}{2}k-q), \quad \varepsilon(q) = \sum_{j=1}^{3}(1-cosq_j).$$

The interaction operator V is the integral operator on $L_2^e(\mathbb{T}^3)$ with the kernel $(2\pi)^{-3/2} v(q-s)$. Under assumption (1) the operator V is positive and belongs to the trace class Σ_1.

EIGENVALUES OF THE HAMILTONIAN $H(k)$

Here we are interested in the discrete spectrum of the operator $H(k)$ and obtain asymptotic formula for the eigenvalues $z_n(k)$ as $n \to \infty$. We recall some known facts. We denote by $m(k), M(k)$, the minimal and maximal values of the function $\varepsilon_k(q)$ respectively. Under assumption (1) the perturbation V of the operator $H_0(k)$ is a trace class operator. Therefore in accordance with invariance of the absolutely continuous spectrum under the trace class perturbations (see [4]) the absolutely continuous spectrum of the operator $H(k)$ fills in the following interval on the real axis

$$\sigma_{\mathrm{ac}}(H(k)) = \sigma(H_0(k)) = [m(k), M(k)].$$

The length of segment $\omega(k) = M(k) - m(k)$ is called the width of the band spectrum of $H(k)$ and it is equal to

$$\omega(k) = 4\cos\frac{k_1}{2} + 4\cos\frac{k_2}{2} + 4\cos\frac{k_3}{2}.$$

The width $\omega(k)$ of the band is symmetric function with respect to the permutation of k_i and k_j, even for each $k_j \in [-\pi, \pi]$ and decreasing on $k_j \in [0, \pi], j \in \{1,2,3\}$. Consequently

$$\min_{k \in \mathbb{T}^3} \omega(k) = \omega(\pi, \pi, \pi) = 0, \quad \max_{k \in \mathbb{T}^3} \omega(k) = \omega(0,0,0) = 12.$$

We also introduce $w_j(k), j \in \{1,2,3\}$ the width of the band spectrum of $H(k)$ for directions e_j by

$$\omega_j(k) = \max_{p_j \in [-\pi,\pi]} \varepsilon_k(p) - \min_{p_j \in [-\pi,\pi]} \varepsilon_k(p).$$

Note that

$$\omega_j(k) = 4\cos\frac{k_j}{2} \quad \textit{and} \quad \omega(k) = \omega_1(k) + \omega_2(k) + \omega_3(k).$$

For any $k \in \mathbb{T}^3$ and $z < m(k)$ we define integral operator $G(k,z) = V^{\frac{1}{2}} r_0(k,z) V^{\frac{1}{2}}$, where $r_0(k,z)$ is the resolvent of the unperturbed Hamiltonian $H_0(k), V^{\frac{1}{2}}$ is the positive square root of the operator $V \geq 0.$ Under assumption (1) the operator $V^{\frac{1}{2}}$ belongs to the Hilbert-Schmidt class Σ_2, therefore $G(k,z)$ belongs to the trace class Σ_1 for all $k \in \mathbb{T}^3$ and $z < m(k)$. The solutions f of the Schrödinger equation

$$H(k) f = zf$$

and motionless points φ of the operator $G(k,z)$ are connected by the parity

$$f = r_0(k,z) V^{\frac{1}{2}} \varphi, \qquad \varphi = V^{\frac{1}{2}} f.$$

Since the operator $H(k)$ is self-adjoint the spectrum $\sigma(H(k))$ is subset of $\mathbb{R}$. It follows from positivity of V that $\sigma(H(k)) \cap (M(k), \infty) = \varnothing$ and therefore $\sigma_{disc}(H(k)) \subset (-\infty, m(k))$.

Lemma. *A number $z < m(k)$ is eigenvalue of the operator $H(k)$ if and only if the number $\lambda = 1$ is eigenvalue of the operator $G(k,z)$.*

We put

$$M(j) \equiv \{k \in \mathbb{T}^3 : k_j = \pi\} := \{k \in \mathbb{T}^3 : \omega_j(k) = 0\}, \quad j \in \{1,2,3\}. \tag{3}$$

Denote by $W(j), j \in \{1,2,3\}$ a class of potentials $\hat{v}$ such that

$$W(j) = \{\hat{v} : \operatorname{supp} \hat{v} \subset e_j \mathsf{Z} \quad \textit{and} \quad \hat{v}(ne_j) > \hat{v}((n+1)e_j), n \in \mathsf{Z}_+\}. \tag{4}$$

Theorem 1. *Let $(k,\hat{v}) \in M(j) \times W(j)$. Then the operator $H(k)$ has infinite number eigenvalues $z_n(k) < m(k), n \in \mathbb{Z}_+$. Them only $z_0(k)$ is simple and the others $z_n(k), n \in \mathbb{N}$ are of multiplicity two. In addition the corresponding eigenfunctions f_0 and $f_n^{\pm}, n \in \mathbb{N}$ to the eigenvalues $z_0(k)$ and $z_n(k), n \in \mathbb{N}$ are equals to*

$$f_0(p) = \frac{1}{\varepsilon_k(p) - z_0(k)} \quad \textit{and} \quad f_n^{\pm}(p) = \frac{e^{\pm inp_j}}{\varepsilon_k(p) - z_n(k)}, \quad n \in \mathbb{N}.$$

Theorem 2. *Let $(k,\hat{v}) \in M(1) \times W(1)$ and $(k_2,k_3) \in (-\pi,\pi)^2$. Then the operator $H(k) = H(\pi,k_2,k_3)$ has infinite number eigenvalues $z_n(k) = z_n(\pi,k_2,k_3) < m(k), n \in \mathbb{Z}_+$ having asymptotics*

$$z_n(k) = m(k) - c(k_2, k_3)\exp\{-\frac{2\sqrt{\cos\frac{k_2}{2}\cos\frac{k_3}{2}}}{\hat{v}(ne_1)}\}[1+o(1)] \quad as \quad n \to \infty,$$

where $c(k_2, k_3)$ is a positive continuous function in $(-\pi;\pi)^2$.

REFERENCES

[1] P.A. Faria da Viega, L.Ioriatti and M.O'Carrol. Energy-momentum spectrum of some two-particle lattice Schrödinger Hamiltonian. Phys. Rev. E(3) **66**, 016130, 9 pp. (2002).

[2] J.I. Abdullaev, S.N. Lakaev. Asymptotics of the discrete spectrum of the three-particle Schrodinger difference operator on a lattice, Theoretical and Mathematical Physics, **136**, 2, 1096-1109. (2003).

[3] S. Albeverio, S.N. Lakaev and J.I.Abdullaev. On the spectral properties of two-particle discrete Schrodinger operators. Preprint, Bonn, Bonn University, pp.14, 2004.

[4] M. Reed and B. Simon. Methods of modern mathematical physics. IV: Analysis of operators, Academic Press, New York, 1978.

SOLUABLITY OF NON-LINEAR POLYSINGULAR INTEGRAL EQUATION IN THE SPACE $\tilde{Z}_\omega^p$

Absalamov T.

(Samarkand)

We prove soluablity of non-linear polysingular integral equation

$$u(x_1, x_2,..., x_n) = \lambda\int_{a_1}^{b_1}\int_{a_2}^{b_2}...\int_{a_n}^{b_n}\frac{f(s_1, s_{2,},..., s_n, u(s_1, s_2,..., s_n))}{\prod_{k=1}^{n}(s_k - x_k)}ds_1ds_2...ds_n \qquad (1)$$

by using approximation sequence method in the $\tilde{Z}_\omega^p$ (see [1] for the definition of $\tilde{Z}_\omega^p$), where $f(s_1, s_2,..., s_n, u)$ is defined on $(a_1,b_1)x(a_2,b_2)x...\times(a_n,b_n)x(-\infty;+\infty)$ and λ is a real parameter.

Lemma 1. Let $f(s_1, s_2,..., s_n, u)$ be a function satisfying the conditions:

1. for almost every $s_k \in (a_k, b_k)$, $k = \overline{1,n}$ and for any

$$u_1, u_2 \in (-\infty;+\infty), \left| f(s_1, s_2,..., s_n, u_1) - f(s_1, s_2,..., s_n, u_2)\right| \le D\left|u_1 - u_2\right|,$$

where D is a positive constant;

2. $f(s_1, s_2, ..., s_n, 0) \in \tilde{Z}^p_\omega$. Then,

   a) the operator $(fu)(s_1, s_2, ..., s_n) = f(s_1, s_{2,}, ..., s_n, u(s_1, s_2, ..., s_n))$ acts in $\tilde{Z}^p_\omega$,

   b) for any $u_1, u_2 \in \tilde{Z}^p_\omega$, $\|fu_1 - fu_2\|_{\tilde{Z}^p_\omega ,} D\|u_1 - u_2\|_{\tilde{Z}^p_\omega}$.

Consider the following operators:

$$(Bu)(x_1, x_2, ..., x_n) = \int_{a_1}^{b_1}\int_{a_2}^{b_2}...\int_{a_n}^{b_n} \frac{f(s_1, s_{2,}, ..., s_n, u(s_1, s_2, ..., s_n))}{\prod_{k=1}^{n}(s_k - x_k)} ds_1 ds_2 ... ds_n ,$$

$$(Av)(x_1, x_2, ..., x_n) = \int_{a_1}^{b_1}\int_{a_2}^{b_2}...\int_{a_n}^{b_n} \frac{v(s_1, s_2, ..., s_n)}{\prod_{k=1}^{n}(s_k - x_k)} ds_1 ds_2 ... ds_n .$$

Lemma 2. Let $f(s_1, s_2, ..., s_n, u)$ be a function satisfying the conditions 1) and 2) of lemma 1. Then,

a) $B : \tilde{Z}^p_\omega \to \tilde{Z}^p_\omega$

b) for any $u_1, u_2 \in \tilde{Z}^p_\omega$, $\|Bu_1 - Bu_2\|_{\tilde{Z}^p_\omega} \le D\|A\|_{\tilde{Z}^p_\omega}\|u_1 - u_2\|_{\tilde{Z}^p_\omega}$

holds.

Proof. The first part of lemma 2 follows from lemma 1 and theorem [1] on invariantness property of $\tilde{Z}^p_\omega$ with respect to bisingular operator A.

Let's prove the second part of lemma 2. By using the lemma 1 and the equality $Bu = Afu$, where

$$\|Bu_1 - Bu_2\|_{\tilde{Z}^p_\omega} = \|Afu_1 - Afu_2\|_{\tilde{Z}^p_\omega} \le \|A\|_{\tilde{Z}^p_\omega}\|fu_1 - fu_2\|_{\tilde{Z}^p_\omega}$$
$$\le D\|A\|_{\tilde{Z}^p_\omega}\|u_1 - u_2\|_{\tilde{Z}^p_\omega}.$$

By using lemma 2 and contraction maps principal follows.

Theorem 1. Let $f(s_1, s_2, ..., s_n, u)$ be a function satisfying the conditions 1) and 2) of lemma 1.

If $|\lambda| < \dfrac{1}{D\|A\|_{\tilde{Z}^p_\omega}}$, the equation (1) has a unique solution in $\tilde{Z}^p_\omega$. The solution can be find by using method of contraction maps begining from any element of $\tilde{Z}^p_\omega$.

By using the boundedness property of the operator A ([2]) in *Lp(* ρ *)* can be proved the following theorem 1.

Theorem 2. Let $f(s_1, s_2, ..., s_n, u)$ be a function satisfying the conditions 1) of lemma 1 and

$$f(s_1, s_2, ..., s_n, 0) \in \tilde{Z}_{\omega}^{p}.$$

If

$$|\lambda| < \frac{1}{D\|A\|_{\tilde{Z}_{\omega}^{p}}},$$

then the equation (1) has a unique solution *Lp(* ρ *)* and the solution can be find by using contraction sequence method begining from any element *Lp(* ρ *)*. The sequence converges in metric of *Lp(* ρ *)*.

REFERENCES

[1] Absalamov T., Goziev A. Multiple singular integral in the space of summable functions. Abstracts of talks, Urgench -1995.

[2] Fefferman R./ Ap weights and singular integrals // Amer. F. Math.//, 1998, 110, 5, p. 975-987.

NEURO-FUZZY SYSTEM OF THE TEXTS MISTAKES CORRECTION ON THE BASIS OF PROCEDURES OF PARALLEL COMPUTING

Akhatov A.R.,.Zaripova G.I

Samarkand state university
akmalar@rambler.ru

One of the perspective ways to develop systems for processing text information for detecting and correcting errors is the use of neural networks (NN) and the concept of parallel computing. The effectiveness of the proposed approach is particularly enhanced when combined capacity of NN models and of fuzzy inference methods, which allows to take into account the conditions of incompleteness, inconsistency and misrepresentation of information [1]. Unlike traditional spelling control systems, neuro-fuzzy system (NFS) allows you to control the accuracy of information in teleprocessing mode, where during the solution of control tasks as objects are considered transmitted signal characteristics of the text elements image and the problem of text elements images recognition and dynamic filtering are solving.

In this paper we presented results of developing the basic approaches and the conceptual principles of designing the neuro-fuzzy system of signal characteristics control for correcting errors in text messages based on semantic hypernetwork through the use of procedures and technologies of parallel computing [2].

Let's assume that the NFS for detecting and correcting errors in texts is described in a fuzzy environment by following parameters:

- vector of states (input parameters) $X = \{x_i\}$, $i = 1, n$;
- external perturbations $Z = \{z_j\}$, $j = 1, k$;
- output parameters $Y = \{y_p\}$, $p = 1,1$;
- managing influences $U = \{u_q\}$, $q = 1, m$..

Input vector characterizes the quantitative or qualitative characteristics of the object and their condition. External disturbances - is difficult to predictable situations, factors, or noises affecting to the transmission channels of information. Sources of initial information of system may be a system of documents circulating in the industry.

To control the authenticity of the information the system uses fuzzy data and developer designes the alleged actions of the operator, consider the nature, patterns and models of external exposure factors. In the process of solving problems of controlling the accuracy of information, NFS is based on a formal knowledge base (KB) containing the declarative and procedural components.

The output of NFS is based on algorithms of formalization of fuzzy inference in accordance with the rules <IF X , THEN Y, ELSE Z> to implement the execution of a conditional fuzzy operator of system:

$$\Phi(X \to Y(Z)) = \{\mu_x / \Phi(Y),\ (1-\mu_x)/\Phi(z)\},$$

where X, Y, Z are fuzzy sets defined on universal sets U, V, W ($X/U, Y/V, Z/W$); μ_x is membership function (MF), characterizing the degree of truth for condition X .

Suggested combination of NN's and fuzzy logic models enabled to conduct the global optimization at training of NFS, to implement the adaptation of NN's parameters on the basis of statistical and dynamical properties of the images signal characteristics.

Semantic hypernetwork that is based on NFS, includes mechanisms for generalizing groups of algorithms that extract hidden regularuties from the data flow into a database and knowledge base. Peculiarity of the designed NFS were algorithms implementation of the nonlinear mapping of previous signal values over time and implementation to adapt the placement of information in memory using the standard and specially developed procedures of parallel computing based on GPU models and CUDA technologies.

The proposed algorithms also allow: to optimize the choice of the rational architecture of NN; to find the parameters used in the adaptation for achieve the given accuracy of approximation of signals and their robustness; to provide sustainability of NN learning under various conditions of noise exposure and processing of information.

The developed system based on fuzzy semantic hypernetwork consists of four main units: fuzzifikator, fuzzy knowledge base, mechanisms and rules of fuzzy inference and defuzzifikator.

During the testing of system as input continuous variables were examined the frequency - amplitude characteristics (AFC), values of which were semantically represented in a view of linguistic variables. For fuzzy sets we conducted characterization and determined the appropriate membership function.

Fuzzifikator of system involves complex of procedures and algorithms that constitute the sequence of measuring transformations of the input, where the optical amplifier produces a bitmap raster signal coupling with modal characteristics.

Discretization of continuous input influence is performed in an optical encoder. Transmitted signal is modeled as an element with a time lag. Static characteristics of fuzzifikator corresponding equation in operator form. Fuzzifikators error is evaluated by determining the boundaries of accuracy of signal characteristics reproduction and by determining MF corresponding them.

To improve the efficiency of direct conversion of physical quantities to the fuzzy values (sets) we developed sensors with fuzzy output and new software procedure, based on the use of opportunities CUDA technology. The effectiveness of the implementation of the system increases by using of specially oriented software which were represents the fragments of expert systems and implementations in a view of NN.

REFERENCES

[1] Holland J.H. Adaptation in natural and artificial systems. An introductory analysis with application to biology, control and artificial intelligence. - London: Bradford book edition, 1994. - 211 p.

[2] Akhatov A.R., Jumanov I.I., Fuzzy Semantic Hypernet for Information Authenticity Controlling in Electronic Document Circulation Systems//4-th International Conference AICT-2010, Tashkent, 2010. p.1-5.

FINITENESS OF DISCRETE SPECTRUM OF THREE-PARTICLE SCHRÖDINGER OPERATORS ON LATTICE

Aliyev N.

Samarkand State University

In this paper we consider a system of three-particles, which move in the three-dimensional integer lattice Z^3 and interact through a pair zero-range potentials. The Hamiltonian of the system acts in $\ell_2((Z^3)^3)$ by form

$$(H\psi)(n_1,n_2,n_3) = H_0\psi(n_1,n_2,n_3) - [\mu_1\delta_{n_2n_3} + \mu_2\delta_{n_2n_3} + \mu_3\delta_{n_1n_2}]\psi(n_1,n_2,n_3)$$

where $H_0 = \frac{1}{2m_1}\Delta\otimes I\otimes I + \frac{1}{2m_2} I\otimes\Delta\otimes I + \frac{1}{2m_3} I\otimes I\otimes\Delta$, I is identical operator, Δ is the lattice Laplacian, $m_\alpha > 0$ is the mass of the particle α, $\mu_\alpha > 0$ and δ_{mn} is the Kronecker delta.

Using the Fourier transform and the decomposition into the direct operator integrals we reduce the investigation of the spectral properties of the operator H to the analysis of the family of self-adjoint, bounded operators (the three-particle discrete Schrödinger operators) $H(K)$, $K \in T^3$, acting in the Hilbert space $L_2((T^3)^2)$ (where T^3 is a three-dimensional torus) by form

$$H(K) = H_0(K) - V, \quad V = \mu_1 V_1 + \mu_2 V_1 + \mu_3 V_3,$$

$$(V_1 f)(p,q) = \frac{1}{(2\pi)^3} \int_{T^3} f(p,s)ds, \quad (V_2 f)(p,q) = \frac{1}{(2\pi)^3} \int_{T^3} f(s,q)ds$$

$$(V_1 f)(p,q) = \frac{1}{(2\pi)^3} \int_{T^3} f(s, p+q-s)ds,$$

Where $H_0(K)$ is operator multiplication by function

$$E_K(p,s) = \frac{1}{m_1}\varepsilon(p) + \frac{1}{m_2}\varepsilon(q) + \frac{1}{m_3}\varepsilon(K-p-q), \quad \varepsilon(p) = \sum_{i=1}^{3}(1-\cos p_i).$$

Let $m_K = \min_{p,s} E_K(p,s)$, $M_K = \max_{p,s} E_K(p,s)$.

Lemma 1. *Let* $m_K = E_K(p_K, q_K)$. *Then the point* (p_K, q_K) *is non-degenerative minimum point of* $E_K(\cdot,\cdot)$.

By lemma 1 the integral $\int_{T^3} \frac{ds}{E_K(s,p) - m_K}$ converges.

We set $\mu^* = \sup_{p,K}[\frac{1}{8\pi^3}\int_{T^3}\frac{ds}{E_K(s,p)-m_K}]^{-1}$. Analyzing as in [1] we get

Lemma 2. Let $\mu_\alpha < \mu^*$, $\alpha = 1,2,3$. Then for the essential spectrum $\sigma_{ess}(H(K))$ of $H(K)$ the equality $\sigma_{ess}(H(K)) = [m_K, M_K]$ holds.

Using methods of integral equations and analyzing as [2] we obtain

Theorem. *For any fixing* $\mu_\alpha < \mu^*$, $\alpha = 1,2,3$ *and fixing* $K \in T^3$ *discrete spectrum of* $H(K)$ *is finite set.*

REFERENCES

[1] Albeverio Sergio, Saidakhmat N. Lakaev and Zahriddin I. Muminov. Schrodinger Operators on Lattices. The Efimov Effect and Discrete Spectrum Asymptotics. Journal Annales Henri Poincare. Volume 5, 2004, 743-772.

[2] Muminov M. I. Formula for the number of eigenvalues of a three-particle Schro"dinger operator on a lattice Teoret. Mat. Fiz., 164:1 (2010), 46–61 4.

ON SIMULTANEOUS REPRESENTATION OF TWO NATURAL NUMBERS BY SUM OF THREE PRIMES

Allakov I., Abrayev B.

Termez State University, Uzbekistan
iallakov@mail.ru, babrayev@mail.ru

Let a_{ij} $(i = 1,2;\ j = 1,2,3)$, b_1, b_2 be integer numbers and p_1, p_2, p_3 be primes. Consider the problem of solvability of the system

$$b_i = a_{i1}p_1 + a_{i2}p_2 + a_{i3}p_3,\ i = 1,2 \tag{1}$$

under conditions:

a) for arbitrary prime p there exist such integers l_1, l_2, l_3, $1 \le l_1, l_2, l_3 \le p-1$, which satisfy the system of the linear congruence:

$$a_{i1}l_1 + a_{i2}l_2 + a_{i3}l_3 \equiv b_i (mod p),\ i = 1,2;$$

b) there exist such real positive numbers y_1, y_2, y_3, for which the equalities:

$$a_{i1}y_1 + a_{i2}y_2 + a_{i3}y_3 = b_i,\ i = 1,2.$$

are satisfied.

Set $B = 3\max|a_{ij}|$, $i = 1,2;$ $j = 1,2,3$. Let $U(X)$ be the set of pairs $\vec{b} = (b_1, b_2)$, $1 \le b_1, b_2 \le X$, which satisfy the conditions a) and b). Let $M(X)$ be the set pairs $\vec{b} = (b_1, b_2) \in U(X)$, which cannot be represented in the form (1). $E(X)$=card $M(X)$ and $J(\vec{b})$ - numbers of solutions of the equation (1).

$$X \ge B^{\exp(10\delta^{-2})},\quad N = 18B^3X,\quad Q = N^{\delta},\quad T = Q^{c_2},\quad L = NQ^{-c_1},$$

where $c_1 = \frac{1}{6} + 0{,}1\delta$, $c_2 = \frac{100}{3} + 4\varepsilon$, $0 < \varepsilon < 1$. From (2) it follows $B < Q^{0{,}1\delta}$.

In this paper the following theorem is proved:

Theorem 1. *If X is a sufficiently large positive integer and* δ *(* $0<\delta<0{,}0001169$*) is a sufficiently small positive real numbers, then for all* $\vec{b} \notin M(X)$, $\vec{b} \in U(X)$ *with the exception of at most* $E(X) < X^{2-\frac{1}{25}\delta}$ *from them, we have*

$$J(\vec{b}) > \frac{\left(9\sqrt{2}B^3|\vec{b}|\right)^{1-\frac{\delta}{25}}}{loq\left(9\sqrt{2}B^3|\vec{b}|\right)^4}.$$

The result of this theorem is generalized I. Allakov's [1] and Wu Fang's [2] results. The conditions a) and b) correspond to the conditions of congruence solvability and of the positive solvability in Tarry's problem.

In the proof of theorem, the scheme of proof of the I.Allakov's paper is used [1].

REFERENCES

[1] Allakov I.A., Israilov M.I. About Simultaneous Representation of Two Natural Numbers by Sum of Three Primes // Computer Algebra in Scientific Computing, CASC-2000: Springer-Verlag Berlin Heidelberg New York. -2000. - P.13-20.

[2] Wu Fang. On the solutions of the systems of linear equations with prime variables // Acta Math. Sinica. - 1957. -№ 7. - P.102-121.

ABOUT ITEM RESPONSE THEORY MODELS

Arzikulov A. U.
Samarkand State University
abdukholik@rambler.ru

The modern theory is understood existing on West Item Response Theory (IRT), intended for an estimation latent (latent from direct supervision) parameters as examinees and parameters as tasks of the test by means of mathematics application - statistical models of measurement. As against the classical test theory for the IRT is aspiration to the fundamental theoretical approach and the correct solving many practical problems of pedagogical measurement. By the IRT is possible to predict probability of correct test performance any examinees in the sample group before presentation of the test performance results sample group the schoolboys to reveal efficiency various on difficulty of the tasks used for an estimation of knowledge, schoolboys, distinguished on preparation, by tested group.

The most significant advantage IRT usually concern are following:

1. Stability and objectivity of estimations of parameter describing a level preparation of the
   examinees.
2. Stability and objectivity of estimations of parameter, difficulty of the tasks, their independence
   of properties to sample of the examinees the carrying out test.
3. Opportunity of measurement meaning(importance) of parameters of the examinees and task of
   the test in the same scale having properties interval.

The basic assumption IRT:

1. Exist (concealment parameters of the person inaccessible to direct supervision). In testing it is
   a level qualification of the examinee and level of difficulty of the task.
2. Exist indicator variable, connected with latent parameters accessible to direct observation. On values of indicator variable can assume about values of latent parameters.

3. Estimated latent parameter should be one-dimensional. It means, that, should measure knowledge only in one, precisely given, subject domain.

There are also other assumptions carrying special character and connected with mathematics - static method IRT for processing the empirical data.

The basic task IRT is the tranformation from indicator variable to latent parameters.

Within the framework of the basic assumption IRT the relation between latent parameters of the examinees and observable results performance of the test is established. At an establishment of relation it is important to understand, that the first reason are the latent parameters. If to speak more precisely, the interactions of two sets of values of latent parameters are caused observable results by performance of the test. Elements the first set is a values of latent parameter determining a level of preparation of the examinees. The second sets form values of latent parameter equal to difficulties of the tasks of the test. However in practice always to be put a return task: Under the answers of the examinees to the task of the test to estimate values of latent parameters. For the solving this problem is necessary to answer two questions. First is connected to a choice latent parameters relation θ and β. The idea of an establishment of a relation θ and β was declared by the Danish mathematics G. Rasch. Which has offered to enter it as differences $\theta - \beta$, assume the parameters θ and β are estimated in the same scale. Answer the second question, which is central in IRT, is connected to a choice of mathematical model for the description of considered relation between latent parameters and observable results of performance of the test.

Latent parameter are properties of the person inaccessible to direct supervision, for example, patriotic feeling, tolerance, level of knowledge and etc. It is possible to estimate value of latent parameter by the indicator. The main advantage of the indicator - its availability to direct supervision. Measuring importance of the indicator, we can estimate importance of latent parameter, to which it is connected. For example, the indicator can be the test tasks.

The indicator is some method of influence (question, test, task), connected with certain latent parameter, the reaction on which is accessible to direct observation.

Georg Rasch has assumed, what, level of examinees qualification θ and level of difficulty score β layout on one scale and they are measured in same units - logit. Argument of examinees success function is the difference $\theta - \beta$. If it is a difference positively and is great, the probability of achievement of success of the examinee in the tasks is accordingly high. If it is a difference is negative and is great on the module, the probability achievement of success of the examinee in the tasks will be low.

Let's discuss a question on a degree of suitability of models for the purposes of measurement of latent parameters. Characteristics of models Rasch, is that the characteristic curves are not crossed. It means if some of the tasks A is easy than tasks B, that this relation is constant in all interval of change of θ. Completely other view is observed for two and three parametrical models. The characteristic curves are crossed practical always to occur for two or three parametrical models.

Thus, only one-parametrical model Rasñh corresponds by the requirement showed to qualitative measuring tool. The model RASCH MEASUREMENT is most of all suitable for construction of the test, as measuring tool.

REFERENCES

Rasch G. Probabilistic Models for some Intelligence and Attainment Tests, 1960, Copenhagen, Denmark: Danish Institute for Educational Research.

Neiman Y. Hlebnikov V. Introduction in theory of modeling and parameterization of the tests. Moscow, 2000, pp. 168.

Kim V. Testing teaching results, Ussuriysk, 2007.

PROBLEM OF REGULARIZATION FOR GROWING POLYHARMONIC FUNCTIONS OF SOME CLASS.

Ashurova Z.R. Juraeva. N. Yu.

We shall result in this clause the theorem for some polyharmonic functions determined in a unlimited strip.

In the given work is discussed continuations polyharmonic of function $u(x)$, on its meanings, and meanings of its normal border S, derivative on a smooth part, of infinite area *D*.

Let R^m - material space, $x=(x_1,x_2,x_3,...,x_m)$, $y=(y_1,y_2,y_3,...,y_m)$, $x\in R^m, y\in R^m$,

$x'=(x_1,x_2,...,x_{m-1},0)$, $y'=(y_1,y_2,...,y_{m-1},0)$, $r=|x-y|$, $s=|x'-y'|$, $h=\pi/\rho$, $\rho>0$, $\alpha^2=s$

D -the unlimited area lying in a layer $\{y:y=(y_1,y_2,...,y_m),(y_1,y_2,...,y_m)\in R, y_j\in R, j=1,...,m-1, 0<y_m<h\}$ with border, $\partial D=\{y:y=(y_1,y_2,...,y_m), y_m=0\}\cup S, S=\{y:y=(y_1,...,y_m), y_m=f(y_1,...,y_{m-1})\}$ where $f(y_1,...,y_{m-1})$ has limited private derivative of the first order.

THE CAUCHY PROBLEM

Let $u\in C^{2n}(D)$ and

$$\Delta^n u(y)=0,\ y\in D \tag{1}$$

$$u(y)=F_0(y),\quad \Delta u(y)=F_1(y),...,\Delta^{n-1}u(y)=F_{n-1}(y),\quad y\in S$$
$$\frac{du(y)}{d\,\overline{n}}=G_0(y),\quad \frac{d\Delta u(y)}{d\,\overline{n}}=G_1(y),...,\frac{d\Delta^{n-1}u(y)}{d\,\overline{n}}=G_{n-1}(y),\quad y\in S, \tag{2}$$

Where $F_i(y), G_i(y)$ given on ∂D continuous function, $\overline{n}$ -external normal to ∂D. It is required to restore $u(y)$ in *D*.

Let's assume, that the decision of a task (1) - (2) exists and continuously differentiate, $2n-1$ of time down to endpoints of border and satisfies to the certain condition of growth (class of a correctness), which provides uniqueness of the decision.

Function $\Phi_\sigma(y,x)$ we can define by the following equality:

$$\Phi_\sigma(y,x) = C_{n,m}\int_{\sqrt{s}}^{\infty} \mathrm{Im}\left[\frac{\exp(\sigma w + w^2) - achi\rho_1\left(w - \frac{h}{2}\right)}{\omega - x_1}\right](u^2 - s)^{n-k}\,du,\ \omega = iu + y_1 \qquad (3)$$

$$C_{n,m} = (-1)^{\frac{m}{2}-1}\left(\Gamma(n - \frac{m}{2} + 1)2^{2n-1}\pi^{\frac{m}{2}}\Gamma(n)\right)^{-1}$$

Theorem–1. Function $\Phi_\sigma(y,x)$ can define by the following equality (3) - polyharmonic of functions. (s > 0).

Theorem-2. The function $\Phi(y,x)$ is satisfied the following inequality :

$$\sum_{k=0}^{n-1}\int_{\partial D\backslash S}\left[\left|\Delta^k\Phi_\sigma(y,x)\right| - \left|\frac{\partial\Delta^k\Phi_\sigma(y,x)}{\partial\bar{n}}\right|\right]ds_y \le C(x)\varepsilon(\sigma),$$

$C(x)$ - constant , $\varepsilon(\sigma) \to 0$, $\sigma \to \infty$.

Let's designate through $B_\rho(D)$ space polyharmonic functions m-dim in domain *D* having continuous private derivative about *2n-1* , down endpoints of border and satisfying to a condition:

$$\sum_{k=0}^{n-1}\left(\left|\Delta^k u(y)\right| + \left|\ grad\Delta^{n-k-1}u(y)\ \right|\right) \le C\exp(\exp\rho_2(y')). \qquad (4)$$

Theorem-3. Let for function $u \in B_\rho(D)$ in any point $y \in \partial D$ the inequality

$$\sum_{k=0}^{n-1}\left|\Delta^k u(y)\right| + \left|\frac{\partial\Delta^{n-1-k}u(y)}{\partial\bar{n}}\right| \le C\exp\left(a\cos\rho_3\left(y_1 - \frac{h}{2}\right)\exp\rho_3|y'|\right) \qquad (5)$$

holds, where $\rho_1 < \rho_2 < \rho_3 < \rho$.

Then for any point $x_0 \in D$ the equality takes place

$$u(x_0) = \sum_{k=0}^{n-1}\int_{\partial D}\left[\Delta^k\Phi_\sigma(y,x_0)\frac{\partial\Delta^{n-1-k}u(y)}{\partial\bar{n}} - \Delta^{n-1-k}u(y)\frac{\partial\Delta^k\Phi_\sigma(y,x_0)}{\partial\bar{n}}\right]ds.$$

Theorem-4. Let $u(x)$ the decision of a task $(1)-(2)$, having continuous private derivative about $2n-1$ down to final points of border ∂D. If for any $y \in D$ is executed conditions of growth

$$\sum_{k=0}^{n-1}\left(\left|\Delta^k u(y)\right| + \left|\ grad\Delta^{n-k-1}u(y)\ \right|\right) \le C\exp(\exp\rho_2(y'))\,(3)$$

$\rho_1 < \rho_2 < \rho_3 < \rho$, and

$$\forall y \in \partial D \setminus S \qquad \left|\frac{\partial\Delta^{n-1-k}u(y)}{\partial\bar{n}}\right| + \left|\Delta^{n-1-k}u(y)\right| \le M\ ,$$

Then $\left|u(x)-u_\sigma(x)\right|\le MC(\sigma)\exp(-\sigma x_m),\quad \sigma\ge\sigma_0>0,\quad x\in D$ is holds.

Where $$u_\sigma(x)=\sum_{k=0}^{n-1}\int_S\left[G_{n-k-1}(y)\Delta^k\Phi_\sigma(y,x)-F_{n-k-1}(y)\frac{\partial\Delta^k\Phi_\sigma(y,x)}{\partial\bar{n}}\right]ds$$

and $C(\sigma)$ -polynom.

Consequence - 2. Limiting equality

$$\lim_{\sigma\to\infty}u_\sigma(x)=u(x)$$

Takes place in regular intervals in any compact from *D*.

REFERENCES

[1] Yarmukhamedov. Sh. Ya. a Task Cauchy for polyharmonic of the equation. The reports of WOUNDS 2003 volumes 388, №2, with 102-165.

[2] Mazhya. B. G., About the decision of a task Cauchy for the equation Laplace (uniqueness, approximation). Works MMO, 1974, т.30, with 61-114).

[3] Juraeva N.Yu On the integral representation of the polyharmonic functions. DAN AN RU № 3, 2008. pp.18-20.

[4] Juraeva N.Yu On the Cauchy problem for the increasing polyharmonic functions.. Proceedings of the Conference, “Inverse and incorrect problems of Mathematical Physics” Novosibirsk. 2007. pp.65-67.

[5]. Juraeva N.Yu. On the integral representation of the polyharmonic functions. «The second International Conference on Control and Optimization with Industrial Applications» Baku, Azerbaijan, 2-4 June, 2008. pp.44-49.

[6]. Juraeva N.Yu. Growing polyharmonic functions and task Cauchy of some class.Uzbek Math Journal №.2, 2009, pp.70-74.

THE PROBLEM KOSHI FOR HELMHOLTZ EQUATION FOR AREAS OF THE TYPE OF THE CURVILINEAR TRIANGLE

Abdukarimov A.*, Islomov I.**
**Samarkand branch of Tashkent Univercity of information Technology*
***Department of Mechanics and Mathemetics, University of Samarkand, University Boulevard 15, 140104 Samarkand Uzbekistan*
abdukarimov54@mail.ru

Let $x=(x_1,x_2)$ and $y=(y_1,y_2)$ of the point two-dimensional material Euclidean space R^2. We shall consider equation Helmholtz

$$\Delta U+\lambda^2U=0, \qquad (1)$$

where Δ - an operator Laplace, $\lambda>0$.

We shall designate through D_ρ the limited area with border, consisting of length rays $|y_1| = \tau y_2, \tau = tg\frac{\pi}{2\rho}, \ \rho > 1, \ 0 < y_2 \leq y_0 < +\infty$, with beginning in zero, and arcs S smooth curve, lying inwardly corner of the width $\frac{\pi}{\rho}$. We shall consider that $(0, x_2) \epsilon D_\rho, x_2 > 0$. D_ρ - shall name the area of the type of the curvilinear triangle. We shall designate $H(D_\rho)$ - a decision space of the equation (1) in D_ρ. If $U(y) \in H(D_\rho) \cap C^1(\overline{D}_\rho)$ where $\overline{D}_\rho = D_\rho \cup \partial D_\rho$, that equitable Green formula

$$U(x) = \int_{\partial D_\rho} \left\{ \Phi \frac{\partial U}{\partial n} - U \frac{\partial \Phi}{\partial n} \right\} ds, \qquad x \in D_\rho \tag{2}$$

where n - a direction to external normal, Φ - a elementary solution of the equation (1).

In formula (2) together $\Phi(y, x)$ shall substitute

$$\Phi_\sigma(y - x, \lambda) = \frac{1}{2\pi} \int_0^\infty Im \left[\frac{E_\rho[\sigma(w - x_2) \exp[(w - x_2)]^2]}{w - x_2} \right] \frac{I_0(\lambda u) u}{\sqrt{u^2 + \alpha^2}} du,$$

$\sigma > 0, \rho > 1, w = i\sqrt{u^2 + \alpha^2} + y_2$ $E_\rho(z)$- whole function Mittag-Leffler [2].

Theorem 1. *Let* $U(y) \in H(D_\rho) \cap C^1(\overline{D}_\rho), U(y) = f(y), \frac{\partial U}{\partial n}(y) = g(y),$

$y \in S$, *where functions* $f(y)$ *and* $g(y) \in C^1(S)$ *is given on S, satisfies.*

$$|U(y)| + \left|\frac{\partial U}{\partial n}\right| \leq 1, \ y \in \partial D \backslash S, \tag{3}$$

$$U_\sigma(x) = \int_S \left[U \frac{\partial \varphi_\sigma}{\partial n} - \varphi_\sigma \frac{\partial U}{\partial n} \right] ds, \qquad x \in D_\rho. \tag{4}$$

Then it is true

$$|U(x_0) - U_\sigma(x_0)| \leq C_{\lambda,\rho} \exp(-\sigma\gamma), x_0 = (0, x_2) \in D_\rho. \tag{5}$$

$C_{\lambda,\rho}$ - constant dependent from λ and ρ.

Theorem 2. *Let* $U(y) \epsilon H(D_\rho) \cap C^1(\overline{D}_\rho)$ *satisfies the border condition (3) on the whole border* ∂D_ρ. *Then*

$$|U(x_0) - U_\sigma(x_0)| \leq C_{\lambda,\rho}(x_0) \exp(-\sigma\gamma), \qquad x_0 = (0, x_2) \epsilon D_\rho$$

where

$$C_{\lambda,\rho}(x_0) = C_{\lambda,\rho} \cdot \int_S \frac{ds}{r}, \quad r = |y - x|, \gamma = \tau x_2 > 0, \ \sigma \geq \sigma_0 > 0. \tag{6}$$

Let $U(y) \in H(D_\rho) \cap C^1(\bar{D}_\rho)$ and together $U(y)$ and $\frac{\partial U(y)}{\partial n}$ on S is given their unceasing approach f_δ, g_δ with abnormal δ ,

$$\max_s |U - f_\delta| + \max_s \left| \frac{\partial U}{\partial n} - g_\delta \right| \le \delta,\ 0 < \delta < 1$$

We shall define σ *and* R from equality

$$\sigma = \tau^{-\rho} R^{-\rho} \ln \delta^{-1},\ R = \max_s \sqrt[\rho]{\mathrm{Re}(i|y_1| + y_2)^\rho}$$

Under these condition equitable

Theorem 3. Let $U(y) \epsilon H(D_\rho) \cap C^1(\bar{D}_\rho)$ *and on the whole border satisfies the condition (3).Then*

$$|U(x_0) - U_{\sigma\delta}(x_0)| \le C_{\lambda,\rho}(x_0)\delta^{\frac{x_2}{R}},\quad x_0 \in D_\rho.$$

where $C_{\lambda,\rho}(x_0)$ *is defined from (6),*

$$U_{\sigma\delta}(x) = \int_{s^*} \{\mathrm{f}_\delta \frac{\partial L_\sigma}{\partial \mathrm{n}} - \mathrm{g}_\delta L_\sigma\} \mathrm{ds},\quad \mathrm{x}_0 \in D_{\rho.}$$

$L_\sigma(x, y)$ - a elementary solution of the equation (1) with special characteristic.

REFERENCES

[1] Yarmuhammedov Sh. About continuation of the decision of the equation Helmholtz //DAN 1997, 357: 3, pp.320-323.

[2] Dzjarbashyan M.M. The Integral presentation and presentations to functions in complex area. M.: Science, 1966.

INTEGRAL GEOMETRY PROBLEMS ON A PLANE AND THE D'ALEMBERT MAPPINGS OF SYMMETRIC DOMAINS

Begmatov H A., Ochilov Z. H.

In this paper an integral geometry problem of the Volterra type [1-3] is studied over a family of parabolas with a weight function having a singularity. We obtain stability estimates for the solution of the problem in Sobolev spaces, which show the weak ill-posedness of the problem, and derive an inversion formula.

In what follows, we use the notation

$$(x, y) \in R^2,\quad (\xi, \eta) \in R^2,\quad \lambda \in R^1,\quad \mu \in R^1$$

$$\Omega = \{(x, y), x \in R^1, y \in (0, l), l < \infty\}$$

$$\overline{\Omega} = \{(x, y), x \in R^1, y \in [0, l]\ \}$$

In the strip , $\overline{\Omega}$ consider a family of curves that are uniquely parametrized using the coordinates of their vertices (x, y). An arbitrary curve P(x, y) of the family

is defined by the relations

$$P(x, y) = \{(\xi, \eta): \quad (y - \eta) = (x - \xi)^2, 0 \le \eta \le y, y \le l, l < \infty\}.$$

Problem 1. *Determine a double_variable function u(x, y) given the integrals of $u(\bullet)$ along the curves P(x,y) for all (x, y) from* $\overline{\Omega}$

$$\int_{x-\sqrt{y}}^{x+\sqrt{y}} g(x-\xi) u(\xi, y-(x-\xi)^2) d\xi = f(x, y), \tag{1}$$

where

$$g(\bullet) = \begin{cases} 1, x > 0 \\ 0, x < 0 \end{cases} \tag{2}$$

is the Heaviside function.

The function u(x, y) is from the class $\tilde{N}_0^2$,

$$\text{supp } u \subset D = \{(x, y): -a < x < a, 0 < a < \infty, 0 < y < l, \quad l < \infty\}$$

Define the following functions:

$$I(\lambda, \mu) = 2\int_0^\infty \frac{e^{i(\mu\tau + \lambda\sqrt{\tau})}}{\sqrt{\tau}} d\tau, \tag{3}$$

$$I_1(\lambda, \mu) = \int_{-\infty}^{\infty} e^{-i\mu y} \frac{d\mu}{(1+\mu^2) I(\lambda, \mu)}, \tag{4}$$

$$I_2 = \int_{-\infty}^{\infty} e^{-i\lambda x} I_1(\lambda, y) d\lambda. \tag{5}$$

Theorem 1. *Let f (x, y) be given for all y ≥ 0. Then problem 1 has a unique solution in the class U that is representable as*

$$u(x, y) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} I_2(x-\xi, y-\eta)(E - \frac{\partial^2}{\partial \eta^2}) f(\xi, \eta) d\xi d\eta \tag{6}$$

and satisfies

$$\|u\|_{L_2} \le C \|f\|_{W_2^{0,1}(R_+^2)},$$

where C is a constant.

Theorem 2. Suppose that the right_hand side of (1) satisfies the following conditions:

(i) f (x, y) is a compactly supported function of x; (ii) the partial derivatives of f (x, y) are all continuous up to the second order inclusive;

(iii) $\frac{\partial^m}{\partial y^m} f(x,y)\Big|_{y=0} = \frac{\partial^m}{\partial y^m} f(x,y)\Big|_{y=l} = 0 \quad (0 \le m \le 2)$.

Then (2) has a solution in the class of continuous functions that are compactly supported in x and that solution is given by formula (6).

Let S(x,y) denote the part of bounded by a curve P(x, y) and the axis y = 0. Define the strip

$$\overline{\Omega} = \{(x,y), x \in R^1, y \in [0,l]\ \}$$

Problem 2. Determine u(x, y) if we are given, for all $(x,y) \in \overline{\Omega}$ *, its integrals over the curves P(x, y) and the areas S(x, y) with a weight function* $k(x,y,\xi,\eta)$

$$\int_0^y u(x-h,\eta)\frac{d\eta}{h} + \int_0^y \int_{x-h}^{x+h} k(x,y,\xi,\eta)u(\xi,\eta)d\xi d\eta = F(x,y)\,, \tag{7}$$

where $h = \sqrt{y-\eta}$,

The function k(·) is compactly supported; has all continuous partial derivatives up to the second order inclusive; and vanishes, together with its derivatives, on the parabolas $y-\eta = (x-\xi)^2$.

The function F(•) is assumed to be given in the entire half_plane.

Equation (7) corresponds to an integral geometry problem with a perturbation. The first term on the lefthand side is

$$\int_0^y u(x-h,\eta)\frac{d\eta}{h} = f(x,y)\,,$$

where, as before, $h = \sqrt{y-\eta}$ is the collection of integrals of the desired function over the family of halves of parabolas with vertices at (x, y). The second term $f_0(x, y) = F(x, y) - f(x, y)$ is an integral with weight k(·) over the internal parts of the parabolas.

Theorem 3. *Let F(x, y) be given in the strip Ω. Suppose that the weight function k(·) ∈ (Ω . Ω), together with its derivatives up to the second order inclusive, vanishes on the parabolas P(x, y). Then problem 2 has a unique solution in Ω in theclass of twice continuously differentiable compactly supported functions and satisfies the inequality*

$$\|u\|_{L_2} \le C_1 \|F\|_{W_2^{0,1}(R_+^2)}\,,$$

where C_1 is a constant.

As is known, the simplest system of hyperbolic equations is the d'Alembert system

$$\frac{\partial u}{\partial y} = 0,\ \frac{\partial v}{\partial y} = 0\,.$$

Following [4], we refer to mappings performed by means of solutions to this system as d'Alembert mappings. This paper considers mapping of domains bounded by Jordan curves. We prove a theorem on the existence of mappings to a canonical domain for a class of bounded domains symmetric with respect to the OY axis. We also state the corresponding inverse problem and prove the uniqueness of its solution.

Let D be a domain bounded by a curve C consisting of four parts:

$$C = C_1 \cup C_2 \cup C_3 \cup C_4$$

$$C_1 = \{(x,y): y = f_1(x), x \in [0,1],\ f_1(0) = \frac{1}{2},\ f_1(1) = 1\}$$

$$C_2 = \{(x,y): y = f_2(x), x \in [0,1],\ f_2(0) = 0,\ f_2(1) = 1\}$$

$$C_3 = \{(x,y): y = f_3(x), x \in [-1,0],\ f_3(0) = 0,\ f_3(-1) = 1\}$$

$$C_4 = \{(x,y): y = f_4(x), x \in [-1,0],\ f_4(-1) = 1,\ f_4(0) = \frac{1}{2}\}$$

All functions $f_k(x)$ are monotone and continuous. Consider the domain

$$D = \{(x,y): f_1^* < f_2^*,\ x \in [-1,1]\},$$

where the functions $f_k^*(x),\ (k = 1,2)$ with k= 1, 2 satisfy the conditions.

$$1)\ f_1^*(x) < f_2^*(x),\ -1 \le x \le 1 \qquad (8)$$

$$2)\ f_k^*(-x) = f_k^*(x) \qquad (9)$$

$$3)\ f_k^*(-1) = f_k^*(1) = 1, \quad f_2^*(0) = 0, \quad f_1^*(0) = \frac{1}{2} \qquad (10)$$

The functions inverse to the d'Alembert mappings determined by the functions

$u = \varphi(x)$ and $v = \psi(y)$ are $x = \varphi_1(u) and \quad y = \psi_1(v)$ respectively.

The mappings determined by the functions $u = \varphi(x)$ and $v = \psi(y)$, take the domain D to the domain D_1 in theplane (u,v) defined by plane (u,v) defined by

$$D_1 = \{(u,v): \quad F_1(u) < v < F_2(u)\}.$$

Here,the functions $F_k(u),\ k = 1,2$ are defined as

$$v = \psi(y) = \psi(f_k^*(x)) = \psi(f_k^*(\varphi(u))) = F_k(u), \quad k = 1,2.$$

The mapping thus defined takes the domain D to the domain D_1 in the plane (u , v) bounded by the corner $v = F_1(u) = |u|, \quad u \in [-1,1]$

and $v = F_2(u), \quad F_2(-1) = F_2(1) = 1, \quad F_2(0) = \frac{1}{2}, \quad u \in [-1,1]$.

Now, consider the mapping that takes the domain D_1 to the domain D_2 in the plane (u_1, v_1) defined by

$$D_2 = \{(u_1, v_2) : F_1(u_1) < v_1 < F_2(u_1)\}$$

The domain D_2 is bounded by the two corners

$$F_1(u_1) = |u_1|, \ u_1 \in [-1, 1]$$

$$F_1(u_1) = \frac{1}{2}|u_1| + \frac{1}{2}, \ u_1 \in [-1, 1].$$

Suppose that this mapping is determined by functions $u_1 = \varphi_2(u)$ and $v_1 = \psi_2(v)$ The following theorem is valid.

Theorem 4. Suppose that a domain D in the plane (x, y) is such that the function $f_1^*(x)$ satisfies conditions(8)–(10).Then, there exists a d'Alembert mapping of the domain D to the D_2 domain in the plane (u_1, v_1) bounded by the functions $v_1 = |u_1|$, and $v_1 = \frac{1}{2}|u_1| + \frac{1}{2}$.

Now, consider the inverse problem for a domain satisfying the conditions in Theorem 4. Obviously, if the functions $\varphi_k(y)$, $(k = 1, 2)$, where k = 1, 2, are inverse to the functions $f_k^*(x)$, $(k = 1, 2)$ (that is, if $\varphi_k[f_k^*(x)] = x$), then

$$D = \{(x, y) : \varphi_2(y) < x < \varphi_1(y), \quad y \in [0,1]\}$$

We set

$$u^*(x) = f_2^*(x) - f_1^*(x),$$
$$v^*(y) = \varphi_1(y) - \varphi_2(y)$$

Inverse problem A. Given functions $f_k(x)$ and $\varphi_k(y)$, determine the functions u(x) or v(y).

Theorem 5. *Inverse problem A has a unique solution.*

REFERENCES

[1].V. G. Romanov. Certain Inverse Problems for Hyperbolic Equations (Nauka, Novosibirsk, 1972) [in Russian].

[2].Akr. H. Begmatov. Dokl. Math. 57, 21–23 (1998) [Dokl. Akad. Nauk 358, 151–153 (1998)].

[3].Akbar H. Begmatov and Akram H. Begmatov. Proceedings of International Conference on Ill_Posed and Nonclassical Problems of Mathematical Physics and Analysis (VSP, Utrecht, 2003), pp. 1–18.

[4] M. M. Lavrent'ev. Sib. Mat. Zh. **42** (5), 1094–1105(2001).

THE SYMMETRIC PRODUCT OF MATRICES

Bekbaev U.Dj.

Turin Polytechnic University in Tashkent,
INSPEM, Universiti Putra Malaysia.
e-mail: bekbaev2011@gmail.com

This paper is about the symmetric product of matrices. In Mathematics the are different "products" of matrices, for example, ordinary product, Kronecker product, Tracy-Singh and Khatri-Rao products. Such products appear in a natural way when one wants to represent the result of some operation ("product") as a matrix in terms of the matrices of the "factors". Our case also is not exception. For example, representation of symmetric multi-linear maps (tensors) by matrices and consideration the symmetric product of symmetric multi-linear maps lead us to the definition of the symmetric product of matrices which will be denoted by $\square$. As usual, first we introduce a symmetric product of matrices, explore it and then show its applications. It should be noted we deal with matrices entries of which are located by pair of multi-indices, not only by pair of natural numbers. Such approach is more suitable in many cases than location by pair of natural numbers. Of course, to do it we need a linear order in the set of multi-indices. Her we consider one of such linear orders. For other applications of the symmetric product and results one can see [1],[2],[3].

For a positive integer n let I_n stand for all row n-tuples with nonnegative integer entries with the following linear order: $\beta=(\beta_1,\beta_2,...,\beta_n)<\alpha=(\alpha_1,\alpha_2,...,\alpha_n)$ if and only if $|\beta|<|\alpha|$ or $|\beta|=|\alpha|$ and $\beta_1>\alpha_1$ or $|\beta|=|\alpha|$, $\beta_1=\alpha_1$ and $\beta_2>\alpha_2$ etcetera, where $|\alpha|$ stands for $\alpha_1+\alpha_2+...+\alpha_n$. We write $\beta=\alpha$ if $\beta_i\le\alpha_i$ for all $i=1,2,...,n$, $\binom{\alpha}{\beta}$ stands for

$\frac{\alpha!}{\beta!(\alpha-\beta)!}$, $\alpha!=\alpha_1!\alpha_2!...\alpha_n!$.

In future n, n' and n'' are assumed to be any fixed nonnegative integers (In the case of $n=0$ one should assume that $I_n=\{0\}$), F stands for the field of real or complex numbers..

For any nonnegative integer numbers p, p' let $M_{n,n'}(p,p';F)=M(p,p';F)$ stand for all "$p\times p'$" size matrices $A=(A^{\alpha}_{\alpha'})_{|\alpha|=p,|\alpha'|=p'}$ (α presents row, α' presents column and $\alpha\in I_n,\alpha'\in I_{n'}$). Note that the ordinary size of a such matrix is $\binom{p+n-1}{n-1}\times\binom{p'+n'-1}{n'-1}$.

Over such kind matrices in addition to the ordinary sum and product of matrices we consider the following "product" as well:

Definition 1. If $A\in M(p,p';F)$ and $B\in M(q,q';F)$ then
$A\square B=C\in M(p+q,p'+q';F)$ such that for any $|\alpha|=p+q$, $|\alpha'|=p'+q'$, where $\alpha\in I_n,\alpha'\in I_{n'}$,

$$C^{\alpha}_{\alpha'} = \sum_{\beta,\beta'} \begin{pmatrix} \alpha' \\ \beta' \\ \ \end{pmatrix} A^{\beta}_{\beta'} B^{\alpha-\beta}_{\alpha'-\beta'}$$

where the sum is taken over all $\beta \in I_n, \beta' \in I_{n'}$, for which $|\beta| = p$, $|\beta'| = p'$, $\beta = \alpha$ and $\beta' = \alpha'$.

Proposition 1. For the above defined product the following are true.

1. $A \square B = B \square A$.

2. $(A+B) \square C = A \square C + B \square C$.

3. $(A \square B) \square C = A \square (B \square C)$

4. $(\lambda A) \square B = \lambda (A \square B)$ for any $\lambda \in F$

5. If A and B are square upper triangular matrices then $A \square B$ is also an upper triangular matrix.

6. $A \square B = 0$ if and only if $A = 0$ or $B = 0$.

In future $A^{(m)}$ means the m-th power of matrix A with respect to the new product.

Proposition 2. If $h = (h_1, h_2, ..., h_{n'}) \in M(0,1;F)$, $v = (v_1, v_2, ..., v_n) \in M(1,0;F)$, then

$$(h^{(m)})^{0}_{\alpha'} = m! h^{\alpha'}, \quad (v^{(m)})^{\alpha}_{0} = \binom{m}{\alpha} v^{\alpha}$$

where v^{α} stands for $v_1^{\alpha_1} v_2^{\alpha_2} ... v_n^{\alpha_n}$

Proposition 3. For any nonnegative integers p, q, p', q' and matrices $A \in M_{n,n'}(p, p'; F)$, $B \in M_{n,n'}(q, q'; F)$, $h = (h_1, h_2, ..., h_n) \in M_{n,n}(0,1;F)$, $v = (v_1, v_2, ..., v_{n'}) \in M_{n',n'}(1,0;F)$ the following equalities

$$\left(\frac{h^{(p)}}{p!} A\right) \square \left(\frac{h^{(q)}}{q!} B\right) = \frac{h^{(p+q)}}{(p+q)!} (A \square B), \quad \left(A \frac{v^{(p')}}{p'!}\right) \square \left(B \frac{v^{(q')}}{q'!}\right) = (A \square B) \frac{v^{(p'+q')}}{(p'+q')!}$$

are true.

Here are some relations between properties of $A \in M(1,1;F)$ and $\dfrac{A^{(k)}}{k!}$.

Theorem 1. Let $A \in M(1,1;F)$ be a square matrix.

1. If $\{\lambda_1, \lambda_2, ..., \lambda_n\}$ is its all eigenvalues, where each eigenvalue occurs as many times as its multiplicity, then $\{\lambda_1^{\alpha_1} \lambda_2^{\alpha_2} ... \lambda_n^{\alpha_n}\}_{|\alpha|=k}$ represents all eigenvalues of $\dfrac{A^{(k)}}{k!}$ including their multiplicities as eigenvalues of $\dfrac{A^{(k)}}{k!}$. Moreover if $Av_i = \lambda_i v_i$ for $i = 1,2,...,n$ then

$$\frac{A^{(k)}}{k!}(v_1^{(\alpha_1)} \square v_2^{(\alpha_2)} \square ... \square v_n^{(\alpha_n)}) = \lambda_1^{\alpha_1}\lambda_2^{\alpha_2}...\lambda_n^{\alpha_n}(v_1^{(\alpha_1)} \square v_2^{(\alpha_2)} \square ... \square v_n^{(\alpha_n)})$$

2.

$$\det \frac{A^{(k)}}{k!} = (\det A)^{\binom{k+n-1}{n}} \quad \textit{and if} \quad rk(A) = l \quad \textit{then} \quad rk(\frac{A^{(k)}}{k!}) = \begin{pmatrix} k+l-1 \\ l-1 \\ \end{pmatrix}$$

Let V ,V' and V'' be vector spaces with given bases, p and q be any natural numbers. If $A: V^p \to V'$ is a symmetric multi-linear map one can attach to A a matrix A for which the equality

$$A(x^1, x^2, ..., x^p) = A(\frac{x^1 \square x^2 \square ... \square x^p}{p!})$$

is true, where $x^1, x^2, ..., x^p$ are from V .

If $B: V^q \to V'$ and $C: V' \times V' \to V''$ are symmetric multi-linear maps one can consider the "product" $A \times_C B$ of A and B with respect to $C(x', y') = C\frac{x' \square y'}{2!}$ as a symmetric multi-linear map $A \times_C B: V^{p+q} \to V''$ defined by

$$A \times_C B(x^1, x^2, ..., x^{p+q}) = \frac{1}{(p+q)!}\sum_{\sigma \in S_{p+q}} C(A(x^{\sigma(1)}, x^{\sigma(2)}, ..., x^{\sigma(p)}), B(x^{\sigma(p+1)}, x^{\sigma(p+2)}, ..., x^{\sigma(p+q)}))$$

where S_{p+q} stands for the symmetric group.

Theorem 2. The equality

$$A \times_C B(x^1, x^2, ..., x^{p+q}) = \frac{1}{2!}C(A \square B)\frac{x^1 \square x^2 \square ... \square x^{p+q}}{(p+q)!}$$

is valid.

REFERENCES

[1] Bekbaev Ural. *A matrix representation of composition of polynomial maps*. arXiv0901.3179v3 math. AC] 22 Sep.2009

[2] Bekbaev Ural. *A radius of absolute convergence for power series in many variables*. arXiv1001.0622v1 [math.CV] 5Jan.2010

[3] Bekbaev Ural. *Matrix Representations for Symmetric and Antisymmetric maps.* arXiv:1010.2579 math.RA.

PERIODICAL REGIMES IN VOLTERRA MODEL OF THREE POPULATIONS.

Buriyev T.E., Ergashev V.E., Muxtarov Ya.

In the present work problems of existence of periodic models in systems of three differential equations a are considered as a kind of

$$\begin{cases} \dot{x}_1 = a_1x_1 + a_{11}x_1 + a_{12}x_2 + a_{13}x_3 \\ \dot{x}_2 = a_2x_2 + a_{21}x_1 + a_{22}x_2 + a_{23}x_3 \\ \dot{x}_3 = a_3x_3 + a_{31}x_1 + a_{32}x_2 + a_{33}x_3 \end{cases} \qquad (1)$$

This system describes dynamics of number of community of three populations, cooperating by a principle «a predator – a victim» Here - number x_i of populations factors characterize a_i (at) or (at) the $a_i > 0$ given population, factors characterize $a_i < 0$ intrapopulation and interpopulation a_{ij} interactions. Let $A = (a_{ij})$ a matrix made of factors of system of the differential equations (1)

We investigate the following system of the differential equations

$$\begin{cases} \dot{x}_1 = c_1x_1 - c_{12}x_1^{\;2} - c_{12}x_1x_2 + c_{13}x_1x_3 \\ \dot{x}_2 = c_2x_2 - c_{21}x_1x_2 - c_{22}x_2^{\;2} - c_{23}x_2x_3 \\ \dot{x}_3 = c_3x_3 - c_{31}x_1x_3 + c_{32}x_2x_2 - c_{33}x_3^{\;2} \end{cases} \qquad (2)$$

In system (2) $x_i \geq 0, c_{ij} \geq 0, c_i > 0$ elements c_i of a matrix C of system (2) a_{ij} are modules A a of elements of a matrix system (2) – the form $c_{ii} = 0$ of representation of the A system (1), visually showing directions of population streams.

In system (2) each population plays a role as predator and victim. Concerning system (2) in the absence of an intraspecific competition when also a symmetric matrix with ($a_{ij}$to a matrix $a_i < 0$) there is a general a_{ij} theorem. Voltaire according to which at any $A = (a_{ij})$ conditions of number of populations beyond all bounds increase. Here effects of infringement of conditions of the Open-air cage are considered. In system (2) we will make the following replacement

$$x_1 = \frac{c_1x}{c_{11}}, x_2 = \frac{c_2y}{c_{22}},\ x_3 = \frac{c_3z}{c_{33}},\ t = \frac{\grave{l}}{c_1},$$

Then the system (2) is led to a kind;

$$\begin{cases} \dot{x} = x(1 - x - l_1 y + d_1 z) \\ \dot{y} = \gamma_1 y(1 - y - b_2 z + d_2 \lambda) \\ \dot{z} = \gamma_2 z(1 - z - b_3 \lambda + d_2 y) \end{cases} \tag{3}$$

Where

$$\begin{cases} b_1 = \frac{c_{12}c_2}{c_1 c_{22}}; \quad d_1 = \frac{c_{13}c_3}{c_1 c_{33}}; \quad b_2 = \frac{c_{22}c_3}{c_2 c_{33}} \\ d_1 = \frac{c_{21}c_1}{c_2 c_{11}}; \quad b_3 = \frac{c_{31}c_1}{c_3 c_{11}}; \quad d_3 = \frac{c_{32}c_1}{c_3 c_{22}} \\ \gamma_1 = \frac{c_2}{c_1}; \quad \gamma_2 = \frac{c_3}{c_1} \end{cases} \tag{4}$$

The system (3) has following conditions: the beginning of coordinates, and a point lying on axes of coordinates or coordinate planes. The system can have also a spatial equilibrium state In B(x_0, y_0, z_0) in the first octant which $c_1 = c_2 = c_3 = 1$coordinates are from system of the linear equations.

$$\begin{cases} 1 - x - b_1 y \\ 1 - y - b_2 z + d_2 x = 0 \\ 1 + z - d_3 y = 0 \end{cases} \tag{5}$$

Let's show possibility of existence closed integrated curve in a vicinity of a special point. In for this purpose we will consider system (2) at factors. And with a matrix of a A special kind

$$A = \begin{pmatrix} 1+d & -2 & -d \\ -d & 1+d & -2 \\ -2 & -d & 1+d \end{pmatrix} \tag{6}$$

Where parameter $d < -1$ Then point coordinates In are equal Characteristic the $x_0 = y_0 = z_0 = 1$equations for a special point looks like (7)

At values of parameter close to number one of roots of the equation (7) is negative, two other roots awakes in a complex - the interfaced numbers. A material part of these complex numbers negatively at and it is positive at hence in the first case a point steady knot–focus, and in the second case unstable knot – focus. At the equation

$$\lambda^3 - 3(1+d)\lambda^2 + 3(1+d^2)\lambda + 3d^2 + 3d + z = 0 \tag{7}$$

it is represented in $-\frac{4}{3}$ a kind: Thus, a point difficult knot – focus with one $\lambda_1 < 0,$ steady direction. Its first Lyapunov size is the third focal size and is negative. Under the theorem

Andropov – Hopf at value of $d < -\frac{4}{3}$ parameter round $d > -\frac{4}{3}$ unstable knot – focus is born and $d = -\frac{4}{3}$ $(\lambda+1)(\lambda^2+\frac{25}{3}) = 0$ at close values of parameter B in system there is a steady limiting H_3 cycle round a special point system (2) has been investigated $d = -\frac{4}{3}$ also by numerical methods. Complexes of programs for the $d > -\frac{4}{3}$ qualitative analysis of systems of the differential equations, developed in IMPB the Russian Academy of Sciences were thus used.

Numerical calculations have shown that in $(b_1, b_2, b_3, d_1, d_2, d_3)$ space of parameters there is an area to which points $B(x_0, y_0, z_0)$ corresponds existence of a steady limiting cycle near to an equilibrium state from here it follows that in this area in community is realized steady coexistence of all three populations in a self-oscillatory model.

REFERENCE

[1]. A.D.Bazikin. Mathematical biophysics of cooperating populations – M,Nauka. 1985

OPTIMAL POSITIONAL CONTROL BY LINEAR DYNAMIC SYSTEM IN A CLASS OF RELAY FUNCTIONS

Davronov B.E.
Samarkand State University
javdavr@rambler.ru

The problem of optimal feedback controls construction is central in control theory. The one approach to solving of optimal systems synthesis problem was suggested in [1]. In this paper it is developed the problems which the Fullers regimes [2], being measurable controls with infinitely frequenting switching points in finite time interval can be arising. For this a class of admissible controls, consisting of piecewise - constant functions with restricted switch frequent is introduced that called in relay controls [3]. In addition the Fullers regimes already is not arising.

The optimal control problem is considered:

$$J(u) = c'x(t^*) + \int_0^{t^*} x'(t)Dx(t)dt \to \min, \tag{1}$$

$$\dot{x} = Ax + bu, \qquad x(0) = x_0; \tag{2}$$

$$Hx(t^*) = g; \tag{3}$$

$$|u(t)| \le 1, \qquad t \in T = [0, t^*] \tag{4}$$

$$\left(x \in R^n, \quad u \in R, \quad g \in R^m, \quad rankH = m \le n, \quad D = D' \ge 0\right)$$

The approachable controls are chosen in a class of relay functions.

At fixed number $\mu > 0$ the piecewise - constant function $u(t), \quad t \in T$ be called relay function, if it satisfies restriction (4) and the distance of its switch points not less than μ. The approachable control $u(t), \quad t \in T$ be called admissible, if the corresponding trajectory $x(t), \quad t \in T$ of system (2) in finitely time moment t^* get to terminal set $X^* = \{x \in R^n : \quad Hx = g\}$ generated by restriction (3). The admissible control $u^0(t), \quad t \in T$ be called optimal program control, if on it the performance criterion (1) is achieving minimal value $J(u^0) = \min\limits_{-1 \le u \le 1} J(u)$.

Optimal program controls are useful on estimation of dynamic system possibilities, but they not often applied for dynamic systems optimization in regime of real time because of inevitable perturbations. In connection with one it is preferred optimal feedback controls.

Suppose that the actual behavior of system on interval $T^0 = [0, t^0], \quad 0 < t^0 < t^*$, is subjugated on equation

$$\dot{x} = Ax + bu + w(x,u,t), \qquad x(0) = x_0, \tag{5}$$

where $w(x,u,t), \quad t \in T^0, w(x,u,t) = 0, t \in [t^0, t^*]$ is the unknown piecewise-continuous n- vector-function.

The problem (1)-(4) is immersed to the family of problems

$$\begin{aligned}
&J_\tau(u) = c'x(t^*) + \int_\tau^{t^*} x'(t)Dx(t)dt \to \min, \\
&\dot{x} = Ax + bu, \qquad x(\tau) = z; \\
&Hx(t^*) = g; \qquad |u(t)| \le 1, \qquad t \in T_\tau = [\tau, t^*],
\end{aligned} \tag{6}$$

depending on position $\omega = (\tau, z)$.

Let Ω be a set of position ω, for the problem (6) has a solution, $u^0(t) = u^0(t/\omega), \quad x^0(t) = x^0(t, \omega), \quad t \in T_\tau$ are optimal program control and trajectory in problem (6).

The relay function $u^0(\omega), \omega \in \Omega$ is called optimal feedback control, if $\forall \omega \in \Omega$ it satisfies to the restriction $|u^0(\omega)| \le 1$ and $u^0(\omega) = u^0(\tau + 0, \ \omega)$.

Let in some particular control process the perturbation $w^*(x,u,t), \quad t \in T^0$ was realized. In system (6), closed by optimal feedback $u^0(t,x)$ it will be generate trajectory $x^*(t), \ t \in T$. Then the function $u^*(t) = u^0(t, x^*(t)), \quad t \in T$ is the control, circulated in closed system in considered particular process. The device, which for each particular process can be generate the control $u^*(t), \ t \in T$ is called optimal controller.

The operating algorithm of optimal controller is described.

REFERENCES

[1] Gabasov R., Kirillova F.M., Kostyukova O.I. // Techn. Kibern., N 4, 1992 (in Russian).

[2] Fuller A.T. // Proc. I congress of IFAC , M., 1961, T.2

[3] Davranov B.E. // Uzb. Acad. Dokl.,N 12, 1990 (in Russian)

OPTIMIZATION OF NEURONETWORKING PROCESSING OF NON-STATIONARY PROCESSES DATA BY METHODS OF PARALLEL COMPUTING

Djumanov I. O., Kholmonov M. S.
Samarkand state university

In present days the methods of parallel computing based on graphic adapters (GPU, Graphics Processing Unit) are even more often applied to performance of complex(difficult) mathematical accounts and they allows appreciablly to speed up accounts on usual cheap personal computers at the expense of use common memory and significant parallelism. It has the especially large theoretical and practical importance at construction the systems of neuronetworking processing of non-stationary information. Such systems allow to decide well the tasks of microobjects images visualization and recognition, data interpretation and approximation, analysis and forecasting, control of accuracy of information transfer and processing at the expense of use natural redundancy.

At the represented article we were stating results of working out the optimization algorithms for identification, interpretation and smoothing on the basis of parallel computing technologies, so as results of optimization are very important for training subset formation adaptation and for adaptation of self-organizing of neural networks (NN) structural components job.

For task decisions we offered to use the cascade multigrade method having computing expenses $0(N \ln e^{-1})$, where N is total number of elements and ε is accuracy of calculations for maintenance an accuracy of smoothing of non-stationary processes learning data submitted in a vector-spatial view.

Having entered grading space of convex cubes $\Omega_h \in [1,...,M;\ 1,...,N;\ 1,...,K]$ we wrote down

$$L_h U_h = F_h \text{ in } \Omega_h,\ \Omega_h \in R^3,\ \partial\Omega_{h0}(U) = U_0,\ \partial\Omega_{h1}(U) = \frac{\partial U}{\partial n_{\partial\Omega}}. \quad (1)$$

Here L is elliptic operator $LU = \sum_{i,j} \frac{\partial}{\partial x_i}\left(a_{ij}\frac{\partial U}{\partial x_j}\right)$;

- a_{ij} satisfy to elliptic condition $a_{ij} = a_{ji}$;

- there is such constant $\alpha > 0$, for which $\sum a_{ij}\xi_i\xi_j \geq \alpha \sum_i \xi_i^2$, where $\xi \in R^3$ and $i, j \in [1,2,3]$;

- F is a some known function;

- an unknown function U belongs to C^2;
- index h means belonging to grading space;
- M, N, K is a dimensionality of task in R^3.

For calculating the side of cube we used follow:

$$\Delta h_j = \frac{1}{X_j}; \; X_j = \{M, N, K\}; \; j \in [1,2,3].$$

The three-dot and five-dot finite-differenced patterns were applied for approximation of the differential elliptic operator and the cascade multigrade method was used for received system of grids. It was proved, that the speed of calculation of this method is made $O(CN)$, where $C > 1$ is any constant which is proportional to speed of convergence, and N is total of settlement cells.

The cascade multigrade method (KM), modified by us for the decision of tasks of the analysis and forecasting of poorly-formalizable processes submitted as grids, consists of two basic stages.

In the first stage the decision of task (1) with use of approximation L on a grid S_i is carriing out so long, while the mistake of the decision does not become less power norm for given i-th grid.

In the second stage a projection of decision maked for the task (1) is producing for function U from S_i to function U from S_{i-1}, i.e. projection of the decision is producing from more rough grid on one exacter.

At the decision of tasks of the first stage the iterative methods having property to smooth high-frequency components of function U was applied, in particular method of Yakobi for simple iteration, method of Gauss-Seydel, methods of relaxation and connected gradients. Besides, the direct methods were applied for initial steps on a rough grid.

At the second stage the methods of linear interpolation or interpolation of higher order by basic spline-functions were used.

For finding of an optimum account and interpolation method we were carry out the series of accounts for the second task on the basis of algorithm realized KM in C ++ at serial mode with one kernel of AMD Athlon 64X2 4800 + dual-core processor. The results show, that the use of combination of methods such as Gaus-Seidel, time series and connected gradients had become most advantageous on time.

For realization of identification and smoothing algorithm on the basis of parallel computing methods we used GPU hardware-software units and CUDA technology.

The procedures of colour painting for cubes on a Gauss-Seidel method were applied for a graphic illustration of the tasks decision on each of stages of cyclic multigrade method. The file of shared memory was entered inside procedure of calculation designated as GAUS_SEIDEL. It is determined, that the speed of access to shared memory surpasses speed of access to global on two order, and consequently the application of shared memory considerably accelerates account.

On the other hand, we was established the fact, that the size of a task exceeds the size of the allocated shared memory. In this connection, we offered to execute the block loading of the data in shared memory, thus two-dimensional indexation (on parallel flows indexes i and j) was used for reduction of inquiry time. It was defined that the inquiry time in this case was reduced in comparison with three-dimensional indexation approximately in 1,5-1,8 times.

TWO – DIMENSIONAL INVERSE PROBLEM FOR A HYPERBOLIC – TYPE EQUATION

Durdiev D.K.

Faculty of mathematics and physics, Bukhara State University
durdiev65@mail.ru

Safarov J.Sh

Center of information technologies, Tashkent University of Information Technology
jurabek_safarov@mail.ru

Consider the differential equation

$$u_{tt} - u_{xx} - a(x,t)u_x = \delta(x, t-s),\ (x,t) \in R^2,\ s > 0 \qquad (1)$$

and the generalized Cauchy data

$$u\big|_{t<0} \equiv 0. \qquad (2)$$

Here $\delta(x,t)$ is the two - dimensional Dirac delta function, $a(x,t)$ is a continuous function, s is a problem parameter and $u = u(x,t,s)$. We pose the inverse problem as follows: it is required to find coefficient $a(x, t)$ if the solution of the problems (1), (2) is known together with its derivative with respect to x

$$u(0,t,s) = f_1(t,s),\ u_x(0,t,s) = f_2(t,s),\quad t > 0, s > 0. \qquad (3)$$

For the case where $a(x, t) = a(x)$ the solvability problems for different statements of problems close to (1)-(3) were studied in [1] (chapter 2). In works [2], [3] inverse problems in the cases of equation (1) when unknown coefficient is multiplied by functions u and u_t was investigated . In the present paper as distinct from [2], [3] we do not suppose the even of the unknown coefficient.

Let

$$Q_T := \{(t,s) | 0 \le s \le T - s\},\ \Omega_T := \{(x,t) | 0 \le |x| \le t \le T - |x|\},\ T > 0,$$

$A(a_0)$ is the set of functions $a(x,t) \in C(\Omega_T)$, satisfying for $(x,t) \in \Omega_T$ to the inequality $|a(x, t)| \le a_0$ with fixed positive constant a_0. We suppose that $f_1(t,s) \in C_t^1(Q_T)$, $f_2(t,s) \in C(Q_T)$ and $f_1(s+0,s) = \frac{1}{2}$ ($C_t^1(Q_T)$ is the class of functions continuous in s, continuously differentiable in t, and defined on Q_T. We let F denote the set of functions $(f_1(t,s), f_2(t,s))$ satisfying the above requirements. We formulate the theorem of the solution stability of the inverse problem.

Theorem. Let $a^i(x,t) \in A(a_0)$ be the solutions to the inverse problem with the data $(f_1^i(t,s), f_2^i(t,s)) \in F$, $i=1,2$, respectively. Then the following estimate is valid:

$$\left\|a^1(x,t)-a^2(x,t)\right\|_{C(\Omega_T)} \le c_0(\left\|f_1^1(t,s)-f_1^2(t,s)\right\|_{C_t^1(Q_T)} + \left\|f_2^1(t,s)-f_2^2(t,s)\right\|_{C(Q_T)}),$$

where c_0 is a positive constant, depending on $a_0, T>0$.

REFERENCES

[1] V. G. Romanov, "Inverse Problems of Mathematical Physics," in Russian, Publishing House "Nauka", Moscow, 1984.

[2] D. K. Durdiev, "Problem of Determining the Nonstationary Potential in a Hyperbolic-Type Equation," Journal of Theoretical and Mathematical Physics, Vol. 2, No 156, 2008, pp. 1154-1158.

[3] D. K. Durdiev, "Problem of Determining the Two-Dimensional Absorption Coefficient in a Hyperbolic-Type Equation," Journal of Applied Mathematics, Vol. 1, 2010, pp. 124-127.

EXPLORATION OF STABILITY OF HYSTERESIS TYPE DYNAMIC SYSTEMS WHICH ARE PROTECTED FROM VIBRATIONS

Dusmatov O.M.
Jizzax state pedagogical institute

Khodjabekov M.U.
Samarkand state architecture and building institute
07time70@mail.ru

INTRODUCTION.

Nowadays all branches of industry, aircraft, machinery-manufacturing, ship-building and another group of manufacturing are developing very rapidly. It has been implementing complication of the usage of technologies parts. It is counted practical that complicated technologies parts being used should work correct and time-proof. That's way in this article we consider that exploration of stability of the nonlinear vibrations of hysteresis elastic characteristic plate and dynamic absorber.

RESULTS AND DISCUSSIONS.

In the [1] has been received differential equations of dynamic system which is protected from vibrations. Below we can write algebraic equations which wrote by frequency characteristic from differential equations [1] for exploration of stability of the given system:

$$x_{1jk}(i\omega) = -\frac{a_1 A_1 - \omega^2 d_{ik} + i a_1 B_1}{A_2 + iB_2} W_0;\ x_2(i\omega) = -\frac{b_1 \omega^2 - Ap_{ik}^2 + iBp_{ik}^2}{A_2 + iB_2} W_0, \qquad (1)$$

Where $a_1 = n^2 d_{ik} + u_{ik0} d_{3ik}$; $n = \sqrt{\frac{c}{m}}$ - natural frequency of the dynamic absorber; c - stiffness of the elastic element of the dynamic absorber; m - mass of the dynamic absorber; $d_{ik} = \frac{d_{1ik}}{d_{2ik}}$; $d_{3ik} = \frac{c}{\rho h d_{2ik}}$; ρ - density of the plate material; h -thickness of the plate; $d_{1ik} = \iint\limits_s u_{ik} dxdy$; $d_{2ik} = \iint\limits_s u_{ik}^2 dxdy$; $u_{ik} = u_{ik}(x, y)$ - free vibration forms of the plate; $u_{ik0} = u_{ik}(x_0, y_0)$; $O(x_0, y_0)$ - dynamic absorber situated point; $A_1 = 1 - \theta_1 (D_0 + F)$; $B_1 = \theta_2 (D_0 + F)$; $F = \sum_{j=1}^{s_1} D_j x_2^j$ - decrement of the vibration; D_j $(j = 0...s)$, θ_1, θ_2 - numbers depend on elastic element of the dynamic absorber, they are found from experience[3];

$$A_2 = \omega^4 - (Ap_{ik}^2 + A_1 a_4)\omega^2 + (AA_1 - BB_1) p_{ik}^2 n^2;\ B_2 = (AB_1 + A_1 B) p_{ik}^2 n^2 - (B_1 a_4 + Bp_{ik}^2)\omega^2$$

;

$$A = 1 - (c_0 + \frac{3D}{d_{2ik}\rho h p_{ik}^2} \sum_{i_1=1}^{r} c_{i_1} x_{1ikA}^{i_1} \frac{h^{i_1}}{2^{i_1}(i_1+3)} [\iint\limits_s u_{ik} (\frac{\partial^2}{\partial x^2}(\alpha_1 |\alpha_1|^{i_1}) + \frac{\partial^2}{\partial y^2}(\alpha_2 |\alpha_2|^{i_1})) dxdy])\eta_1 -$$

$$-(\frac{6D(1-\mu)}{d_{2ik}\rho h p_{ik}^2} \sum_{i_2}^{s_2} k_{i_2} x_{1ikA}^{i_2} \frac{h^{i_2}}{2^{i_2}(i_2+3)} \iint\limits_s u_{ik} (\frac{\partial^2}{\partial x \partial y}(\alpha_3 |\alpha_3|^{i_3})) dxdy + \frac{2D(1-\mu)}{d_{2ik}\rho h p_{ik}^2} k_0 \iint\limits_s u_{ik} \frac{\partial^2}{\partial x \partial y}(\alpha_3) dxdy)\nu_1;$$

$$B = (c_0 + \frac{3D}{d_{2ik}\rho h p_{ik}^2} \sum_{i_1=1}^{r} c_{i_1} x_{1ikA}^{i_1} \frac{h^{i_1}}{2^{i_1}(i_1+3)} [\iint\limits_s u_{ik} (\frac{\partial^2}{\partial x^2}(\alpha_1 |\alpha_1|^{i_1}) + \frac{\partial^2}{\partial y^2}(\alpha_2 |\alpha_2|^{i_1})) dxdy])\eta_2 +$$

$$+(\frac{6D(1-\mu)}{d_{2ik}\rho h p_{ik}^2} \sum_{i_2}^{s_2} k_{i_2} x_{1ikA}^{i_2} \frac{h^{i_2}}{2^{i_2}(i_2+3)} \iint\limits_s u_{ik} (\frac{\partial^2}{\partial x \partial y}(\alpha_3 |\alpha_3|^{i_3})) dxdy + \frac{2D(1-\mu)}{d_{2ik}\rho h p_{ik}^2} k_0 \iint\limits_s u_{ik} \frac{\partial^2}{\partial x \partial y}(\alpha_3) dxdy)\nu_2;$$

c_{i_1} $(i_1 = 0...r)$, k_{i_2} $(i_2 = 0...s)$ - numbers depend on material of the plate, they are found from experience [3]; $a_4 = n^2 + u_{ik0}^2 d_{3ik}$; x_{1ikA} - amplitude of the $x_{1ik}(t)$; $b_1 = 1 - u_{ik0} d_{ik}$; $\alpha_1 = \frac{\partial^2 u_{ik}}{\partial x^2} + \mu \frac{\partial^2 u_{ik}}{\partial y^2}$; $\alpha_2 = \frac{\partial^2 u_{ik}}{\partial y^2} + \mu \frac{\partial^2 u_{ik}}{\partial x^2}$; $\alpha_3 = \frac{\partial^2 u_{ik}}{\partial x \partial y}$; p_{ik} - natural frequency of the plate; $D = \frac{Eh^3}{12(1-\mu^2)}$ - cylindrical stiffness of the plate; $W_0 = \frac{\partial^2 \varpi_0}{\partial t^2}$ - acceleration of the foundation; μ - Poisson's ratio; E -Young's modulus; $\eta_1, \eta_2, \nu_1, \nu_2$ - linearization coefficients.

We now define condition of stability for the given system from (1). For this we use from vertical tangent method [2]. In the given problem vertical tangents of $\mathrm{mod}(x_{1jk}(i\omega))$ are below:

$$\frac{d(\mathrm{mod}(x_{1jk}(i\omega)))}{d\omega}=\frac{\frac{\partial f_1}{\partial x_2}\frac{\partial f_2}{\partial \omega}+(1-\frac{\partial f_2}{\partial x_2})\frac{\partial f_1}{\partial \omega}}{(1-\frac{\partial f_2}{\partial x_2})(1-\frac{\partial f_1}{\partial x_1})-\frac{\partial f_1}{\partial x_2}\frac{\partial f_2}{\partial x_1}}=\infty \tag{2}$$

where $f_1(\omega,x_2)=\mathrm{mod}(x_{1jk}(i\omega))=\left|x_{1jk}(i\omega)\right|$, $f_2(\omega,x_2)=\mathrm{mod}(x_2(i\omega))=\left|x_2(i\omega)\right|$.

It is a possible that (2) are changed with below equivalent equations:

$$(1-\frac{\partial f_2}{\partial x_2})(1-\frac{\partial f_1}{\partial x_1})-\frac{\partial f_1}{\partial x_2}\frac{\partial f_2}{\partial x_1}=0 \tag{3}$$

If $\frac{\partial f_2}{\partial x_2},\frac{\partial f_1}{\partial x_1},\frac{\partial f_2}{\partial x_1},\frac{\partial f_1}{\partial x_2}$ derivatives are calculated and to be substituted in (3), we will receive following result:

$$x_2^2(a_2\frac{\partial a_5}{\partial x_2}\frac{\partial a_6}{\partial x_1}-a_5\frac{\partial a_2}{\partial x_2}\frac{\partial a_6}{\partial x_1}-a_6\frac{\partial a_5}{\partial x_2}\frac{\partial a_2}{\partial x_1})-2(2a_2+x_2\frac{\partial a_2}{\partial x_2})a_5^{\frac{1}{2}}a_6^{\frac{3}{2}}-2a_5a_6x_2\frac{\partial a_2}{\partial x_1}=0 \tag{4}$$

where

$$a_2=A_2^2+B_2^2;\; a_5=(A_1a_1-d_{ik}\omega^2)^2+(B_1a_1)^2;\, a_6=(Ap_{ik}^2-b_1\omega^2)^2+(Bp_{ik}^2)^2;$$

CONCLUSION.

We can explore stability of the given dynamic system completely which is protected from vibrations when variable ω depend on construction parameters from this received result (4). Namely if (4) equation does not have ω roots , vibration of the given

dynamic system will be stable. Other wise it will not be stable.

REFERENCES.

[1] *Pavlovsky M.A.,Rijkov M.L., Yakovenko V.B., Dusmatov O.M.* Nonlinear problems of dynamics vibro-protected system, Kiev, 1997. pp.– 118.

[2] *Dusmatov O.M.* , Burkhanov X.M. Investigations of stability vibro-protected systems by amplitude of the vibrations. Uzbek Math Journal, 2006, №3, 36-39.

[3] *Pisarenko G.S., Yakovlev A.P., Matveev V.V.*) Vibro-absorbing properties constructive materials, Kiev, 1971.pp327 .

THE ADDITIONAL GIBBS STATES FOR INFINITE TODA CHAIN

Dzhalilov A. Akhadkulov K.
Samarkand State University
a_dzhalilov@yahoo.com, akhadkulov@yahoo.com

We consider the Hamiltonian of the Toda chain, written as an infinite formal series

$$H = \sum_n \left\{ \frac{p_n^2}{2} + \exp(q_n - q_{n-1}) \right\} \qquad (1)$$

The phase-space for such a system first was constructed by Sinai in [1] . As configurations of the model (1) we take countable locally-finite subsets $X \subset R^1$ such that for every $q \in X$ the right and left "neighbors" q^r and q^l are defined. It is not necessary that $q^l < q < q^r$, but we do require that

I. $(q^r)^l = q = (q^l)^r$;

II. the natural graph with the edges $q \to q^l,\ q \to q^r$ is connected;

III. if an order is introduced X by putting $q < q^r$, then $\lim_{q\uparrow} q = +\infty$ and $\lim_{q\downarrow} q = -\infty$.

We denote by Ω the configuration space of X. Let $X \in \Omega$. We consider maps $\tau : X \to R^2$ of the form $\tau(q) = (q, p) \in R^2$ for any $q \in X$. The phase-point is defined as the set $Y = \tau(X)$, together with the graph generated by X. The phase-space is the set $M = \bigcup_{X\in\Omega}\bigcup_{\tau} Y$. The right (or left) *semi-infinite tail* of Y is defined as the subset $Y^r \subset Y$ (or $Y^l \subset Y$) such that from $(q,p) \in Y^r$ (or $(q,p) \in Y^l$) it follows that $(q,p)^r \in Y^r$ (or $(q,p)^l \in Y^l$). The left (or right) *end* of Y^r (or Y^l) consists of those $z \in Y^r$ (or $z \in Y^l$) for which $z^l \notin Y^r$ (or $z^r \notin Y^l$). On M there is a natural σ-algebra of subsets γ generated by all sets the form:

$$A = \left\{ \begin{array}{l} Y \in M \mid Y \cap \Delta_0 = \{(q,\ p)\},\ Y \cap \Delta_1 = \{(q,\ p)^r\},\ \left((q,p)^r\right)^r \in \Delta_2, ..., \\ \underbrace{(...((q,\ p)^r)^r ...)^r}_{k\ \ times} \in \Delta_k,\ (q,\ p)^l \in \Delta_{-1},\ \left((q,p)^l\right)^l \in \Delta_{-2}, ..., \underbrace{(...((q,\ p)^l)^l ...)^l}_{m\ \ times} \in \Delta_{-m} \end{array} \right\}$$

where $\Delta_{-m}, ..., \Delta_{-1}, \Delta_0, ..., \Delta_k$ are Borel subsets in R^2. We consider the set $M(\Delta_0, \Delta_1) = \{Y \in M \mid Y \cap \Delta_0 = \{(q,\ p)\},\ Y \cap \Delta_1 = \{(q,\ p)^r\}$. We introduce the partition $\xi^r(\Delta_0, \Delta_1)$ (or $\xi^l(\Delta_0, \Delta_1)$) of $M(\Delta_0, \Delta_1)$ for which the element $C_{\xi^r}(\Delta_0, \Delta_1)$ (or $C_{\xi^l}(\Delta_0, \Delta_1)$) is given by the semi-infinite tail Y^r (or Y^l) whose left (or right) end lies in Δ_0 (Δ_1), and $z^l \in \Delta_0$ ($z^r \in \Delta_1$) consists of those Y for which $Y \cap \Delta_0 = \{z\},\ Y \cap \Delta_1 = \{z^r\}$ ($Y \cap \Delta_1 = \{z\},\ Y \cap \Delta_0 = \{z^l\}$), and the tail coincides with Y^r (or Y^l) . We define on $\xi^r(\Delta_0, \Delta_1)$ the σ–algebra $\gamma^r(\Delta_0, \Delta_1)$ generated by the sets of the form:

$$A = \left\{ Y \mid Y \cap \Delta_0' = \{(q,\ p)\}, Y \cap \Delta_1' = \{(q,\ p)^r\},\ \left((q,p)^r\right)^r \in \Delta_2, ..., \underbrace{(...((q,\ p)^r)^r ...)^r}_{k\ \ times} \in \Delta_k, \right\}$$

where $\Delta_0^{'} \subset \Delta_0$, $\Delta_1^{'} \subset \Delta_1$, $\Delta_2,...,\Delta_k$ are Borel subsets in R^2. The σ–algebra $\gamma^r(\Delta_0,\Delta_1)$ on $\xi^l(\Delta_0,\Delta_1)$ is defined in a similar way. On the set $M(\Delta_{-2},\Delta_{-1};\Delta_0,\Delta_1) =$ $= M(\Delta_{-2},\Delta_{-1}) \bigcap M(\Delta_0,\Delta_1) = \gamma^l(\Delta_{-2},\Delta_{-1}) \vee \gamma^r(\Delta_0,\Delta_1)$. We take Borel sets $\Delta_k \subset R^2$, $k=-2,-1,0,1$ with $\Delta_i \bigcap \Delta_j = \varnothing,\ i \neq j$. Let fix left tail Y^l with end points $z_1 \in \Delta_{-1},\ z_1^l \in \Delta_{-2}$ and right tail Y^r with end points $z_2 \in \Delta_0,\ z_2^r \in \Delta_1$. We consider the set $M(Y^l,Y^r) \subset M(\Delta_{-2},\Delta_{-1};\Delta_0,\Delta_1)$, consisting of the $Y \subset M(\Delta_{-2},\Delta_{-1};\Delta_0,\Delta_1)$ having Y^l, Y^r as their tails. It is clear that $M(Y^l,Y^r) = \bigcup_{k=0}^{\infty} M_k(Y^l,Y^r)$ where $M_k(Y^l,Y^r)$ consists of those Y which there are k-particles between right end z_1 tail Y^l and left end z_2 tail Y^r (in the sense of the graph corresponding to Y). It is known [2] that the equation of motion for an $m+l+1$-particle system can be written in the form.

$$\frac{\partial L_{m,l}}{\partial t} = \left[L_{m,l},\ A_{m,l}\right] \tag{2}$$

where $A_{m,l}$ and $L_{m,l}$ are matrices of order $m+l+1$ with the entries $a_{k,n} =$ $= \frac{1}{2} i(c_k^{1/2}\delta_{k,n+1} + c_n^{1/2}\delta_{k+1,n})$, $l_{k,n} = i(c_k^{1/2}\delta_{k,n+1} - c_n^{1/2}\delta_{k+1,n}) + p_k\delta_{k,n}$ where $c_k =$ $= \exp(q_k - q_{k-1}),\ k = \overline{(-m+1),\ l}$ and $\delta_{k,n}$ is the kronecker delta. In the case of infinity many particles , A and L in (2) must be regarded as infinite matrices. Denote by $H_{-m,l}^{(r)} = tr\, L_{m,l}^r$, $0 \le r \le m+l+1$. It is known that $H_{-m,l}^{(r)}$ is a first integral of the system (2). We consider the linear combination of four first integral $H_{-m,l}^{(i)}$, $i = \overline{1,4}$.

$$\hat{H}_{-m,l} = H_{-m,l}^{(4)} + \alpha_3 H_{-m,l}^{(3)} + \alpha_2 H_{-m,l}^{(2)} + \alpha_1 H_{-m,l}^{(1)} \tag{3}$$

Definition 1. *A conditional Gibbs distribution* for the inverse temperature $\beta > 0$ and the state Y^l and Y^r is defined as a probability distribution $M(Y^l,Y^r)$ such that for $k, m \ge 0$ its restriction to $M_{l+m}(Y^l,Y^r)$ has the density

$$\Xi_{\beta,\mu}^{-1} \cdot \exp\left\{-\beta(\hat{H}_{-m-2,l+2} + \mu(l+m+1))\right\} \tag{4}$$

where $(q_{-m-1},\ p_{-m-1}) = z_1,\ (q_{-m-2},\ p_{-m-2}) = z_1^l$, $(q_{l+1},\ p_{l+1}) = z_2,\ (q_{l+2},\ p_{l+2}) = z_2^r$, $(q_i,\ p_i) = (q_{i-1},\ p_{i-1})^r$, m, l are the number of particles lying on right and left sides of initial particle. $\Xi_{\beta,\mu}$ is a normalizing factor. According to this definition the integration is over $q_i \notin \bigcup_{k=-2}^{1} \Delta_k$, $i = \overline{-m,l}$. The parameter $\mu \in R^1$ is called the *chemical potential* .

Definition 2. *A limit Gibbs distribution* $\nu(\beta,\mu)$ for $\beta>0$ and μ is defined as a probability distribution on γ such that for any $M(\Delta_{-2},\Delta_{-1};\Delta_0,\Delta_1)$ the conditional distribution induced by it on the σ-subalgebra $\gamma^t(\Delta_{-2},\Delta_{-1};\Delta_0,\Delta_1)$ coincides almost everywhere with (4) a. e. with respect to $\nu(\beta,\mu)$.

Now we formulate our main results.

Theorem 1. For every (r,β,μ), $r\ge 1$, $\beta>0$ and μ satisfying the condition $4e^{\beta\mu}(2\pi/\beta)^{1/2}<1$ there is a limit Gibbs distribution $\nu(\beta,\mu)$.

By using the formal series for H_r $r\ge 1$, we form an infinite chain of Hamilton equations

$$\begin{cases} \dot q_k = \dfrac{\partial H_r}{\partial p_k}, \\ \dot p_k = -\dfrac{\partial H_r}{\partial q_k}, \quad k\in Z^1. \end{cases} \tag{5}$$

Next we consider the space with the probability measure (M,γ,ν_γ) .

Theorem 2. a) There is a subset $\widetilde M\subset M$ of full ν_γ -measure and a flow of automorphims $\{S_r^t\}_{t\in R^1}$ of (M,γ,ν_γ) defined up to sets of measure zero that preserve $\widetilde M$ and are such that for $Y\in\widetilde M$ the functions $Y_\gamma(t)=S_\gamma^t Y$ satisfy (5) for all $r\ge 1$.

b) Every measure ν_γ $\gamma\ge 1$ is invariant under the dynamics $\{S_r^t\}$, $t\in R^1$.

c) The flows $\{S_k^t\}_{t\in R^1}$ and $\{S_l^t\}_{t\in R^1}$ commute for all $k,l\in N$ that is $S_k^{t_1}\circ S_l^{t_2}=S_l^{t_2}\circ S_k^{t_1}$ for any $t_1,\ t_2\in R^1$.

REFERENCES

[1] Ya. G. Sinai, The ”commensurability-incommensurability” transition in one-dimensional chains, Zh. Eksper. Theoret. Physics. 83(1982), 1223-1231. MR 84i:82031. Sovet Physics JETP 56 (1982), 698-702.

[2] S.V. Manakov, Complete integrability and stochasticization of discrete dynamical systems, Zh. Eksper. Teoret. Physics. 67 (1974), 543-555.MR 52. 9938. Sovet Physics JETP 40 (1974), 269-274.

[3] I. P. Kornfel’d, Ya.G. Sinai and S. V. Fomin, *Ergodicheskaya teoriya* (Ergodik Theory), Nauka, Moskow 1980. MR 83a: 28017.

[4] A. A. Dzhalilov, The Toda chain in the thermodynamical limit, Russian Math. Surves. 1983. pp. 213-214 .

SINGULAR INTEGRAL OPERATORS WITH SHIFT ON THE CIRCLE

Dzlalilov A., Mardiyev R.
Samarkand State University, Uzbekistan

We study singular integral operators with shift on the circle S^1 which have several break points.

Consider a singular operator T: $L_p(S^1)$, p>1 defined by formula (see [1])

$$T=A_+P_+ - A_-P_- \quad (1)$$

Where $A_\pm = a_\pm(t)I - b_\pm(t)W$,

I is identity operator, $P_\pm = 1/2(I \pm S)$, S is singular integral operator with kernel Cauchy, W is operator with shift: $W\varphi(t)=\varphi(\alpha(t))$ kernel Cauchy, $a_\pm, b_\pm \in C(S^1)$, and $W\varphi(t)=\varphi(\alpha(t))$, where $\alpha(t)$ is a circle homeomorphism.

Suppose that $\alpha(t)=e^{2\pi i f(t)}$, $t\in[0.1]$ satisfies the following conditions (see [2])

1) the lift f(t) belong to class $C^{2+\varepsilon}(S^1\{f^{n_i}(x_0), i=1,m\})$, $\varepsilon>0$, f^k is the k-th iteration of f;
2) $x_0, f^{n_1}(x_0), f^{n_2}(x_0), \ldots, f^{n_m}(x_0)$ are the break points of f i,e.

$$\frac{Df^{n_i}(x_0-0)}{Df^{n_i}(x_0+0)}=\sigma_i(f)\neq 1 \;;$$

3) $\prod_{i=1}^{m}\sigma_i(f)=1$;
4) $\alpha(t)$ is ergodic w.r.t Lebesgue measure ℓ.

We define the transformation M: $C(S^1)\to[0,+\infty]$:

M(u)=exp$\{2\pi^{-1}\int_{[0,1]}\log|u(t)|d\ell$ if this integral converges , and M(u)=0 , if it deverges.

By definition put $\eta(A_\pm)=M(a_\pm)-M(b_\pm)$. Now we formulate our main result ,

Theorem 1. Assume that the shift $\alpha(t)$ satisfies the conditions 1) -4) and operators $A_\pm$ are invertible. Then the operators T defined by (1) is Neotherian.

Theorem 2. Suppose that $\alpha(t)$ satisfies conditions (1)-(4). Then $A_\pm$ is invertible if and only if one of the following conditions

- $\eta(A_\pm)>0$ and $\inf_{S^1}|a_\pm(t)|>0$;
- $\eta(A_\pm)>0$ and $\inf_{S^1}|b_\pm(t)|>0$;

hold.

REFERENCES

[1] A. B. Ottcher and Yu.I.Karlovich, Cauchy's singular integral operator and its beautiful spectrum. In: "Systems, approximation, singular integral operators, and related topics"(Bordeaux, 2000),Operator Theory: Advances and Applications, 129 (2001), 109–142.

[2] A.Dzhalilov. Piecewise – smooth conjugations of circle homeomorphism with break type singularities.-Theor. and Math. Phys. 1999 v.120, №2 , p.179-192.

INTEGRATION OF THE GENERAL KORTEWEG-DE VRIES EQUATION WITH SELF-CONSISTENT SOURCE OF INTEGRAL TYPE IN THE CLASS OF RAPIDLY DECREASING COMPLEX-VALUED FUNCTIONS

Hoitmetov U.A
Faculty of mathematics, Urgench State University
x_umid@mail.ru

In this paper we consider the following system of equations

$$u_t - Z_p(u) = 2\frac{\partial}{\partial x}\int_{-\infty}^{\infty}\phi(x,\eta)\phi(x,-\eta)d\eta\,, \tag{1}$$

$$L\phi = \eta^2\phi\,, \tag{2}$$

$$u(x,0) = u_0(x), \quad x \in R, \tag{3}$$

where L - Sturm-Liuouville's operator and the initial function $u_0(x)$ is a complex-valued and has the following properties:

1) for some $\varepsilon > 0$

$$\int_{-\infty}^{\infty}|u_0(x)|e^{\varepsilon|x|}dx < \infty\,, \tag{4}$$

2) operator $L(0)$ has exactly N complex eigenvalues $\lambda_1(0), \lambda_2(0), ..., \lambda_N(0)$ with multiplicities $m_1(0), m_2(0),..., m_N(0)$ and has no spectral features.

In this problem the function $\phi(x,\eta,t)$ solution of equation (2) determined by asymptotic

$$\phi(x,\eta,t) = h(\eta,t)e^{-i\eta x} + \bar{\bar{o}}(1), \quad x \to \infty \tag{5}$$

where $h(\eta,t)$ - initially given continuous function satisfying

$$h(-\eta,t) = \overline{h(\eta,t)}, \quad \int_{-\infty}^{\infty}|h(\eta,t)|^2 d\eta < \infty\,, \tag{6}$$

for all nonnegative values of t.

Suppose that the function $u(x,t) = \mathrm{Re}\,u(x,t) + i\,\mathrm{Im}\,u(x,t)$ is sufficiently smooth and sufficiently rapidly tends to their limits when $x \to \pm\infty$, so that

$$\int_{-\infty}^{\infty}\left(\left|\frac{\partial^j u(x,t)}{\partial x^j}\right| e^{\varepsilon|x|}\right)dx < \infty, \quad j = 0,1,2,3. \qquad (7)$$

The main result of this paper is the following

Theorem. If $u(x,t)$, $\phi(x,\eta,t)$ are the solution of problem (1) - (7), then the evolution of the scattering data of the operator $L(t)$ with the potential $u(x,t)$ is as follows

$$S_t = \left[ik\sum_{l=0}^{p} c_l (2k^2)^l - 2\pi|h(k,t)| + 2iV.p.\int_{-\infty}^{\infty}\frac{|h(\eta,t)|^2}{k+\eta}d\eta \right] S,$$

$$\lambda_n(t) = \lambda_n(0),$$

$$\frac{\partial\chi_r^n}{\partial t} = \sum_{l=0}^{r}\left[\sum_{q=0}^{p}\left(ic_q 2^q \frac{1}{l!}\frac{(2q+1)!}{(2q+1-l)!}k_n^{2q+1-l} + 2\int_{-\infty}^{\infty}\frac{(-1)^l |h(\eta,t)|^2}{i(k_n+\eta)^{l+1}(1-S(\eta)S(-\eta))}d\eta\right)\right]\chi_{r-l}^n,$$

$$n = 1, 2, ..., N, \; r = 0,1,2,...,m_n - 1.$$

Resulting equalities completely determine the evolution of scattering data that allows us to apply the inverse scattering method for solving the problem (1) - (7).

REFERENCES

Blaschak V.A. (1968) Analogue of the inverse scattering problem for nonself-adjoint operator. I //Diff. eq. v IV, # 8, 1519-1533.

Blaschak V.A. (1968) Analogue of the inverse scattering problem for nonself-adjoint operator. I //Diff. eq. v IV, # 10, 1915-1924..

Gardner C.S., Green I.M., Kruskal M.D., Miura R.M. (1967) Method for solving the Korteweg-de Vries equation // Phys.Rev. Lett. v.19. – pp. 1095-1097.

Hoitmetov U.A. (2007) Integration of the general KdV equation with self-consistent source in the class of rapidly decreasing complex-valued functions. Dokl. AN - Uzbekistan, 5, 16-20.

ASYMPTOTICS OF THE EIGENVALUES DISCRETE SCHRODINGER OPERATORS

Holmatov Sh.Yu.
Samarkand State University, Samarkand, Uzbekistan,
Shohruhhon1@mail.ru

INTRODUCTION

Let $T^d = \left(R/2\pi Z\right)^d = (-\pi,\pi]^d$ be the d - dimensional torus (Brillouin zone), and $L_2(T^d)$ be the Hilbert space of square-integrable functions on T^d.

We consider a family $h_\mu(k),\ k\in T^d$ of the discrete Schrodinger operators acting in Hilbert space $L_2(T^d)$ as

$$h_\mu(k) = h_0(k) - \mu V.$$

Here $h_0(k),\ k\in T^d$ is the multiplication operator by the function $E_k(q)$, where $E_k(\cdot)$ – real-analytic function on T^d and there exists such open connected set $G\subset T^d$ that for any $k\in T^d$ the function $E_k(\cdot)$ has a unique non degenerated minimum. The perturbation v is integral operator of rank one:

$$(v\,f)(q) = \int_{T^d} f(q)\,dq,\ \ f\in L_2(T^d).$$

The essential spectrum $\sigma_{ess}(h_\mu(k))$ of $h_\mu(k)$ fills the following segment:

$$\left[\min_{q\in T^d} E_k(q),\ \max_{q\in T^d} E_k(q)\right].$$

The operator $h_\mu(k)$ has no more than one eigenvalue below $E_{\min}(k) = \inf\sigma_{ess}(h_\mu(k))$.

RESULTS AND DISCUSSIONS

Let $d\ge 3$, $k\in G$ and $\nu(k,z) = \int_{T^d} \frac{dq}{E_k(q)-z},\ z\le E_{\min}(k)$.

In order to study the dependance of the eigenvalue $z(\mu,k)$ of $h_\mu(k)$ on the quasi-momentum $k\in T^d$ we introduce the sets

$$S_<(\mu) = \{k\in G:\ \mu(k)<\mu\},\quad S_=(\mu) = \{k\in G:\ \mu(k)=\mu\},$$

where $\mu(k) = (\nu(k))^{-1} = [\nu(k,E_{\min}(k))]^{-1}$. Since the function $\mu(k)$ is not constant there exists such $\mu^*\in R$ that $\inf_{k\in G}\mu(k) < \mu^* < \sup_{k\in G}\mu(k)$ and all above introduced sets are not empty at $\mu=\mu^*$.

Theorem 1. For any $k \in S_{<}(\mu^*)$ the operator $h_{\mu^*}(k)$ has a unique eigenvalue $z^*(k) = z(\mu^*, k)$ below the essential spectrum. Moreover, $z^*(k)$ is analytic in $S_{<}(\mu^*)$ and for any $k^* \in S_{=}(\mu^*)$ the following asymptotics hold as $k \to k^*, k \in S_{<}(\mu^*)$:

(i) if $d = 3$, then $z^*(k) = E_{\min}(k^*) + \sum_{j=1}^{d} \frac{\partial E_{\min}(k^*)}{\partial k_j}(k_j - k_j^*) + O(|k - k^*|^2)$

where $|k - k^*|^2 = \sum_{j=1}^{d}(k_j - k_j^*)^2$;

(ii) if $d = 4$, then $z^*(k) = E_{\min}(k^*) + \sum_{j=1}^{d} \frac{\partial E_{\min}(k^*)}{\partial k_j}(k_j - k_j^*) + o(|k - k^*|)$;

(iii) if $d \geq 5$, then $z^*(\mu) = E_{\min}(k^*) +$

$$+\sum_{j=1}^{d}\left[\frac{\partial E_{\min}(k^*)}{\partial k_j} - \left(\frac{\partial \nu}{\partial z}(k,z)\Big|_{z=E_{\min}(k^*)}\right)^{-1}\frac{\partial \nu(k^*)}{\partial k_j}\right](k_j - k_j^*) + O(|k - k^*|^2).$$

We remark that the order of the asymptotics depends not only on k^*, but also the dimension *d* of the torus T^d and the function $E_k(\cdot)$.

Corollary. If $d = 3$ (resp. $d = 4$) the asymptotics of $z^*(k)$ as $k \to k^*$ has the order $E_{\min}(k^*) + O(|k - k^*|^2)$ (resp. $E_{\min}(k^*) + o(|k - k^*|)$) if and only if when the gradients of the functions $E_{\min}(k)$ and $\nu(k)$ are equal to zero at $k = k^*$.

Remark. Note that in case $d \geq 5$ asymptotics of $z^*(k)$ as $k \to k^*$ has the order $E_{\min}(k^*) + O(|k - k^*|^2)$ either: (i) if the gradients of $E_{\min}(k)$ and $\nu(k)$ are equal to zero at $k = k^*$; or: (ii) if the gradients of $E_{\min}(k)$ and $\nu(k)$ are not equal to zero at $k = k^*$ simultaneously, but the following relations hold:

$$\frac{\partial E_{\min}(k^*)}{\partial k_j} - \left(\frac{\partial \nu}{\partial z}(k,z)\Big|_{z=E_{\min}(k^*)}\right)^{-1}\frac{\partial \nu(k^*)}{\partial k_j} = 0, \; j = 1,\dots,d.$$

REFERENCES

[1] S.Albeverio, S.N.Lakaev, K.A. Makarov, Z.I.Muminov: Comm.Math.Phys. **262** (2006), 91--115.

[2] S.N.Lakaev, Sh.Yu.Holmatov: J. Phys. A: Math. Theor. **44** (2011)

SYNTHESIS OF ALGORITHMS GENERALIZED ESTIMATION DYNAMIC SYSTEMS ON THE BASIS OF REGULAR METHODS

Igamberdiyev H.Z., Abdurakhmanova Y.M., Zaripov O.O.
Faculty of electronics and automatics,
Tashkent State Technical University

uz3121@rambler.ru

INTRODUCTION

In the management theory extensive areas of theoretical researches and technical applications where for result reception sharing of methods estimation is necessary, identifications and checks of hypotheses [1] exist difficult dynamic systems. The considered approach leads to some subclass of problems optimum nonlinear estimation. The traditional approach to working out of algorithms of management and a filtration consists in transformation of the equations of dynamics of object and measurements to a demanded initial form a method of expansion of space of conditions or a division method. Such association of diverse systems on the dynamic properties can lead to a resultant to system with "rigid" character of movement and, as consequence, to bad conditionality of computing algorithms. The analysis of the scientific and technical literature of last years, concerning researches on working out of methods and algorithms of the steady generalized condition of dynamic systems, testifies to achievement of considerable theoretical and practical results in this area. Various ways of construction of the high-precision filters functioning in the conditions of various degree of aprioristic uncertainty exist and are developed. Despite importance of a considered problem, now the uniform scientifically proved methodology of synthesis of adaptive filters in the conditions of aprioristic uncertainty has not completely developed yet, that is connected with an imperative need of the decision of some methodological problems, in particular, with attraction of concepts of return problems of dynamics of operated objects and regular methods [2].

RESULT AND DISCUSSION

In work on the basis of concepts of the system analysis, identification, a dynamic filtration and methods of the decision of incorrect problems are developed regularization uniform algorithms co-ordinate and parametrical estimation dynamic objects of management in the conditions of aprioristic uncertainty. As a result following results are received:

Various correct statements on Tikhonov of a problem of minimization material functional are analyzed. Regular algorithms estimation dynamic systems in the conditions of presence of errors in model of object of management are offered. Are offered regularization algorithms of correction of results joint estimation a condition and parameters of dynamic objects of management. Algorithms of the decision of a problem generalized estimation on the basis of a method of updating of monotonous displays are offered. Algorithms of the decision of a problem estimation on the basis of a method of monotonous variation inequalities both in the form of return problems, and in the form of a problem of calculation of value of the nonlinear unlimited operator are offered. Are developed iterative regularization algorithms generalized estimation on the basis of base methods of a zero order and methods of type of Newton. Iterative algorithms steady estimation conditions of dynamic systems and an estimation of speed of convergence of approximation iterative regularization are offered.

CONCLUSION

Developed regularization algorithms generalized estimation dynamic systems can be taken as a principle synthesis of an information subsystem uniform estimation and identifications in the general structure of systems of adaptive optimum control of the dynamic objects, promoting increase of efficiency of their functioning.

REFERENCES

[1] Beloglazov I.N., Kazarin S.N. Joint optimum estimation, identification and check of hypotheses in discrete dynamic systems // News Academy of sciences. Theory and control systems, №4, 1998. – pp.26-43.

[2] Bakushinskiy A.B., Goncharsky A.V. Iterative methods of the decision of incorrect problems. - M: Science, 1989. – 128 p.

REGULAR SYNTHESIS ALGORITHMS IT IS ADAPTIVE INVARIANT CONTROL SYSTEMS OF DYNAMIC OBJECTS

Igamberdiyev H.Z.,Yusupbekov A.N.
Faculty of electronics and automatics,Tashkent State Technical University
uz3121@rambler.ru

INTRODUCTION

The decision of problems of optimum control of many technological objects with use of their mathematical model can be carried out a control system working on the basis of a principle of indemnification of indignations. Maintenance of normal functioning of control systems with dynamic objects is rather frequent becomes complicated that revolting influences in most cases do not give in to direct measurement. The conditions of absolute or full invariance providing exact indemnification of any indignations, as a rule, physically not realized. Easing of these conditions of the invariance, providing indemnification of a known class of indignations, leads to concept of selective or selective invariance of control systems. Presence of the aprioristic information on the indignations, following their those or other models of indignations, has allowed to create the systems, capable to counteract wider classes of indignations. At insufficient level of the aprioristic information on indignations it is expedient to use the adaptive approach which allows to receive an estimation of these classes of indignations in the conditions of uncertainty and to use it for their indemnification. In connection with above noted rather tempting the way of construction of the operating systems, based on concepts of designing of invariant systems in the conditions of uncertain indignations is represented, and working out of effective synthesis algorithms is adaptive the invariant systems which are not demanding full aprioristic knowledge of object of management and conditions of its functioning.

RESULT AND DISCUSSION

In work on the basis of concepts of the system analysis, a dynamic filtration and methods of the decision of incorrect problems [1,2] regular synthesis algorithms are developed is adaptive invariant control systems of technological objects in the presence of uncertain indignations. Synthesis algorithms of the adapting regulators are developed, allowing to make steady estimation operating influences in control systems taking into account the correlated properties of real external indignations without expansion of the equations of dynamics of object and measurements. Algorithms estimation operating influences on the basis of concepts of the adaptive filtration are developed, allowing regularization problem of adaptive fine tuning and to realize steady estimation elements of optimum factor of strengthening Kalman filter in the conditions of aprioristic uncertainty. Algorithms estimation on a basis decomposition the approaches are developed, allowing to raise accuracy of calculation of operating influences and quality indicators of processes of regulation in the synthesized is adaptive to an invariant control system. Regular synthesis algorithms of suboptimum regulators with operational indignations on an exit and management and the limited external indignation are offered. Algorithms of research of convergence of the adaptive filter are developed at unknown statistical characteristics of a useful signal and a hindrance. Algorithms optimum estimation a vector of errors of indignant filter Kalmana on the basis of theory methods of conditionally-Gauss filtration are offered at use as a signal of supervision of a vector of target signals of the indignant filter. Algorithms of suppression of the limited external indignations on the basis of a method invariant ellipsoid analyzed. Algorithms of restoration of entrance signals of dynamic system by results of exit measurement in conditions when errors of measurement of an exit are limited by the set number in Gilbert space are offered.

CONCLUSION

Developed regularization synthesis algorithms and computing schemes of their practical realization promote increase of efficiency of functioning of the industrial is adaptive invariant control systems of dynamic objects in the presence of uncertain indignations.

REFERENCES

[1] Sinitsyn I.N. Filter of Kalman and Pugachev. - M: University book, Logos, 2006. - 640 p.

[2] Bakushinskiy A.B., Kokurin M.Yu. Iterative methods of the decision of the irregular equations. - M: Lenand, 2006. - 214 p.

ON THE FOURIER TRANSFORM OF MEASURES SUPPORTED ON CURVES WITH TORSION[1]

Ikromova D., Soleeva N.
Department of Mathematics, Samarkand State University
dikromova_89@rambler.ru

In this article, we derive sharp estimates for the Fourier transform of smooth measures concentrated on smooth curves with torsion in $\mathbb{R}^3$. Moreover, we prove that the Fourier transform belongs to the Lorentz space $L^{7\infty}(\mathbb{R}^3)$. The result is sharp in the sense that it does not belong to the space $L^{7q}(\mathbb{R}^3)$ for any $q<\infty$.

If $\gamma \subset \mathbb{R}^3$ is a smooth curve and $d\mu$ is the measure given by $d\mu = \psi(x)dl$, then $d\mu$ is called to be a measure supported on the curve γ, where $\psi \in C_0^\infty(\mathbb{R}^3)$ is a smooth function with compact support, dl is an element of curve length. So, for any function $f \in C(R)$ we have a number given by

$$< f, d\mu >:= \int_{\mathbb{R}^3} f d\mu = \int_\gamma f\psi dl.(1)$$

Thus by (1) we get a linear functional. It is easy to see that the functional is a distribution on $\mathbb{R}^3$, e.g. a continuous linear functional on the Schwartz class of functions [3]. Actually, it is a distribution with compact support. The Fourier transform of the measure is denoted by

$$d\mu(\xi) = \int_\gamma e^{i(x,\xi)}\psi dl.$$

It is well-known that if γ is a smooth curve with torsion, more precisely if for any $x \in \sup(\psi) \cap \gamma$ curvature and torsion of the curve are nonzero, then we have the relation

$d\mu \in L^p(\mathbb{R}^3)$ [5] for any $p>7$. Moreover, if $\psi \neq 0$, then $d\mu \notin L^7(\mathbb{R}^3)$ [5] (also see [1]). Where $L^p(\mathbb{R}^3)$ is the of class integrable functions with degree p.

We show that the function $d\mu$ belongs to the space $L^{7\infty}(\mathbb{R}^3)$.

Firstly, we define the space $L^{p\infty}(\mathbb{R}^3)$ [6]. Assume that f is a measurable function defined on $\mathbb{R}^3$, then for any $s>0$ the function $\lambda(s) = |x \in \mathbb{R}^3 : |f(x)| > s|$, where $|A|$ denotes the Lebesgue measure of the set A. A space $L^{p\infty}(\mathbb{R}^3)$ is the class of functions satisfying the condition:

$$\sup_{s>0}\{s[\lambda(s)]^{1/p}\} < \infty.$$

By the classical Tchebychev inequality we have the relation $L^p(\mathbb{R}^3) \subset L^{p\infty}(\mathbb{R}^3)$ [6].

[1] We acknowledge the support for this work by the Uzbek Consul Sciences grant No. F1.006

The following Theorem is the main result of this paper.

Theorem 0.1 *If $\gamma \subset \mathrm{R}^3$ is a smooth curve with torsion, then $\hat{d\mu} \in L^{7\infty}(\mathrm{R}^3)$.*

Remark 0.2 *It is easy to show that if $\psi \not\equiv 0$ then $\hat{d\mu} \notin L^{7q}(\mathrm{R}^3)$ for any $q < \infty$. See [6] for the definition of the Lorentz space $L^{pq}(\mathrm{R}^3)$.*

Firstly, we we prove the theorem for the model curve with torsion $\gamma = (t, t^2, t^3)$. Then we discuss a proof for general smooth curves with torsion.

Proof. Without loss of generality we may assume that ψ is concentrated in a sufficiently small neighborhood of $(0,0,0) \in \mathrm{R}^3$. For functions ψ with compact support the result follows from usual partition of unity arguments. So, we dealt with oscillatory integral

$$d\mu(\xi) := \int_{\mathrm{R}} a(t) e^{i(\xi_1 t + \xi_2 t^2 + \xi_3 t^3)} dt,$$

where $a(t) = \psi(t, t^2, t^3)\sqrt{1 + 4t^2 + 9t^4}$ by our assumption a is a smooth function concentrated in a sufficiently small neighborhood of the origin. If $\xi \in A_1 := \{\xi : |\xi_1| \geq \max\{|\xi_2|, |\xi_3|\}\}$, then the phase function

$$\phi := \xi_1 t + \xi_2 t^2 + \xi_3 t^3 = \xi_1 \left(t + \frac{\xi_2}{\xi_1} t^2 + \frac{\xi_3}{\xi_1} t^3 \right)$$

has no critical points on the set $\{4 |t| \leq 1\}$.

By our assumption we may suppose that $a \in C_0^\infty(-1/4, 1/4)$. So, if $|t| \geq 1/4$ then $a(t) \equiv 0$.

Now, we use integration by parts arguments and obtain $d\mu(\xi) = O(|\xi|^{-3})(as |\xi| \to \infty)$.

Since $d\mu$ is a bounded function then we have the relation $d\mu \in L^p(A)$ for any $p \geq 1$.

Now, we will investigate behavior of the function on the set $A_2 := \{\xi : |\xi_2| \geq \max\{|\xi_1|, |\xi_2|\}\}$. The we have

$$\phi = \xi_2 \left(\frac{\xi_1}{\xi_2} t + \frac{\xi_3}{\xi_2} t^3 + t^2 \right).$$

If $|t| < 1/4$ then we have the inequality $|\phi''(t)| \geq |\xi_2|/2$. So by classical Van-Der Corput theorem we have the inequality [1]:

$$|d\mu(\xi)| \leq \frac{c}{|\xi|^{1/2}}.$$

Therefore for any $p > 6$ we have the relation $d\mu \in L^p(A_1 \cup A_2)$. Now for the set $A_1 \cup A_2$ a required result follows from the classical Tchebychev's inequality [6]. Now, we will consider the oscillatory integral on the set $A_3 := \{\xi : |\xi_3| \geq \max\{|\xi_1|, |\xi_2|\}\}$. For this reason we write the phase function in the form:

$$\phi = \xi_3(t^3 + \sigma_2 t^2 + \sigma_1 t),$$

where $\sigma_j = \xi_j/\xi_3$, here $j = 1,2$. Now, we use the following change of variables $t + \sigma_2/3 \mapsto t$ and have

$$d\mu(\xi) = e^{i\xi_3 b(\sigma)} \int_{\mathrm{R}} a(t - \sigma_2/3) e^{i\xi_3(t^3 + (\sigma_1 - \sigma_2^2/3)t)} dt,$$

where $b(\sigma) := 2\sigma_2^3/27 - \sigma_1\sigma_2/3$. We consider a subset of the set A_3 denoting by

$$A_M = \{\xi \in A_3 : |\sigma_1 - \sigma_2^2/3| < M |\xi_3|^{-2/3}\},$$

where M is a sufficiently big but, fixed positive real number. Due to the Van-der Corput theorem we have the bound [1]:

$$|d\mu(\xi)| \leq \frac{c}{|\xi_3|^{1/3}}$$

for any ξ_3. If s is a number bigger than a fixed positive number $\varepsilon > 0$, then we have a required bound by using standard arguments. Therefore we have to estimate $\lambda(s)$ for sufficiently small positive number s. The obtained estimates give the following bound:

$$|A_M \bigcap \{\xi : |d\mu(\xi)| > s\}| = \int_{|\widehat{d\mu(\xi)}| > s} d\xi_1 d\xi_2 d\xi_3 = 2\int_0^{c/s^3} \xi_2^3 d\xi_3 \int_{A_\mu} d\sigma_1 \sigma_2 \leq$$

$$2M \int_0^{c/s^{-3}} \xi_3^{2-2/3} d\xi_3 = \frac{6Mc^{7/3}}{7s^7}.$$

Now we take a natural number N satisfying the condition $2^N < \delta |\xi_3|^{2/3}$, where δ is a sufficiently small fixed positive real number. Now we will introduce the following sets:

$$A^n = \{\xi : 2^n |\xi_3|^{-2/3} < |\sigma_1 - \sigma_2^2/3| < 2^{n+1} |\xi_3|^{-2/3}$$

where n is a natural number satisfying the condition $\log_2^M \leq n \leq N$. We use the following inequality for the oscillatory integral [4]:

$$|d\mu(\xi)| \leq \frac{c}{|\xi|^{1/2} |\sigma_1 - \sigma_2^2/3|^{1/4}}.$$

Then the measure of the set $\tilde{A}_n := A^n \bigcap \{\xi : |d\mu(\xi)| > s\}$ is estimated by the following:

$$|\tilde{A}_n| = \int_{\tilde{A}_n} d\xi_1 d\xi_2 d\xi_3 \leq 2^{n+1} \int_0^{c/s^3 2^{3n/4}} |\xi_3|^{2-2/3} \, d\xi_3 \leq c 2^{-3n/4}/s^7.$$

Thus $|\bigcup_n A^n \bigcap \{\xi :|\hat{d\mu}(\xi)|> s|\le c/s^7$. Now, we will consider the behavior of the integral on the set $A_\delta = \{\xi :|\sigma_1 - \sigma_2^2/3|\ge \delta\}$. Then we have the following estimate:

$$|d\mu(\xi)|\le \frac{c}{|\xi_3|^{1/2}}$$

for the oscillatory integral. Since the relation $d\mu \in L^p(A_\delta)$ holds for any $p > 6$ the measure of the set $A_\delta \bigcap \{\xi :|d\mu(\xi)|> s\}$ can be estimated by the classical Tchebychev's inequality to have a required bound. Thus the main Theorem is proved for the model curve with torsion.

Now, we will discuss the general case. If γ is a curve with torsion then it by linear change of the space $\mathbb{R}^3$, can be written as $(t, t^2\varphi_1(t), t^3\varphi_2(t))$, where φ_1, φ_2 are smooth functions satisfying the condition $\varphi_1(0)\varphi_2(0) \ne 0$. For the analogical sets A_1, A_2 the above-mentioned estimates hold. To estimate the oscillatory integral on the set A_3 we use the following result about normal form of the functions.

If σ_1, σ_2 are sufficiently small numbers then there exists a change of variables $t = t(T, \sigma_1, \sigma_2)$ such that the function

$$\phi(t, \sigma_1, \sigma_2) := t^3\varphi_2(t) + \sigma_2 t^2 \varphi_1(t) + \sigma_1 t$$

can be reduced to the form [2]:

$$\phi(t(T, \sigma_1, \sigma_2), \sigma_1, \sigma_2) := T^3 + \Sigma(\sigma_1, \sigma_2)T + g(\sigma_1, \sigma_2),$$

where Σ is a smooth function satisfying the conditions:

$$\Sigma(0,0) = 0, \quad \frac{\partial \Sigma(0,0)}{\partial \sigma_1} \ne 0.$$

Therefore we can use our method to such oscillatory integral. The main Theorem is proved.

REFERENCES

[1] G. I. Arhipov, A. A. Karacuba, and V. N. Čubarikov. Trigonometric integrals. *Izv. Akad. Nauk SSSR Ser. Mat.*, 43(5):971--1003, 1197, 1979.

[2] V. I. Arnol'd, S.M.Guseyn-zade, A.N. Varchenko. Singularity of differentiable maps V.1 *Moscow. Nauka*, 1982.

[3] I.M. Gel'fand and G. E. Shilov Generalized functions. Academic Press, New York, 1964.

[4] I.A. Ikromov, Invariant estimates of two-dimensional trigonometric integrals. Math. USSR Sb. Vol. 76 (1990), No. 2: 473-488.

[5] I. A. Ikromov, Summability of Oscillatory Integrals over Parameters and the Boundedness Problem for Fourier Transform on Curves, Mathematical Notes, 2010, Vol. 87, No. 5, 734-755.

[6] E. M. Stein and Weiss G. *Introduction to Fourier analysis on Euclidean Spaces Princeton Mathematical Series*. Princeton University Press, Princeton, NJ, 1971.

ON ASYMPTOTIC PROPERTIES OF TOTAL PROGENY IN Q-PROCESSES

Imomov A.
Chair of Mathematical Analysis and Algebra
Faculty of Physics and Mathematics, Karshi State University
imomov_azam@mail.ru

INTRODUCTION

In this paper we research asymptotic properties of so-called Q-processes. Such model of Branching Processes is allocated from others, that its trajectory never to die out. Therefore the Q-process can be connected with Galton-Watson Branching Process (GWP) allowing Immigration of a special form. We establish the Law of Large Numbers and Central Limit Theorem analogue for the total progeny in Q-processes .

The number $Z_n,\ n\in\mathbf{N}_0\left(\mathbf{N}_0=\{0\}\cup\{\mathbf{N}=1,2,...\}\right)$, of individuals in the nth generation of GWP is given recursively by

$$Z_0=1,\ Z_{n+1}=\sum_{k=1}^{Z_n}\zeta_{nk}$$

where random variables ζ_{nk} are i.i.d. and denotes the offspring of the ith individual in the ith generation; let $A=\mathbf{E}\zeta_{nk}$ is finite. The evolution law of GWP is regulated by generating function (g.f.) $F(s):=\sum_{k\in\mathbf{N}_0}p_k s^k$, where $p_k=\mathbf{P}\{Z_1\equiv\zeta_{01}=k\in\mathbf{N}_0\}$. The g.f. $F_n(s)=\mathbf{E}s^{Z_n}$ determined by n-step iteration of $F(s)$; see, e.g. [1, pp.1-2].

In fact the Q-process $\{W_n,\ n\in\mathbf{N}_0\}$ is the homogeneous Markov chain with states on $\mathbf{N}$ and transition probabilities $Q_{ij}^{(n)}:=\mathbf{P}\{W_{n+k}=j\,|\,W_k=i\},\ \ n,i,j,k\in\mathbf{N}$ defined by

$$Q_{ij}^{(n)}=\lim_{m\to\infty}\mathbf{P}\{Z_{n+k}=j\,|\,Z_k=i,\ Z_{n+k+m}>0\}=\mathbf{P}\{Z_{n+k}=j\,|\,Z_\infty>0\};$$

these are (on details see [1, pp.56-58])

$$Q_{ij}^{(n)}=\frac{jq^{j-i}}{i\beta^n}\mathbf{P}\{Z_{n+k}=j\,|\,Z_k=i\},$$

where $\beta = F'(q) < 1$ and $q \in (0,1]$ is the smallest root of $s = F(s)$ in $s \in [0,1]$. Denote $W_n^{(i)}(s) := \sum_{j \in \mathbf{N}_0} Q_{ij}^{(n)} s^j$. Considering iteration for $F_n(s)$ we will obtain the relation

$$W_n^{(i)}(s) = \left[\frac{F_n(qs)}{q}\right]^{i-1} W_n(s);$$

see [2]. Where the g.f. $W_n(s) = W_n^{(1)}(s) = \mathbf{E}s^{W_n}$ and is

$$W_n(s) = s\frac{F_n'(qs)}{\beta^n}, \quad n \in \mathbf{N} \tag{1}$$

By differentiation (1), we calculate the mean of the W_n in the form of

$$\mathbf{E}W_n = \begin{cases} (\alpha-1)n+1, & A=1 \\ 1+\gamma\left(1+\beta^n\right), & A \neq 1 \end{cases},$$

where $\gamma := (\alpha-1)/(1-\beta)$, and $\alpha = W_1'(1) = 1 + qF''(q)/\beta$.

MAIN RESULTS

Let's put into consideration variables $Y_n := \sum_{k=0}^{n-1} Z_k$, $Y := \lim_{n\to\infty} Y_n$. The variable Y_n is interpreted as the total progeny of zero particle till the moment of time n of process $\{Z_n,\, n \in \mathbf{N}_0\}$. The total progeny till the moment of time n in Q-processes we designate as $S_n = W_0 + W_1 + \ldots + W_{n-1}$, $S_0 = 0$.

We will need to following g.f.s:

$$H_n(y;x) := \mathbf{E}\left[y^{Z_n} x^{Y_n}\right], \quad h(x) := \mathbf{E}x^Y, \quad \Delta_n(y;x) := h(x) - H_n(y;x);$$

$$G_n(y;x) := \mathbf{E}\left[y^{Z_n} x^{Y_n} \,\middle|\, Z_n > 0\right], \quad R_n(y;x) := \mathbf{E}\left[y^{W_n} x^{S_n}\right],$$

in set of $\mathbf{D} := \left\{(y;x) \in \mathbf{R}^2 : |y| \leq 1,\ |x| \leq 1,\ \sqrt{(y-1)^2 + (x-1)^2} \geq r \in \mathbf{R}_+\right\}$.

The further reasoning gives us the formula

$$R_n(y;x) = -\frac{y}{A^n}\frac{\partial \Delta_n(y;x)}{\partial y}. \tag{2}$$

From (2) we will immediately receive, that

$$\mathbf{E}S_n = \begin{cases} (1+\gamma)n + \gamma\dfrac{1-A^n}{1-A}, & A < 1 \\[2ex] \dfrac{\alpha-1}{2}n(n+1) + n, & A = 1 \end{cases}. \tag{3}$$

Introduce a function $u(x) = xF'(h(x))$. The following lemmas we will write without the proofs which will be useful further.

Lemma 1. *Let* $A < 1$. *Then as* $x \to 1$

$$u(x) \sim Ax\left[1 - \gamma(1-x)\right]; \tag{4}$$

Lemma 2. *Let* $A < 1$. *Then lengthways small vicinity of the point* $x = 1$ *the following asymptotic representation holds:*

$$\left.\frac{\partial \Delta_n(y;x)}{\partial y}\right|_{y=1} \sim -u^n(x), \quad n \to \infty. \tag{5}$$

Theorem. *If* $A < 1$, *then* $(2\gamma n)^{-1/2}\left[S_n - \mathbf{E}S_n\right]$ *converges in distribution as* $n \to \infty$ *to a Gaussian random variable with zero mean and variance of one.*

Proof. From (2) and (5) we receive the asymptotic formula for g.f. of variable S_n as $x \to 1$ in the form of

$$\mathbf{E}x^{S_n} \sim \frac{1}{A^n}u^n(x), \quad n \to \infty. \tag{6}$$

Let $\psi_n(\theta)$, $\theta > 0$ be Laplace transform of $(2\gamma n)^{-1/2}\left[S_n - \mathbf{E}S_n\right]$ and put $\theta_n := e^{-\theta/\sqrt{2\gamma n}}$. Since $\theta_n \to 1$, $n \to \infty$, it is directly checked from (3), (6) that

$$\psi_n(\theta) \sim \left[\frac{1}{A}\theta_n^{-(1+\gamma)}u(\theta_n)\right]^n, \quad n \to \infty. \tag{7}$$

Further in (7) we use representation (4) and through elementary reasoning it is concluded that

$$\psi_n(\theta) \sim \left[1 + \frac{\theta^2}{2n}\right]^n \longrightarrow e^{\theta^2/2}, \quad n \to \infty.$$

The theorem is proved.

REFERENCES

[1] Athreya K.B., Ney P.E. (**1972**). Branching Processes. Springer, New York.

[2] Imomov A.A. (**2010**). Q-processes as Galton-Watson branching processes with immigration. Proc. of IX FAMET Conference, Russia. 148–152. (in Russian)

CONTINUATION OF THE SOLUTION OF THE INHOMOGENEOUS CAUCHY – RIEMANN EQUATION

Ishankulov T.
Department of Mechanics and Mathematics, Samarkand State university,
itolib@rambler.ru

Solvability conditions and continuation formulas of boundary value problem for the inhomogeneous Cauchy – Riemann equation are obtained.

Let D_i is a simple connected domain on the plane of complex variable $z = x + iy$, bounded with simple closed rectifiable Jordan curve L. Consider the Dirichlet problem for the inhomogeneous Cauchy – Riemann equation [3]

$$\partial_{\bar{z}} W(z) = f(z), \quad z \in D_i, \partial_{\bar{z}} = \frac{1}{2}\left(\frac{\partial}{\partial x} + i\frac{\partial}{\partial y}\right), \tag{1}$$

$$W(z) = \gamma(z), \quad z \in L. \tag{2}$$

This problem is over determined that is ill posed [4].

We establish the analog of Golubev – Privalov criterion [2] for the equation (1).

Theorem 1. *The Dirichlet problem (1), (2) for* $f \in L_p(D_i, \mathbb{C}), \ (p > 2), \quad \gamma \in C(L, \mathbb{C})$ *is solvabe if only if*

$$\frac{1}{2\pi i}\int_L \gamma(\zeta)\zeta^n d\zeta - \frac{1}{\pi}\int_{D_i} f(\zeta)\zeta^n d\xi\, d\eta = 0 \quad (n = 0,1,2,...).$$

If these conditions are fulfulled, the unuqe solution is given by

$$W(z) = \frac{1}{2\pi i}\int_L \gamma(\zeta)\frac{d\zeta}{\zeta - z} - \frac{1}{\pi}\int_{D_i} f(\zeta)\frac{d\xi\, d\eta}{\zeta - z}.$$

Now we consider the problem of continuation of the solution of the Cauchy – Riemann equation to the domain by it values on a part of boundary i.e. Cauchy problem. Let D is a bounded simple connected domain on the complex plane $z = x + iy$ with piece – smooth boundary, consisting from segment AB of the real axis and a smooth curve S lying on the upper half plane. We consider the problem of description of functions $\varphi \in C(S)$ which are traces of a solution of the inhomogineous Cauchy – Riemann equation (1).

Theorem 2. *Let* $f \in L_p(D), \ p > 2$ W *- a regular solution in the domain* D *of the equation (1), continuos on closed domain* $\overline{D} = D \cup \partial D$ *and* $W(z) = \varphi(z), \ z \in S$ *. Then helds the following equivalent formulas of continuation:*

$$W(z) = \lim_{\sigma\to\infty}\{\frac{1}{2\pi i}\int_S \exp[-i\sigma(\zeta - z)]\varphi(\zeta)\frac{d\zeta}{\zeta - z} -$$

$$\frac{1}{\pi}\iint_D \exp[-i\sigma(\zeta - z)]f(\zeta)\frac{d\xi d\eta}{\zeta - z}\},\ \ z\in D \qquad (3)$$

$$W(z) = \frac{1}{2\pi i}\int_S \frac{\varphi(\zeta)}{\zeta - z}d\zeta - \frac{1}{\pi}\iint_D \frac{f(\zeta)d\xi d\eta}{\zeta - z} + \int_0^\infty [-\frac{1}{2\pi i}\int_S exp[-i\sigma(\zeta - z)]\varphi(\zeta)d(\zeta) +$$

$$\frac{i}{\pi}\iint_D \exp[-i\sigma(\zeta - z)]f(\zeta)d\xi d\eta]d\sigma,\ \ z\in D. \qquad (4)$$

Theorem 3. *Let* $f\in L_p(D),\ p>2$ *and* $\varphi\in L(S)\cap C(S^\circ)$, $S^\circ = \mathrm{Int}S$. *Then:*

1) If there exists a solution of equation (1) in domain D *continuous on* $\overline{D}$ *and it is equal to the* $\varphi(\zeta)$ *on* S, *then the improper integral*

$$|\int_0^\infty [-\frac{1}{2\pi}\int_S \exp[-i\sigma(\zeta - z)]\varphi(\zeta)d\zeta + \frac{i}{\pi}\iint_D \exp[-i\sigma(\zeta - z)]f(\zeta)d\xi d\eta\,| < \infty \qquad (5)$$

uniformly converges on each compact $K\subset\{\mathrm{Imz}>0\}$.

2) If the function φ *satisfies to the condition (5), then there exists a solution* $W(z)$ *of equation (1) in domain* D, *continuous on up to* S° *and it is equal to the* $\varphi(\zeta)$ *on* S. *This solution is given by equivalent formulas (3) and (4).*

Remark 1. The Dirichlet problem for the inhomogeneous polyanalytic equation in other way was considered in [3].

Remark 2. The Fok – Kuni theorem [1] for the generalized analytic functions was obtained in [4].

Remark 3. The conditions under which the Green type integral for the harmonic functions transforms into Green integral was investigated in [2].

REFERENCES

[1] Fok V.A., Kuni F.M. : On inputting a quenching function into variance ratios. Dokl. Akad.Nauk. SSSR, 127 (6) : 1195 – 1198, 1959 (Russian).

[2] Solomentsev E.D. : On the basis Green formula for the harmonic functions. Sibirsk. Math. zh. 7(6): 1432 – 1434, 1966 (Russian).

[3] Begehr H., Kumar A. : Boundary value problems for the inhomogeneous polyanalytic equation I. Analysis 25, 55 – 71 (2005).

[4] Ishankulov T. : On the possibility generalized analytical continuation in the domain functions given on a pies of the boundary. Sibirsk. Math. zh. 1(6): 1350 – 1356, 2000 (Russian).

REGULARIZATION OF THE SINGULAR OPERATOR EQUATION WITH ADDITIVE OPERATORS

Izatullaev N.
Faculty of Mechanics and Mathematics, Samarkand State University, Uzbekistan

In this paper we generalize one of the results investigation of singular equations on a normalized and unitary ring with the help of which we prove equivalence study of the singular and regularized equations.

Let a unitary ring with identity. Consider the equation

$$Tx = Ux + \upsilon S_1 x + WS_2 x + \lambda_1 p_1 x + \lambda_2 p_2 x = y \qquad (1)$$

where $u, \upsilon, w, y -$, given elements, $R; \lambda_1, \lambda_2 -$ the required element of the complex parameters; P_1, P_2 Linear regular operators (which use the theory of Riesz-Schauder and S_1, S_2 the additive singular operators acting in a unitary R ring.

1) $S_1^2 = S_2^2 = E$; 2) For any $u \in R$ operator $S_i u - uS_i$, $i = \overline{1,n}$ regular; under the operators understand Su and uS , $Su(y) = S(uy)$ и $Su(y)$; 3) S_1, S_2 - commuting operators $S_1, S_2 = S_2 S_1$; 4)additive singular operators S_1, S_2 are of the view

$$S_1 x = S_{11} x + S_{12} \overline{x} \text{ and } S_2 x = S_{21} x + S_{22} \overline{x} \qquad (2)$$

Following, we perform a regularization of (1). Denote $y_o = y - \lambda_1 P_1 x - \lambda_2 P_2 x$. Then by (2) we have.

$$Tx = Ux + \upsilon S_{11} x + \upsilon S_{12} \overline{x} + wS_{21} x + wS_{22} \overline{x} = y_{.0} \qquad (3)$$

Equation (3) investigate the scheme proposed A.Sh.Gabib-Zadeh (1978).

It is easy to see that (3) the equivalent system (4). Consider the topological product $R^1 = RxR,$ by entering the following vectors: $u_O = (u, \overline{u}),$ $\upsilon_O = (\upsilon, \overline{\upsilon}),$ $\omega_O = (\omega, \overline{\omega}),$ $\alpha = (\varphi, \psi),$ $g_0(y_o, \overline{y}_0)$ and the operator - the matrix $\delta_1 = \begin{pmatrix} S_{11} & S_{12} \\ \overline{S}_{12} & \overline{S}_{11} \end{pmatrix}$, $\delta_2 = \begin{pmatrix} S_{21} & S_{22} \\ \overline{S}_{22} & \overline{S}_{21} \end{pmatrix}$.

Since $\alpha = (\varphi, \psi) \in R,$ then $u_O \alpha = (u\varphi, \overline{u}\psi),$ $\upsilon_O \delta_1 \alpha = (\upsilon S_{11} \varphi + \upsilon S_{12} \psi, \overline{\upsilon} \overline{S}_{12} \varphi + \overline{\upsilon} \overline{S}_{11} \psi),$ $\omega_O \delta_2 \alpha = (\omega S_{21} \varphi + \omega S_{22} \psi), \overline{\omega} \overline{S}_{22} \varphi + \overline{\omega} \overline{S}_{21} \psi),$ and

$$g_O = h - \lambda_1 H_1 \alpha - \lambda_2 H_2 \alpha - \overline{\lambda}_1 H \alpha - \overline{\lambda}_2 H_2 \alpha; \qquad (5)$$

here $h=(y,\bar{y}), H_1\alpha=(P_1x,o),\ H_2\alpha=(P_2x,o),\ H_3\alpha=(O,\overline{P_1}x), H_4\alpha(0,\overline{p_1x})$, and H_3H_4 - additive, and H_1H_2 - linear operators. After these definitions, system (4) can be written as

$$U_O\alpha+\upsilon_O\zeta\alpha+\omega_O\delta_2\alpha=g_O \qquad (6)$$

By virtue of equality $S_1^2x=x,\quad S_2^2x=x$ and additivity of the operators S_1,S_2 can prove that $\delta_1^2=E,\ \ \delta_2^2=E$. Thus, the operators $\delta_1\delta_2$, act in R^1 and have the property $\delta_1^2=E$, $\delta_2^2=E$, where E the identity operator in R^1.

Acting sequences of operators $\delta_1,\delta_2,\delta_1\delta_2$ on both sides of equation (6), we obtain

$$\delta_1(U_0\alpha+\upsilon_0\delta_1\alpha+\omega_0\delta_2\alpha)=\delta_1g_0 \qquad (7)$$

$$\delta_2(u_0\alpha+\upsilon_0\delta_1\alpha+\omega_0\delta_2\alpha)=\delta_2g_0 \qquad (8)$$

$$\delta_1\delta_2(u_0\alpha+\upsilon_0\delta_1\alpha+\omega_0\delta_2\alpha)=\delta_1\delta_2g_0 \qquad (9)$$

Now equation (7), (8) and (9) we rewrite the transformed image

$$\delta_1U_0\delta_1\delta_2\alpha+\delta_1\upsilon_0\delta_1\alpha+\delta_1\omega_0\delta_2\delta_1\delta_2\alpha=\delta_1g_0, \qquad (10)$$

$$\delta_2U_0\delta_2\delta_2\alpha+\delta_2\upsilon_0\delta_1\delta_1\delta_2\delta_1\delta_2\alpha+\delta_1\omega_0\delta_2\alpha=\delta_2g_0 \qquad (11)$$

$$\delta_1\delta_2U_0\delta_1\delta_2\delta_1\delta_2+\delta_1\delta_2\upsilon_0\delta_1\delta_2\delta_2\alpha+\delta_1\delta_2\omega_0\delta_2\delta_1\delta_1\alpha=\delta_1\delta_2g_0 \qquad (12)$$

Let $\alpha=\varphi,\ \ \delta_1\alpha=\varphi_2,\ \ \delta_2\alpha=\varphi_3,\ \ \delta_1\delta_2\alpha=\varphi_4$. Then from (6), (10), (11) and (12)

$$\left.\begin{aligned}
&U_0\varphi_1+\upsilon_0\varphi_2+\omega_0\varphi_3=g_0\\
&U_0\varphi_2+Q_1^{(2)}\varphi+\upsilon_0\varphi_1+Q_1^{(1)}\varphi_1+\omega_0\varphi_4+Q_1^{(4)}\varphi_4=\delta_1g_0\\
&U_0\varphi_3+Q_2^{(3)}\varphi_3+\upsilon_0\varphi_4+Q_2^{(4)}\varphi_4+\omega_0\varphi_1+Q_2^{(1)}\varphi_1=\delta_2g_0\\
&U_0\varphi_4+Q_3^{(4)}\varphi_4+\upsilon_0\varphi_3+Q_3^{(3)}\varphi_3+\omega_0\varphi_2+Q_3^{(2)}\varphi_2=\delta_1\delta_2g_0
\end{aligned}\right\} \qquad (13)$$

Thus, we have a system of equations (13), equivalent to (6). Consider the topological product $\widehat{R}=R^1xR^1xR^1xR^1$ and introduce the operator - the matrix and $\theta=(\varphi_1,\varphi_2,\varphi_3,\varphi_4)$, $f_0(g_0\delta_1g_0,\delta_2g_0,\delta_1\delta_2g_0)$ vectors. Then (13) can be written as follows: $\Omega\theta+K_1\theta=f_0$. So system (13) were reduced to an operator equation with a regular operator K_1 acting in $\overline{R}$. If the operator is invertible, then we have. $\theta-K\theta=f$ (14) where $K=-\Omega^{-1}K_1$, $f=\Omega^{-1}f_0$. Proved the following theorem.

Theorem. Each equation of the form (6) in the ring R^1 with two singulyarnvmi operator satisfying the conditions 1)-3), can be mapped (14) in the ring $\overline{R}=R^1xR^1xR^1xR^1$ with a regular operator K, so that every solution α of equation (6) vector($\alpha,\delta_1\alpha,\delta_2\alpha,\delta_1\delta_2\alpha$) is the solution (14) and, conversely, if the $(\varphi_1,\varphi_2,\varphi_3,\varphi_4)-$ solution of equation (14), the element $\alpha=0{,}25(\varphi_1+\delta_1\varphi_2+\delta_2\varphi_3+\delta_1\delta_2\varphi_4)$ is a solution of the original equation (6).

ON EXISTENCE AND UNIQUENESS OF THE SOLUTION OF PLASTICITY PROBLEMS FOR TRANSVERSELY ISOTROPIC MATERIALS

Khaldjigitov A.A.[1], Adambaev U.[1] , N. M.A. Nik Long[2],

[1] *Faculty of Mechanics and mathematics, National University of Uzbekistan*
Vuzgorodik UzNU, 100074 Tashkent, Uzbekistan
E-mail: uchqunbek@yandex.ru

[2] *Department of Mathematics and INSPEM, University Putra Malaysia,*
43400, Serdang, Selangor, Malaysia
E-mail: nmasri@math.upm.edu.my

INTRODUCTION

This work is devoted to formulating of the boundary value problems for stress space plasticity theory of transversely isotropic materials and to prove the theorem on existence and uniqueness of the weak solution.

The boundary value problem for stress space plasticity theory for transversely isotropic materials is as follows(Khaldjigitov ,1995)

$$\sum_{k,l,j=1}^{3} \frac{\partial}{\partial x_j}\left(D_{ijkl}^{p} \frac{\partial \dot{u}_k}{\partial x_l} \right) + \dot{X}_i = 0 \tag{1}$$

$$\dot{u}_i\Big|_{\Sigma_1} = 0 \ , \tag{2}$$

$$\sum_{j,k,l=1}^{3} D_{ijkl}^{p} \frac{\partial \dot{u}_k}{\partial x_l} n_j \Bigg|_{\Sigma_2} = \dot{S}_i \tag{3}$$

where

$$D_{ijkl}^{p} = \begin{cases} \mathrm{C_{ijkl}}, \quad at \;\; f_1 < 0 \; u \;\; f_2 < 0 \\ \mathrm{C_{ijkl}} - \dfrac{h_1 \mathrm{C_{ijmn}} \dfrac{\partial f_1}{\partial P_{mn}} \mathrm{C_{klrs}} \dfrac{\partial f_1}{\partial P_{rs}}}{1 + h_1 \dfrac{\partial f_1}{\partial P_{mn}} \mathrm{C_{mnrs}} \dfrac{\partial f_1}{\partial P_{rs}}} - \dfrac{\mathrm{C_{ijmn}} \dfrac{\partial f_2}{\partial Q_{mn}} \mathrm{C_{klrs}} \dfrac{\partial f_2}{\partial Q_{rs}}}{1 + h_2 \dfrac{\partial f_2}{\partial Q_{mn}} \mathrm{C_{mnrs}} \dfrac{\partial f_2}{\partial Q_{rs}}} \\ \qquad at \;\; f_1 = 0, \; \dfrac{\partial f_1}{\partial P_{ij}} dP_{ij} > 0 \;\; u \;\; f_2 = 0, \;\; \dfrac{\partial f_2}{\partial Q_{ij}} dQ_{ij} > 0 \end{cases} \tag{4}$$

$$f_1(P_{ij}, \chi_1) = 0, \qquad f_2(Q_{ij}, \chi_2) = 0 \tag{5}$$

f_1, f_2 are the loading functions; P_{ij}, Q_{ij} - stress tensors , C_{ijkl} - a symmetric tensor, X_i, S_i - body and surface forces, h_1, h_2 - hardening functions, n_i .unit vector,

$$\chi_1 = \int P_{ij} dp_{ij}^{p} \,, \;\; \chi_2 = \int Q_{ij} dq_{ij}^{p} \tag{6}$$

Multiplying Eq.(1) to $\dot{v}_i \in W_2^{'}$ and integrating on domain V after some transformations we find that

$$a(\dot{u},\dot{v}) = A(\dot{v}) \tag{7}$$

where

$$a(\dot{u},\dot{v}) = \int_V D_{ijkl}^p \frac{\partial \dot{u}_k}{\partial x_l} \frac{\partial \dot{v}_i}{\partial x_j} dv$$

$$A(\dot{v}) = \int_V \dot{x}_i \cdot \dot{v}_i dv + \int_{\Sigma_2} \dot{S}_i \dot{v}_i ds \tag{8}$$

Theorem 1. Let loading surfaces $f_1(P_{ij}, \chi_1) = 0$ and $f_2(Q_{ij}, \chi_2) = 0$ are the continuously differentiable and homogeneous function of a degree $n\ (n \geq 1)$. Then for anyone $\dot{v}_i \in W_2^{'}$ satisfies the condition (2) there is the unique generalized solution of equation (7) for hardening materials.

Proof: First we show the positive definiteness of the elastic-plastic matrix D_{ijkl}^p. Accounting (4) we find that

$$D_{ijkl}^p \dot{\varepsilon}_{kl} \dot{\varepsilon}_{ij} = C_{ijkl} \dot{\varepsilon}_{kl} \dot{\varepsilon}_{ij} -$$

$$- \frac{\left(\varepsilon_{ij} \mathrm{C}_{\mathrm{ijmn}} \dfrac{\partial f_1}{\partial P_{mn}}\right)\left(\mathrm{C}_{\mathrm{klrs}} \dfrac{\partial f_1}{\partial P_{rs}} \varepsilon_{kl}\right)}{\dfrac{1}{h_1} + \dfrac{\partial f_1}{\partial P_{mn}} \mathrm{C}_{\mathrm{mnrs}} \dfrac{\partial f_1}{\partial P_{rs}}} - \frac{\left(\varepsilon_{ij} \mathrm{C}_{\mathrm{ijmn}} \dfrac{\partial f_2}{\partial Q_{mn}}\right)\left(\mathrm{C}_{\mathrm{klrs}} \dfrac{\partial f_2}{\partial Q_{rs}} \varepsilon_{kl}\right)}{\dfrac{1}{h_2} + \dfrac{\partial f_2}{\partial Q_{mn}} \mathrm{C}_{\mathrm{mnrs}} \dfrac{\partial f_2}{\partial Q_{rs}}} \tag{9}$$

Using the following inequality(Cen, 1988.)

$$\left(a_{ij} C_{ijkl} b_{kl}\right)^2 \leq \left(a_{ij} C_{ijkl} a_{kl}\right) \cdot \left(b_{ij} C_{ijkl} b_{kl}\right) \tag{10}$$

the expression (9) can be reduced to the following form:

$$D_{ijkl}^p \dot{\varepsilon}_{kl} \dot{\varepsilon}_{ij} \geq C_0 C_{ijkl} \dot{\varepsilon}_{kl} \dot{\varepsilon}_{ij} \tag{11}$$

where

$$C_0 = \frac{1 + h_1 h_2 \left(\frac{\partial f_1}{\partial P_{ij}} \mathrm{C}_{\mathrm{ijkl}} \frac{\partial f_1}{\partial P_{kl}} \right) \left(\frac{\partial f_2}{\partial Q_{ij}} \mathrm{C}_{\mathrm{ijkl}} \frac{\partial f_2}{\partial Q_{kl}} \right)}{\left(1 + h_1 \frac{\partial f_1}{\partial P_{ij}} \mathrm{C}_{\mathrm{ijmn}} \frac{\partial f_1}{\partial P_{mn}} \right) \left(1 + h_2 \frac{\partial f_2}{\partial Q_{ij}} \mathrm{C}_{\mathrm{ijmn}} \frac{\partial f_2}{\partial Q_{mn}} \right)} \tag{12}$$

In the elastic region, i.e. when, $h_1 = 0$, and $h_2 = 0$ and receive that . $C_0 = 1$

Then, from (11)we can find the well-known inequality

$$D^p_{ijkl} \dot{\varepsilon}_{kl} \dot{\varepsilon}_{ij} \geq C_0 C_{ijkl} \dot{\varepsilon}_{kl} \dot{\varepsilon}_{ij} \geq 2\mu C_0 C_{ijkl} \dot{\varepsilon}_{kl} \dot{\varepsilon}_{ij} \tag{13}$$

Inequality (13) and (12) show the positive definiteness of the matrix D^p_{ijkl} .

The boundedness of the bilinear form $a(\dot{u}, \dot{v})$ and the functional $A(\dot{v})$ can be proved similar to Cen (1988). Using inequality (13) from (7) we find that

$$a(\dot{u}, \dot{v}) \geq 2\mu C_0 \int_V \dot{\varepsilon}_{ij} \dot{\varepsilon}_{ij} dv \tag{14}$$

Further, using well-known inequalities of Korn's and Poincare, after some transformations we receive the estimate(Pobedrya,1996)

$$\|\dot{u}\|_{L_2} \leq \frac{\mu_1 \mu_2}{2\mu C_0} \left(\|\dot{x}\|_{L_2(v)} + \|\dot{s}\|_{L_2(\Sigma)} \right) \tag{15}$$

which proves the existence and uniqueness of the solution of the boundary problem (1-3).

CONCLUSIONS

By Khaldjitov(1995) was proposed the strain and stress type of constitutive relations for transversely isotropic materials. In this work formulated plasticity boundary value problem and proved the theorem on existence and uniqueness of the weak solution of the plasticity problem for transversely isotropic materials.

REFERENCES

Pobedrya B. E. 1996. Computational methods in elasticity and plasticity. -M.: MSU, 343p.

Khaldjigitov A.A. 1995. Some questions of lasticity theory for anosotropic materials.Dsc.Thesis., Tashkent State University.

Cen Y. 1988. Fundamental proporties of alasto-plastic matrix and existence and uniqueness of elasti-plastic stress and displasement.Acta mech.solida. V.1,1, p.1-13

ASYMPTOTICS OF EIGENVALUES OF THE DISCRETE SCHRÖDINGER OPERATORS

Khalkhujaev A.M., Lakaev Sh.S.
Samarkand State University, Samarkand, Uzbekistan,
e-mail: ahmad_x@mail.ru.

In [1,2] it has been studied that the continuous Schrödinger operator $h(\lambda) = -\Delta + \lambda V$ on $\mathsf{R}^d, d = 1,2$ has a unique eigenvalue when the potential $V(x)$ satisfies some conditions and $\lambda > 0$ is sufficiently small. Moreover the asymptotics of this eigenvalue is found as $\lambda \to 0$.

Let Z^d be the d - dimensional hybercubic lattice and $\mathsf{T}^d = (\mathsf{R}/2\pi\mathsf{Z})^d = (-\pi,\pi]^d$ be the d - dimensional torus (Brillouin zone), the dual group of Z^d.

Discrete Schrödinger operator $h_\mu(k), k \in \mathsf{T}^d, d = 1,2,$ associated with a system of two particles on lattice Z^d, parametrically depends on the quasi-momentum $k \in \mathsf{T}^d$. Consequently the spectra of the family $h_\mu(k)$ will rather sensitive to variations in the quasi-momentum k.

In the present paper it is considered a family of discrete Schrödinger operators $h_\mu(k), k \in \mathsf{T}^d, d = 1,2,$ associated with the hamiltonian of the system of two quantum particles (bosons), moving on the $d = 1,2$ dimensional lattice Z^d, interacting via zero-range pair potential $\mu > 0$. Moreover the expansion of the unique eigenvalue asymptotic of this eigenvalue $z(\mu,k), k \in \mathsf{T}^d$ for small $\mu > 0$.

Let $L_2^e(\mathsf{T}^d) -$ the Hilbert space of square-integrable even functions on T^d.

The operator $h_\mu(k), k = (k_1,\ldots,k_d) \in \mathsf{T}^d$ acts on $L_2^e(\mathsf{T}^d)$ as follows [4,5]:

$$h_\mu(k) = h_0(k) - \mu v.$$

The non-perturbed operator $h_0(k)$ on $L_2^e(\mathsf{T}^d)$ is multiplication operator by the function

$$\mathsf{E}_k(q) = \sum_{j=1}^{d}\left[2 - 2\cos\frac{k_j}{2}\cos q_j\right].$$

The perturbation v is an integral operator of rank one

$$(vf)(p) = \frac{1}{(2\pi)^d}\int_{\mathsf{T}^d} f(t)dt, \quad f \in L_2^e(\mathsf{T}^d).$$

The essential spectrum $\sigma_{ess}(h_\mu(k))$ of $h_\mu(k)$ fills the segment $[\mathsf{E}_{\min}(k), \mathsf{E}_{\max}(k)],$ where

$$\mathrm{E}_{\min}(k) = \min_{q\in \mathrm{T}^d} \mathrm{E}_k(q) \geq 0, \mathrm{E}_{\max}(k) = \max_{q\in \mathrm{T}^d} \mathrm{E}_k(q) \leq 4d.$$

For the discrete Schrödinger operators $h_\mu(k), k\in \mathrm{T}^d, d=1,2,$ the following results have been proven.

Theorem 1. *Let* $k\in(-\pi,\pi)^d$. *Then there is* $\mu_0>0$ *and for* $\mu\in(0,\mu_0)$ *the following equality holds: if* $d=1$, *then*

$$z(\mu,k) = \mathrm{E}_{\min}(k) - \mu^2\left[\sum_{n=0}^{\infty} a_n(k)\mu^n\right]^2,$$

where $a_n(k),\ k=1,2,\ldots$ – some real numbers and

$$a_0(k) = \frac{1}{2\sqrt{\cos\frac{k}{2}}} > 0;$$

if $d=2$, then

$$z(\mu,k) = \mathrm{E}_{\min}(k) - b(k)\exp\{-\frac{c(k)}{\mu}\} + \sum_{n,m\geq 1,\, n+m>2} c(n,m;k)\mu^n\sigma^m,$$

where

$$b(k)>0; c(k) = 4\pi\sqrt{\cos\frac{k_1}{2}\cos\frac{k_2}{2}} > 0; \sigma = \frac{1}{\mu}\exp\{-\frac{c(k)}{\mu}\}$$

and $c(n,m;k), n,m=1,2,\ldots$ -- some real numbers.

Theorem 2. *For any* $k\in \mathrm{T}^d$ *the function* $z(\mu,k)$ *has the following asymptotics as* $\mu\to+\infty$

$$z(\mu,k) = -\mu + z^{(1)}(\mu,k),$$

where $z^{(1)}(\mu,k) = O(1)$ uniformly for $k\in \mathrm{T}^d$.

Remark. *In case* $\mu<0$ *analogous results hold for the unique eigenvalue* $z(\mu,k)$ *of the operator* $h_\mu(k),\ k\in \mathrm{T}^d$ *lying above the essential spectrum.*

REFERENCES

[1] B. Simon : The Bound State of Weakly Coupled Schroedinger Operators in one Dimension.Annals of Physics 108, 288-300 (1977).

[2] M.Klaus : On the Bound State Schroedinger Operators in one and two Dimensions.Annals of Physics 97, 279-288 (1976).

[3] M.Klaus and B.Simon: Coupling Constant Thresholds in Nonrelativistic Quantum Mechanics. I. Short-Range Two-Body Case. Annals of Physics **130**, 251-281, New Jersey, 1980.

[4] S. Albeverio , S. N. Lakaev , K. A. Makarov , Z. I. Muminov: The Threshold effects for the Two-Particle Hamiltonians on Lattices. Commun.Math.Phys. 262(2006), 91-115.

[5] P. A. Faria da Veiga, L. Ioriatti, and M. O'Carroll, Phys. Rev. E, 66, 6130 (2002).

ESTIMATES FOR TWO-DIMENSIONAL TRIGONOMETRIC INTEGRALS WITH SPECIAL PHASE

Khasanov G.A.
Samarkand state university
khasanov_g75@mail.ru

We consider both upper and lower estimates for oscillatory integrals. First, we define a class of amplitude functions. For this reason we consider a family of smooth curves $K=\{k=(x_1(\eta,\,\xi),x_2(\eta,\,\xi)):\,(\eta,\,\xi)\in[u,\,\vartheta]\times[h,\,\omega]\}$, where (x_1,x_2) is a pair of fixed smooth functions [1,3].

The function $a\in A(K)$ if and only if there exists a fixed positive $C(a)$ such that for any $k\in K$ the following inequality: $\mathrm{V}[a\circ k]\le C(a)$ holds, where $\mathrm{V}[a\circ k]$ is a total variation of the function $a\circ k$ on the interval $[u,\,\vartheta]$. The class $A(K)$ is a normed space with respect to norm

$$\|a\|_{\mathrm{A}}=\sup_{\xi}\left(|\,a(x_1(u,\,\xi),x_2(u,\,\xi))\,|+\mathrm{V}[a\circ k]\right)$$

The class of amplitude functions A is defined by $\mathcal{A}=\bigcap A(K)$.

Let (r_1,r_2) be a pair of positive rational numbers. We define the norm in the space $C^\infty(\overline{U})$ вy the following

$$\left\|f\middle|\overline{U}\right\|_r=\max_{k_1r_1+k_2r_2\le1}\max_{x\in\overline{U}}\left|\frac{\partial^{|k|}f(x)}{\partial x_1^{k_1}\partial x_2^{k_2}}\right|$$

Let $D=\left\{r^1,r^2,\,.\,.\,.\,,r^k\right\}$ be a finite number of pairs of positive rational numbers, U be a bounded neighborhood of the origin of R^2. We define a norm in the space $C^\infty(\overline{U})$ by the following

$$\left\|f\middle|U\right\|_D=\max_{r^k\in D}\left\|f\middle|U\right\|_{r^k}$$

Let $f:(R^2,0)\to(R,0)$ be a smooth function in a neighborhood of the point (0,0). We assume the local coordinates system is fixed at the origin.

We construct the Newton polygon $N(\tilde{f})$ in this coordinates system.

The union of all compact edges of Newton polygon is called to be a Newton diagram. It is denoted by $D(\tilde{f})=\{\gamma_1,\gamma_2,\ .\ .\ .\ ,\gamma_k\}$. We can define a pair of rational numbers $r_{\gamma\ell}=(r_{1\ell},r_{2\ell})$ corresponding to the edge γ [1,2].

Let γ^p be the principal edge of the Newton polygon of f and (s,m) type of singularity of the polynomial P. Let F be a smooth function and $\operatorname{supp}\tilde{F}\subset\operatorname{supp}\tilde{f}$. Let $m=(m_1,m_2),\ \ (M=(M_1,M_2))$ be point corresponding to the least (gratest) compact edge of Newton polygon of F . We suppose that the function F can be written as

$$F(x_1,x_2)=x_1^{m_1}x_2^{M_2}F_1(x_1,x_2)$$

where F_1 is a smooth function.

If $\operatorname{supp}\tilde{F}\subset\operatorname{supp}\tilde{f}$ and F satisfies the above-mentioned condition, then we will write $\operatorname{supp}\tilde{F}\subset\subset\operatorname{supp}\tilde{f}$.

If $\operatorname{supp}\tilde{F}\bigcap D\neq\varnothing$, then the function F can be written as

$$F(x_1,x_2)=F_{\gamma_1}(x_1,x_2)+\tilde{F}(x_1,x_2)\,,$$

where $F_{\gamma_1}(x_1,x_2)$ is a weighted homogeneous part of the function F and $\tilde{F}\in I_{r_1}$. We can write the function $\tilde{F}(x_1,x_2)$ in the form

$$\tilde{F}(x_1,x_2)=\sum_{r_{11}i+r_{12}j=1}x_1^ix_2^jb_{ij}(x_1,x_2)$$

where $b_{ij}(x_1,x_2)\in\mathfrak{M}$, $b_{ij}(0,0)=0$. $\mathfrak{M}$ is the maximal ideal of ring of germs of smooth functions at the origin.

Lemma. For any positive number ε there exists a positive number $\delta>0$ such that for any $|x_1|<\delta,\ \ |x_2|<\delta$ and for any $r_{11}i+r_{12}j=1$ the inequality

$$|b_{ij}(x_1,x_2)|<\varepsilon$$

holds.

The following theorem shows that the oscillation exponent does not change under small perturbation of the Newton polygon [4].

Theorem. Let $f:(R^2,0)\to(R,0)$ be a smooth function in a neighborhood of the point (0,0). Then there exist a neighborhood *U* of the origin and positive numbers $\varepsilon,\ C$ such that for any function *F* satisfying the following conditions:

1) $\operatorname{supp}\tilde{F}\subset\subset N(\tilde{f})$

2) $\left\| f|U \right\|_{D(f)}<\varepsilon$

3) $a\in\mathcal{A}(U)$

then the followers inequality

$$\left|\int_{R^2} a(x)\exp(it(f+F)(x))dx\right|\le C\|a\|_V\ |t|^{-s}|\ln t|^m$$

holds, where (s,m) is the type of the principal part of the function f at the origin.

REFERENCES.

[1] Varchenko A.N. Newton polyhedra and estimates for oscillating integrals //Funct. and applications., 1976, V.10, № 5, 13-38 p.

[2] Karpushkin V.N. Theorem on uniform estimates for oscillating integrals with phase on two variables. Tr.Sem.I.G.Petrovsky, M.MSU 1984, V.10, 150-169 p.

[3] Ikromov I.A. and Khasanov G.A. Newton polyhedrons and estimates for oscillatory integrals with smooth phases. // Preprint Italy IC/2000/114. p. 1-19.

[4] I.A.Ikramov, G.A.Khasanov. Uniform estimates for some trigonometric integrals with smooth phase. /Uzbek Math. Jour. 2003 RUzb. № 2. 21-29 p.

ON THE BEST APPROXIMATION OF FUNCTIONS WITH DERIVATIVE OF GENERALIZED FINITE VARIATION BY POLYNOMIAL SPLINES

Khatamov A.
Faculty of mechanics and mathematics
Samarkand State University, Uzbekistan
khatamov@rambler.ru

ABSTRACT.

The report is devoted to the exact (in the sense of the order of smallness) estimates of the best spline approximations of functions with derivative of generalized finite variation given on a finite segment of the straight line in uniform and integral metrics.

DEFINITIONS AND NOTATIONS.

Let N be the set of all natural numbers, $Z_+ = N \cup \{0\}$, $\Delta = [a,b]$ a finite segment of the straight line with the length $|\Delta| = b-a$. Let $L_p(\Delta)$ be the space of all measurable by Lebesque real-valued on Δ functions f whose pth-power is integrable. The space is equipped with the ordinary quasi-norm $\|f\|_{p,\Delta}$. Let $\Phi(u)$ be a continuous, increasing, convex to down function, defined on the interval $[0,\infty)$ and such that $\Phi(0)=0$. For a function $f(x)$ defined and finite on Δ the value

$$V_\Phi(f,\Delta) = \sup\left\{\sum_{k=0}^{n-1}\Phi\left[\left|f(x_{k+1}) - f(x_k)\right| : a = x_0 < x_1 < \cdots x_n = b \quad (n=1,2,\cdots)\right]\right\}$$

we call

Φ-variation of the function f on the segment Δ [1]. Let for $M = const > 0$, $r \in Z_+$

$V_\Phi^{(r)}(M,\Delta) = \left\{ f : V_\Phi(f^{(r)},\Delta) \le M \right\} \quad \left(f^{(0)} \equiv f \right).$

The value $$\chi(f,n) = \sup\left\{\sum_{k=0}^{n-1}\left|f(x_{k+1}) - f(x_k)\right| : a \le x_0 \le x_1 \le \cdots \le x_n \le b\right\}$$

is called the modulus of variation of the function f on a segment Δ ([2]).

A function s is called a polynomial spline (or shorter spline) of degree m of minimal defect with free $n+1$ knots on a segment Δ if

1) s is polynomial of the degree non-exceeding m on each segment $[x_k, x_{k+1}]$, $k = \overline{0, n-1}$;

2) the $(m-1)th$ order derivative of the function s is continuous on Δ.

We denote by $S(m,n,\Delta)$ the set of all splines of degree m of minimal defect with free $n+1$ knots on a segment Δ. Let $S_n^m\left(f,\Delta\right)_p = \inf\left\{\|f-s\|_{p,\Delta} : s \in S(m,n,\Delta)\right\}$.

Let $S_n^m\left(V_\Phi^{(r)}\left(M,\Delta\right)\right)_p = \sup\left\{S_n^m(f,\Delta)_p :\ f \in V_\Phi^{(r)}\left(M,\Delta\right)\right\}$.

Everywhere below $C(\alpha,\beta,\cdots)$, $C_1(\alpha,\beta,\cdots),\cdots$ denote positive constants depending on the parameters indicated in parenthesis and on the subscripts only.

MAIN RESULTS.

In the articles [3] of author the exact (in the sense of the order of smallness) estimates for the best piecewise-linear approximations in the metrics $L_p(\Delta)$ at all $0 < p < \infty$ for a function either measurable to respect of Lebesque's measure and bounded on a segment Δ or with the finite Φ-variation on a segment Δ are proved. The following theorems are the generalizations of the theorems just now mentioned above and the main results of the report.

Theorem 1. *If at* $r \in Z_+$ $f^{(r)}$ *is measurable by Lebesque and bounded on* Δ, *then for arbitrary p*, $0 < p \le \infty$, $n \in N$ $S_n^{r+1}(f,\Delta) \le C(r)\left|\Delta\right|^{r+1/p} \chi(f^{(r)},n)/n^{r+1}$.

At $r = 0$ *the estimate holds* for $0 < p < \infty$ *only.*

On the other hand, for each $n \in N$ *and* $r \in Z_+$ *there is a function* $f = f_{n,r} \in V_\Phi^{(r)}(M,\Delta)$ *such that* $S_n^{r+1}(f,\Delta)_p \ge C(r,p)\left|\Delta\right|^{r+1/p} \chi(f^{(r)},n)/n^{r+1}$.

Theorem 2. *For all* p, $0 < p \le \infty$, $n \in N$, $r \in Z_+$ *the estimates*

$$C(r,p)\left|\Delta\right|^{r+1/p} n^{-r}\Phi^{-1}(M/n) \le S_n^{r+1}(V_\Phi^{(r)}(M,\Delta))_p \le C(r)\left|\Delta\right|^{r+1/p} n^{-r}\Phi^{-1}(M/n)$$

hold, where for $r = 0$ *the upper estimate is valid at* $0 < p < \infty$ *and* $C(r)$, $C(r,p)$ *are the same as in Theorem 1.*

REFERENCES

[1]. J. Musielac and W. Orlich. (1959). On generalized variation . Studia Math., 18: 11-41.

[2]. V. A. Popov. (1975). On the connection between rational and spline approximation. Serdica Rendos Acad. Bulg. Sci. 27 (5): 623-626.

[3]. A. Khatamov. (2006). On the exact estimates of the best spline approximations of functions. ETNA, Kent State University (USA). 25: 446-453.

ON INVERSE PROBLEMS FOR HYPERBOLIC EQUATIONS

Khaydarov A.
Samarkand state university
akramkh@rambler.ru

Rustamov M.
Jizzax state pedagogical institute

Inverse problems for differential equations consist of determination of coefficients or right hand side by same additional condition on solution.

Problem on discovering properties of elastic medium by observations the wave field on the boundary of the domain is one of the basic problems in geophysics. In this paper we consider proposition of the problem in the multidimensional case. The problem consists of determination of the wave coefficient c of the operator by Cauchy dates in the original time and on the part of lateral area of the convex cylinder with respect to the space variables.

The method of proof is based on Carleman’s type estimates and application to the inverse problem.

Necessary Carleman’s type estimates with weight function ψ obtained by L. Hörmander satisfying pseudo-convexity conditions.

Uniqueness and stability of coefficient c are obtained by using Carleman type estimates.

Notation: $x=(x_0,..,x_n)$ is a vector in $R^{n+1}, t=x_0$, $x'=(0,x_{1,..,}x_{n-1})$, $<x,y>=x_0y_0+...+x_ny_n$, B_δ is the set $B\cap\{\psi>\delta\}$, where ψ is a given function , N is the exterior normal to the boundary of the domain , $\|.\|_{(k)}(Q)$ - is the norm of the Sobolev's space $W_2^k(Q)$, $|.|^k(Q)$ - is the norm of the space $C^k(Q)$ of k times continuous differentiable functions, $\overset{0}{W}{}_2^k$ is the closure of $C_0^\infty(Q)$ with respect to the norm $\|\cdot\|_{(k)}(Q)$.

Let Ω be a domain in R^n with boundary consisting of the part of hyper-plane $x_n=-d$ and part of S from class C^2 which lies on the shell $-d<x_n\le 0$, $Q=(-T,T)\times\Omega$, $\tilde{A}=(-T,T)\times S$, T, d are positive numbers $R=\sup|x'|$ where $x'\in\Omega$.

We obtain Carleman's type estimates for wave equation

$$\square_C\, u=cu_{tt}-\Delta u,$$

with coefficient c satisfying the conditions $c\in C^1(\overline{Q})$; $c>\overline{Q}$.

Define the weight function $\psi(x)$ by the formula

$$\psi(x)=\exp\left[x_n+d\left(1-x_0^2T^{-2}\right)\right]-1$$

Theorem 1. *There exists a number* M *depending only on* $|c|^1(\varphi)$ *such that, if*

$$c_{x_n}\cdot T>M\left(1+d^4\right),$$

then the following Carleman's type estimate

$$\tau\int|\nabla u|^2e^{2\tau\psi}dx+\tau\int|u|^2e^{2\tau\psi}dx\le$$

$$\le K\int\ \square_C\, u\ \ e^{2\tau\psi}dx,\ \ \tau\ge K$$

with some constant K *for any function* $u\in\overset{0}{W}{}_2^2(Q_0)$..

Theorem 1 follows from Carleman's type estimates proved by L. Hörmander.

Uniqueness of construction of the coefficient $c(x)$ is reduced to the uniqueness of solution (u,q) of the following problem

$$\square_C\, u+\sum_{j=1}^n a^ju_{x_j}+au=pq \text{ on } Q,$$

$$u=0,\ \ u_N=0 \text{ on } \tilde{A}_0,\ u=0 \text{ on } \{0\}\times\Omega..$$

Theorem 2. *If*

$$\rho > 0 \;\; on \;\; \{0\} \times \overline{\Omega}, \qquad \rho,\, \rho_t \in C^1(\overline{Q}),\; \square C\, \rho_t \in C(\overline{Q}),$$

and conditions of the Theorem 1 are hold and also $u,\, u_t \in W_2^2(Q)$, *then* $u = 0,\; q = 0$ *on* Q_0.

Theorem 2 has been proved by the authors under the additional condition $c_t = 0$ [1].

REFERENCES.

[1] Khaydarov A. Carleman’s estimates and inverse problems for the second order hyperbolic equations. // Math. Ud. Sb. 1986 V. 130 № 2. pp. 265- 274.

MODELLING AND ANALYSIS OF TORSIONAL VIBRATIONS OF ROTATING CYLINDRICAL SHELL

Khudoynazarov Kh.Kh., Burkutboyev Sh.M.
Faculty of mathematics and mechanics
Samarkand State University
khayrulla@samdu.uz , sherzod-b@samdu.uz

INTRODUCTION

The problems of vibrations of rotating circular cylindrical shells are of great importance in the context of aerospace and mechanical engineering applications. There are various engineering applications of rotating cylindrical shells such as drilling works and gas turbines for high-power aircraft engines. Analysis of rotating shells have been further investigated by many researchers. Huang et al. (1990) used the Love-Timoshenko theory to analyze the bifurcation frequencies of rotating cylindrical shells. Rotating composite cylindrical shells have also been studied by Rand et al. (1991) and Liew et al. (2002). The natural frequencies analysis of rotating laminated cylindrical shells was carried out by Lam et al. (1995) using different thin shell theories. The frequencies were obtained by solving the partial differential equations of motion.

In this work is considered torsional vibrations of circular cylindrical shell rotating with constant angular velocity.

MODELLING AND SOLUTION

Torsional vibration equations that has developed by Khudoynazarov (2003) is improved by taking into account of centrifugal forces for rotating elastic circular cylindrical shell. Deduced equations are indefinitely high order partial differential equations relatively displacements of the cylindrical shell and are impossible for solution of applied problems, so rejected high order terms in the vibration equations and received second order PDE.

We consider a cylindrical shell rotating about its axis at a constant angular velocity Ω. The periodic displacement is given from first end and second end is fixed. For solution of this problem was carried out numerically by finite difference method. In this case length is separated on 400 sections. Received results are presented in Figure 1. and Figure 2. The computations are carried out for the shell at values h=0.001m (thickness), *l*=10m (length).

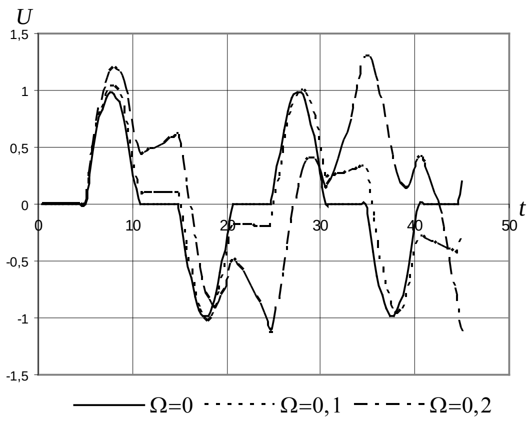


Figure 1. Variation of the displacement with time for different values of the

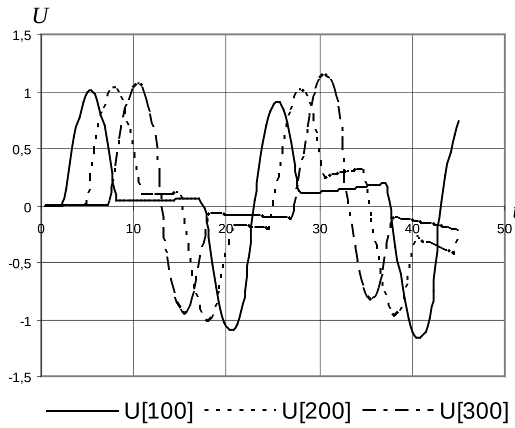


Figure 2. Variation of the displacement with time for different cross sections

CONCLUSION

Numerical solution has been presented for the problem of torsional vibrations of the rotating cylindrical shell under influence periodic loading. Results show, that the increasing of the angular velocity reduce to increasing and partially distortion of displacement amplitude. Influence of the angular velocity is greater on closer cross sections to second end than first one.

REFERENCES

Huang S.C., Hsu B.S. (1990) Resonant phenomena of a rotating cylindrical shell subjected to harmonic moving load. Journal of sound and vibration, 136, 215-228.

Lam K. Y., Loy C. T.(1995) Analysis of rotating laminated shell by different thin shell theories. Journal of Sound and Vibration. 186(1), 23-35.

Liew K. M., Ng T. Y., Zhao X. (2002) Vibration of axially loaded rotating cross-ply laminated cylindrical shells via Ritz method. Jour. eng. mech.128 (9), 1001-1007.

Rand O. and Stavsky Y.(1991) Free Vibrations of Spinning Composite Cylindrical Shells. International Journal Solids and Structures. 28. (7), 831–843.

Khudoynazarov Kh.(2003) Nonstationary interaction cylindrical shells and columns with deformable medium. Tashkent: Abu Ali ibn Sino. 326 p.

NONLINEAR MODELLING ELASTIC DEFORMATION OF RIBBED PLATE

Khudoynazarov Kh.Kh., Nishonov U.A.
Faculty of mathematics and mechanics
Samarkand State University
utkirn@rambler.ru

Karshiev A.B.
Samarkand branch of Tashkent University of Information Technologies

INTRODUCTION

At the present in researching of SSS of ribbed structural elements, in particular of shells and plates are solved various axisymmetric and nonaxisymmetric problems, both in geometrically, and such in physically linear statements. Andrianov U.B. et al. (1985) are considered some general problems of accounting of a statics and dynamics of ribbed shells. Galiev Sh. U. et al. (1990) are studied the numerical analysis of known models of a ribbed dome, under pulse loadings and substantiation of a new way of account of ribbed plates. Influence of change of the geometrical characteristics of ribs on a dynamic SSS of the circular plate is investigated below.

STATEMENT OF PROBLEM AND ITS SOLUTION

Elastic reaction of supported with ring rib circular plate on nonstationary pulse loading is investigated. Pulse loading on outer surface changes with time according exponential law.

Properties of plate and ring are equal. Border of plate is fixed. Nonlinear Timoshenko type theory of flexible plate is used for description of plate SSS (Galiev Sh. U. et al. 1990; Kholmuradov R.I. et al. 2002).

The plate equations complemented by initial and boundary conditions, are integrated by finite difference method (Galiev Sh. U. et al. 1990). Midsurface of the shell is divided into ring elements. Thus the explicit scheme is used

$$V_{ik}^{m+1} = 2V_{ik}^{m} - V_{ik}^{m-1} + \tau^2 U[(N_1, N_2, M_1, M_2, Q)_{i\pm1/2,k\pm1/2}^{m}, P_{ik}^{m}],$$

where V_{ik}^{m} – vector with components u_{ik}^{m}, w_{ik}^{m} φ_{ik}^{m}; function U – finite difference expression of the left hand side of the plates equations. In the scheme displacements and angles of turn are defined on nodes of mesh, and deformation, efforts and moments in the centre of cells. Exactness and stability of computation of this scheme were investigated in the work Galiev Sh. U. et al. (1990).

CONCLUSIONS

On the basis of the plate equations designed the rigidly jammed ribbed circular plate under pulse loading $P = P_0 e^{-t/\alpha}$, where $P_0 = 2{,}5$ *MPa*, $\alpha = 10^{-3}\sec$. *R=0,5m*; *h=0.01m; E=75600 MPa;* $\nu = 0{,}3$; $\rho = 2640\,\kappa g / m^3$.

Let's plate is supported with two ribs with height $h^p = 0.04\,m$ and width $\xi = 0.0333\,m$. Considered three cases of rib disposition:

1) near to centre of plate with coordinates $r_1 = 0{,}1m$ and $r_2 = 0{,}2m$;
2) in the middle part of plate with coordinates $r_1 = 0{,}2m$ and $r_2 = 0{,}3m$;
3) near to border of plate with coordinates $r_1 = 0{,}3m$ and $r_2 = 0{,}4m$.

Results corresponding theese cases of computation are presented in the Figure 1.

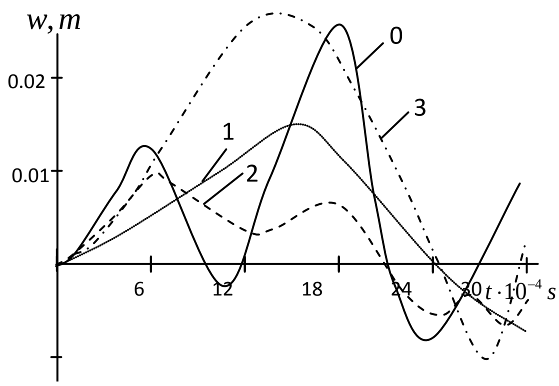


Figure. Bendings in the central point nonsupported (curve 0) and supported with two ribs, situated near to centre (curve 1), in the middle part (curve 2) and near to border (curve 3).

Analyzing of Figure allows to notice influence of changing disposition ribs to maximal value of vibration amplitude. In the first and second cases situation ribs maximal bending approximately twice smaller, and in the third case a little larger than maximal value of nonsupported plate. Thus disposition of rib near to border can reduce decreasing strength of structures than nonsupported smooth plates.

REFERENCES

Andrianov U.B., Lesnichaya B. A., Manevich L. U. (1985). Method of averaging in a statics and dynamics of ribbed shells.-M.:Science,-221p.

Kholmuradov R.I., Karshiev A.B. (2002). Account of elemets of designs with Infringements of a regularity structures.- Tashkent: Abu Ali ibn Sino, -285p.

Galiev Sh. U., Karshiev A.B. (1990) The numerical analysis of accuracy of models of a ribbed spherical dome subject pulse loading. Problems of Strength.-No 5,-pp. 83-86.

THE SPECTRAL PROPERTIES OF THE ONE-PARTICLE SCHÖDINGER OPERATOR ON THE TWO-DIMENSIONAL LATTICE

Kuljanov U.N.
Faculty of mathematics and mechanics
Samarkand State University

INTRODUCTION

Some spectral properties of the Schrödinger operators, corresponding to energy operators - Hamiltonians of one and the system of two quantum particles moving on lattices, are studied by S.Albeverio, S.N.Lakaev, Z.I.Muminov, Faria da Veiga P. A., Ioriatti L., and O'Carroll M.

The earliest results relating positivity and the nondegeneracy of an eigenvalue go back to a fundamental theorem of Perron and Frobenius: a finite matrix with strictly positive elements always has its spectral radius as an eigenvalue of multiplicity one with the corresponding eigenvector strictly positive. The Perron-Frobenius theorem first appeared in O. Perron and then F. G. Frobenius.

The idea of applying a theorem of Perron-Frobenius type to quantum systems is due to J. Glimm and A. Jaffe. The idea ofusing the irreducibility which simplifies the proof is due to I. Segal. The application to nonrelativistic systems is due to B. Simon and R. Hoegh-Krohn.

In the book of M.Reed and B.Simon the theorem of Perron-Frobenius type for the Hamiltonian of an N-body Schrödinger system with center of mass motion removed.

RESULT AND DISCUSSION

In the present paper we consider the Hamiltonian $h = h_0 - V_{\mu\lambda}$, describing the energy of one quantum particle on the two-dimensional lattice Z^2 and moving in the potential field $V_{\mu\lambda}$.

Here the operators h_0 and $V_{\mu\lambda}$ are defined by

$$(h_0\psi)(x) = \sum_{s\in Z^2} \tilde{\varepsilon}(s)\psi(x+s), \quad (V_{\mu\lambda}\psi)(x) = v_{\mu\lambda}(x)\psi(x), \ \psi \in l_2(Z^2),$$

where we assume the functions $\tilde{\varepsilon}(s)$ and $v_{\mu\lambda}(x)$ to be defined by

$$\tilde{\varepsilon}(s) = \begin{cases} 2, \ s = 0 \\ -\dfrac{1}{2}, s \in \{\pm e_1, \pm e_2\} \\ 0, \ otherwise \end{cases}, \qquad v_{\mu\lambda}(x) = \begin{cases} \mu, \ x = 0 \\ \lambda, \ x = e_1 \\ 0, otherwise \end{cases}.$$

Here e_1, e_2 are the elements of the canonical basis of Z^2 and μ, λ are arbitrary positive numbers.

Theorem. For any $\forall \mu, \lambda > 0$, *$\mu \neq \lambda, (\mu = \lambda)$ the operator h has two simple eigenvalues (an eigenvalues of multiplicity two), lying below the bottom of the essential spectrum, with the (positive) eigenfunctions belong to $l_2(Z_0^2)$ and $l_2(Z_1^2)$,* where Z_0^2 resp. Z_1^2 the subset $Z \times 2Z$ resp. $Z \times (2Z+1)$ of two-dimensional lattice Z^2.

CONCLUSION

The main result of the present paper refers to the one-particle Schrödinger operator that has an eigenvalue as the lowest point in its spectrum. Under certain conditions, we show that the eigenspace corresponding to this eigenvalue may be one or two dimensional and that the corresponding eigenvector is a positive function.

REFERENCES

Albeverio S., Lakaev S.N., and Muminov Z.I.: Ann. Henri Poincaré. **5**, 743--772 (2004).

Faria da Veiga, P. A. Ioriatti L., and O'Carroll M.: Phys. Rev. E (3) **66**, 016130, 9 pp. (2002).

F. G. Frobenius: Preus. Akad. Wiss Berlin (1908); 471-476.

J.Glimm, A.Jaffe,.Ann. Math. **91** 362-401 (1970).

O.Perron. -Math. Ann. 64 (1907), 248-263.

Reed M. and Simon B.: Methods of modern mathematical physics. IV: Analysis of

Operators, Academic Press, New York, 1979.

B. Saymon, R. Hoegh-Krohn, J. Func.Anal. 121-180 (1972).

ON ASIMPTOTICAL THEOREMS FOR DISTRIBUTIONS QUEUE LENGTH OF DUAL SYSTEMS $M|G|1|N$ AND $GJ|M|1|N-1$

Kurbanov H.
Samarkand State University

We consider two $\left(F_1 - M|G|1|N \ \ and \ \ F_2 GJ|M|1|N-1\right)$ single-server queuing systems as duals of each other and use common notations for dual characteristics.

In F_1, let the customers arrive is a Poisson process with parameter λ_i in the dual queue system F_2 this means that the service–time distribution is exponential with mean λ^{-1}. Similarly, let the service–times in the queue system F_1 and inter arrival times in the queue system F_2 be independent and identically distributed random variables with mean μ^{-1}.

Let $N(N-1)$ the maximal number of customers that may be accommodated in queue system $F_1(F_2)$.

We shall denote by $\xi_1(\xi_2)$ the stationary queue length for the $F_1(F_2)$ system and use the notations

$$\rho_1 = \lambda\mu^{-1}, \ \ \rho_2 = \rho_1^{-1} = \mu\lambda^{-1},$$

$$b = 1 - 2(1-\rho_1)(\lambda\sigma)^{-2} = 1 - 2(1-\rho_2)(\lambda\sigma)^{-2}$$

$$a_x = \begin{cases} 0, & x \le 0, \\ [x]+1, & x > 0, \end{cases}$$

where $[x]$ - integer part of x.

$$F_{m,M} = \left(1 - e^{-\frac{m}{M}x}\right)\left(1-e^{-m}\right)^{-1}, \quad 0 \le x \le M, \ 0 \le m \le \infty .$$

Theorem 1. If $N \to \infty$, $\rho_1 \to 1$ and $x \to \infty$ $(0 < x \le N+1)$, them

$$P(\zeta_1 < x) = \frac{1-b^{-a_x}}{1-b^{-N-1}} + \mathrm{O}\left(\frac{1}{x} + |1-\rho_1|\right).$$

Theorem 2. If $N \to \infty$, $\rho_2 \to 1$ and $x \to \infty$ $(0 < x \le N+1)$, then

$$P(N-\zeta_2 < x) = \frac{1-b^{-a_x}}{1-b^{-N-1}} + \mathrm{O}\left(\frac{1}{x} + |1-\rho_2|\right).$$

Theorem 3. If $N \to \infty$, $\rho_1 \uparrow 1$ $(\rho_2 \downarrow 1)$ and $N(1-\rho_1) \to \alpha$ $[N(\rho_2 - 1) \to \alpha]$, $0 \le \alpha \le \infty$, then

$$\lim P(\xi_1 < x \cdot E\xi_1) = \lim P(N - \xi_2 < xE\xi_2) = F_{\alpha,A}(x), \tag{3}$$

where $A = \alpha(1 - e^{-\alpha})^{-1}$.

Theorem 4. If $N \to \infty$, $\rho_1 \downarrow 1$ $(\rho_2 \uparrow 1)$ and $N(\rho_1 - 1) \to \beta$ $[N(1-\rho_2) \to \beta]$, $0 \le \beta \le \infty$, then

$$\lim P(N - \xi_1 < xE\xi_1) = \lim P(\xi_2 < xE\xi_2) = F_{\beta,B}(x), \tag{4}$$

where $B = \beta(e^{-\beta} - 1)^{-1}$.

The results of [1] are used in the prove of theorems 1 and 2. The results (3) and (4) has been obtained in [3] for the $M \mid M \mid 1 \mid N$ system.

Theorem 3 u 4 has been obtained in [2] by other methods.

REFERENCES

[1] Kurbanov H. On a duality relation in queuing systems. Jzv. AN UzSSR, serija f.m.n., 3, 1988, 18-21.

[2] Kurbanov H. Systems and its Applications. The International Conference on some Topicsof Mathematics, Samarkand State University, 1996, 173-174.

[3] Tashmanov H.T. The limiting laws in the system with finite queue. Jzv. ANUzSSR, Serija f.m.n., 1.

NUMBER OF EIGENVALUES OF THE FAMILY OF FRIEDRICHS MODELS

Kurbonov Sh.H.
Samarkand State University, Samarkand, Uzbekistan,
shaxzod_kurbanov@mail.ru

INTRODUCTION

Let $\mathrm{T}^d = (\mathrm{R}/2\pi\mathrm{Z})^d = (-\pi,\pi]^d$ be the d - dimensional torus (Brillouin zone) and $\mathbf{C}^1$ – be the complex plane and $L_2(\mathrm{T}^d)$ be the Hilbert space of square-integrable functions on T^d. Consider the Hilbert space H consisting of the direct sum of the Hilbert spaces $\mathbf{C}^1$ and $L_2(\mathrm{T}^d)$, i.e. $\mathrm{H} = \mathbf{C}^1 \oplus L_2(\mathrm{T}^d)$. The Friedrichs model operator $H_{\lambda\mu}(p)$, $p \in \mathrm{T}^d$ is of the form [1]:

$$H_{\lambda\mu}(p) = H_0(p) + V_{\lambda\mu}(p) = \begin{pmatrix} 0 & 0 \\ 0 & h_0(p) \end{pmatrix} + \begin{pmatrix} u(p) & \sqrt{\mu}B \\ \sqrt{\mu}B* & -\lambda\Phi^*\Phi \end{pmatrix}. \qquad (1)$$

Where

$$\Phi : L_2(\mathrm{T}^d) \to \mathbf{C^1}, \quad \Phi f = (f, \varphi)_{L_2(\mathrm{T}^d)},$$

$$\Phi^* : \mathbf{C^1} \to L_2(\mathrm{T}^d), \quad \Phi^* f_0 = \varphi(q) f_0$$

$$B : L_2(\mathrm{T}^d) \to \mathbf{C^1}, \quad Bf = (f, b)_{L_2(\mathrm{T}^d)},$$

$$B^* : \mathbf{C^1} \to L_2(\mathrm{T}^d), \quad B^* f_0 = b(q) f_0$$

and the non perturbed operator $h_0(p)$ on $L_2(\mathrm{T}^d)$ is multiplication operator by the function $\omega(p,q)$:

$$(h_0(p) f)(q) = \omega_p(q) f(q),$$

where $\omega_p(q) := \omega(p,q)$ and $,\varphi(q), b(q), u(p)$ are analytic functions on $(\mathrm{T}^d)^2$ and T^d respectively and $\lambda, \mu \in \mathrm{R}^1$ – non negative numbers and $\lambda + \mu > 0$.

Hypothesis 1. *Let function $\omega(p,q)$ has a inique non-degenerated minimum in* $(0,0) \in (\mathrm{T}^d)^2$.

The perturbation $V_{\mu\lambda}(p)$ of the operator $H_0(p)$ is a self-adjoint operator of rank no more three. Therefore in accordance Weil's theorem (see [2]), the essential spectrum of $H_{\mu\lambda}(p), p \in \mathrm{T}^d$ fills the following interval on the real axis:

$$\sigma_{ess}(H_{\mu\lambda}(p)) = \sigma(H_0(p)) = \sigma_{ess}(h_0(p)) = [m(p), M(p)],$$

where

$$m(p) = \min_{q \in \mathrm{T}^d} \omega_p(q) = \min_{q \in \mathrm{T}^d} \omega(p,q), \qquad M(p) = \max_{q \in \mathrm{T}^d} \omega_p(q) = \max_{q \in \mathrm{T}^d} \omega(p,q)$$

Notice that in [1] the existence of eigenvalues of the operator $H_{\lambda\mu}(p)$ have been studied for $d \geq 3$.

RESULTS AND DISCUSSIONS

Let $\lambda = 0$. Then the operator definded by (1) is of the following form

$$H_{0\mu}(p)=H_0(p)+V_{0\mu}(p)=\begin{pmatrix}0 & 0\\ 0 & h_0(p)\end{pmatrix}+\begin{pmatrix}u(p) & \sqrt{\mu}B\\ \sqrt{\mu}B^* & 0\end{pmatrix}=\begin{pmatrix}u(p) & \sqrt{\mu}B\\ \sqrt{\mu}B^* & h_0(p)\end{pmatrix}.$$

Lemma 1.1 *Let the hypothesis* 1 *is true. Then there exists such* $\delta-$*neighborhood* - $U_\delta(0)\subset \mathrm{T}^d$ *of point* $p=0$ *and analitic function* $q_0: U_\delta(0)\to \mathrm{T}^d$ *wich for any* $p\in U_\delta(0)$ *function* $w_p(\cdot)$ *has a unique non-degenerated minimum in* $q_0(p)$

Let $d=1,2.$ We remark that if $b(q_0(p))=0$ then there exists

$$\mu(p)=\left(\int_{\mathrm{T}^d} b^2(s)(w_p(s)-m(p))^{-1}ds\right)^{-1}>0$$

The following result is on the existence of eigenvalues of the operator $H_{0\mu}(p), p\in U_\delta(0).$

Theorem 12. ***a****) Let* $u(p)\le m(p)$. *Then the operator* $H_{0\mu}(p),\ \ p\in U_\delta(0)$ *has a unique eigenvalue* $E(\mu,p)\in(\infty,u(p)).$

b) Let $u(p)>m(p).$ Then the following results are hold true:

(***i***) If $b(q_0)\ne 0$ or $b(q_0)=0,\nabla b(q_0)\ne 0$ and $\mu>(u(p)-m(p))\mu(p)$ then the operator $H_{0\mu}(p),\ \ p\in U_\delta(0)$ has a unique eigenvalue $E(\mu,p)\in(\infty,m(p)).$

(***ii***) If $b(q_0(p))=0,$ $\nabla b(q_0(p))\ne 0$ then $\mu(p)>0$ and for any $\mu<(u(p)-m(p))\mu(p)$ the operator $H_{0\mu}(p),$ $p\in U_\delta(0)$ has none eigenvalue in $(-\infty,m(p)]$;

(***iii***) If $b(q_0(p))=0,$ $\nabla b(q_0(p))\ne 0$ and $\mu=(u(p)-m(p))\mu(p)$ then the equation

$$H_\mu(p)f=m(p)f,\qquad p\in U_\delta(0)$$

has non-zero solution

$$f=(f_0,f_1),\quad 0\ne f_0\in\mathbf{C}^1,\quad f_1=\frac{-\sqrt{\mu}b(q)f_0}{\omega_p(q)-m(p)}\notin L_1(\mathrm{T}^d)\setminus L_2(\mathrm{T}^d);$$

(***iv***) If $b(q_0(p))=0,$ $\nabla b(q_0(p))=0$ and $\mu=(u(p)-m(p))\mu(p)$ then the number $z=m(p)=w_p(q_0(p))$ is eigenvalue of $H_{0\mu}(p),$ $p\in U_\delta(0)$ and the corresponding eigenfunction has a form:

$$f=(f_0,f_1)\in\mathrm{H},\quad 0\ne f_0\in\mathbf{C}^1,\quad f_1=\frac{-\sqrt{\mu}b(q)f_0}{\omega_p(q)-m(p)}.$$

REFERENCES

[1] Albeverio, Sergio, Lakaev, Saidakhmat N., Muminov, Zahriddin I.: The threshold effects for a family of Friedrichs models under rank one perturbations. J. Math. Anal. Appl. **330** (2007), no. 2, 1152 -- 1168.

[2] M. Reed and B. Simon: Methods of modern mathematical physics. IV: Analysis of operators. Academic Press, N.Y., 1978

THRESHOLD EFFECTS FOR THE TWO AND THREE PARTICLE DISCRETE SCHRÖDINGER OPERATORS

Lakaev S. N.

INTRODUCTION

The main goal of this report is to give *new threshold phenomena* those are not presents for the two and three-particle discrete Schrödinger operators $H_2(k), k \in \mathrm{T}^d$, and $H_3(K), K \in \mathrm{T}^d$, associated to the Hamiltonians H_2 and H_3 of systems of two and three identical particles on the d-dimensional lattice Z^d interacting via pair short-range potentials.

The spectral properties for the two-particle lattice Hamiltonians on the d-dimensional lattice $\mathrm{Z}^d, d \geq 1$ studied intensively [1, 5, 6, 7, 8, 10, 11, 12].

The kinematics of quantum quasi-particles on lattices, even in the two-particle sector, is rather exotic. For instance, due to the fact that the discrete analogue of the Laplacian or its generalizations are not rotationally invariant, the Hamiltonian of a system does not separate into two parts, one relating to the center-of-mass motion and the other one to the internal degrees of freedom. In particular, such a handy characteristics of inertia as mass is not available. Moreover, such a natural local substituter as the effective mass-tensor (of a ground state) depends on the quasi-momentum of the system and, in addition, it is only semi-additive (with respect to the partial order on the set of positive definite matrices). This is the so-called *excess mass* phenomenon for lattice systems [11, 12] the effective mass of the bound state of an N-particle system is greater than (but, in general, not equal) to the sum of the effective masses of the constituent quasi-particles.

The n-body problem on lattices can be reduced to the effective n-particle Schrödinger operators by using the Gelfand transform. The underlying Hilbert space $\ell^2((\mathrm{Z}^d)^n)$ is decomposed as a direct von Neumann integral associated with the representation of the discrete group Z^d by shift operators on the lattice and the total n-body Hamiltonian turns out to be decomposable. In contrast to the continuous case, the corresponding fiber Hamiltonians $H_n(K,V)$ associated with the direct decomposition depend parametrically on the quasi-momentum $K \in \mathrm{T}^d = (-\pi, \pi]^d$, which ranges over a cell of the dual lattice. Due to the loss of the spherical symmetry of the problem, the spectra of the family $H_n(K,V)$ turn out to be rather sensitive to the quasi-momentum $K \in \mathrm{T}^d$.

The appearance of negative bound states for critical (non-negative) Schrödinger operators under infinitesimally small negative perturbations is especially remarkable: it is the presence of zero-energy resonances in at least two of the two-particle subsystems that leads to the existence of infinitely many bound states for the corresponding three-body system, the Efimov effect [2,3,4,10, 13, 15, 16].

It turns out that in the two-body lattice case there exists an extra mechanism for the bound state(s) to emerge from the threshold of the critical Hamiltonians which has nothing to do with additional (effectively negative) perturbations of the potential term. The role of the latter is rather played by the adequate change of the kinetic term which is due to the nontrivial dependence of the fiber Hamiltonians $H_2(k)$ on the quasi-momentum k and is related to the excess mass phenomenon for lattice systems mentioned above.

In the case of the three-particle lattice Schrödinger operators $H_{\mu_0}(K)$, $K \in \mathrm{T}^3$ -- three-particle quasi-momentum, associated to the Hamiltonian of a system of three particles on Z^3 interacting via zero-range pair potentials $\mu > 0$ the following *phenomenon* is also deeply related to the appearance below the bottom of the essential spectrum of two-particle Schrödinger operators under infinitesimally small negative perturbations: for $\mu = \mu_0$ the corresponding three-particle lattice Schrödinger operator $H_{\mu_0}(0)$ has infinitely many eigenvalues, whereas $H_{\mu_0}(K)$, $K \neq 0$ has only finitely many [3, 4, 10].

THE DISCRETE SCHRÖDINGER OPERATORS.

Let Z^d be the d - dimensional hybercubic lattice and $\mathrm{T}^d = (\mathrm{R}/2\pi\mathrm{Z})^d = (-\pi,\pi]^d$ be the d - dimensional torus (Brillouin zone), the dual group of Z^d. Let $L_2^e(\mathrm{T}^d) -$ the Hilbert space of square-integrable even functions on T^d. The two and three-particle Hamiltonians H_2 and H_3 (in the momentum representation) are unitary equivalent to the direct integrals

$$H_2 = \int_{k \in \mathrm{T}^d} \oplus H_2(k) d\mu^*(k) \,\text{and}\, H_3 = \int_{k \in \mathrm{T}^d} \oplus H_3(K) d\mu^*(K),$$

where μ^* is the (normalized) Haar measure on the torus T^d. In the physical literature the parameter $k \in \mathrm{T}^d$ resp. $K \in \mathrm{T}^d$ is called the *two-particle resp. tree-particle quasi-momentum* and the corresponding operators $H_2(k)$ $k \in \mathrm{T}^d$ resp. $H_3(K)$ $K \in \mathrm{T}^d$ are called the *fiber operators (discrete Schrödinger operators).*

The discrete Schrödinger operator $H_2(k), k \in \mathrm{T}^d$ acts on $L^{2,e}(\mathrm{T}^d)$ and is of the form

$$H_2(k) = H_2^0(k) - v, K \in \mathrm{T}^d. \qquad (1)$$

The operator $H_2^0(k), k \in \mathrm{T}^d$ is defined on the Hilbert space $L^{2,e}(\mathrm{T}^d)$ by

$$(H_2^0(k) f)(q) = \mathrm{E}_k(q) f(q), \quad f \in L^{2,e}(\mathrm{T}^d),$$

where

$$\mathrm{E}_k(p)=\varepsilon(\frac{k}{2}-p)+\varepsilon(\frac{k}{2}+p),\varepsilon(p)=\sum_{j=1}^{d}[1-\cos(p_j)],\vec{p}=(p_1,...,p_d)\in \mathrm{T}^d.$$

and v is the integral operator of convolution type, i.e.,

$$(vf)(p)=(2\pi)^{-\frac{d}{2}}\int_{\mathrm{T}^d} v(p-q)f(q)dq, \quad f\in L_2^e(\mathrm{T}^d).$$

where $v(\cdot)$ is even continuous function on T^d.

The three-particle discrete Schrödinger operators $H_3(K), K\in\mathrm{T}^d$, associated to the Hamiltonian H_3 of a system of three identical particles on the d-dimensional lattice T^d interacting via short-range pair potentials act on the Hilbert space $L_2^e((\mathrm{T}^3)^2)\cong L_2(\mathrm{T}^3)\otimes L_2^e(\mathrm{T}^3)$ and given by

$$H_3K)=H_3^0(K)-V_1-V_2-V_3.$$

The operators $H_0(K)$ and $V_\alpha\equiv V,\alpha=1,2,3,$ are defined on the Hilbert space $L_2^e((\mathrm{T}^3)^2)\cong L_2(\mathrm{T}^3)\otimes L_2^e(\mathrm{T}^d)$ and in the coordinates $(k,q)\in(\mathrm{T}^d)^2$ are of the form

$$(H_0(K)f)(k,q)=\mathrm{E}_K(k,q)f(k,q),\mathrm{E}_K(k,q)=\varepsilon(K-k)+\varepsilon(\frac{k}{2}-q)+\varepsilon(\frac{k}{2}+q),$$

where $V=I\otimes v$ and $I=I_{L_2(\mathrm{T}^d)}$ is the identity operator on $L_2(\mathrm{T}^d)$.

The essential spectrum $\sigma_{\mathrm{ess}}(H_2(k))$ of $H_2(k),k\in\mathrm{T}^d$ fills the segment $[\mathrm{E}_{\min}(k),\mathrm{E}_{\max}(k)]$, where

$$\mathrm{E}_{\min}(k)=\min_{p\in\mathrm{T}^d}\mathrm{E}_k(p)=\mathrm{E}_k(0)=0,\mathrm{E}_{\max}(k)=\max_{p\in\mathrm{T}^d}\mathrm{E}_k(p).$$

From the positivity of v and the min-max principle we further obtain that all isolated eigenvalues of finite multiplicity lie below the bottom $\mathrm{E}_{\min}(K)$ of the essential spectrum $\sigma_{\mathrm{ess}}(H_2(k))$.

CONCEPTS OF REGULAR AND SINGULAR POINTS

In order to introduce the concept of a *virtual level* (*threshold resonance*) for the (lattice) two particle operators $H_2(k)$, we define a compact self-adjoint non-negative *limit Birman-Schwinger operator* $B(K,\mathrm{E}_{\min}(K))$ on $L^{2,e}(\mathrm{T}^d), d\geq 3$ with the kernel function

$$B(K,\mathrm{E}_{\min}(k);p,q)=\int_{\mathrm{T}^d}\frac{v^{\frac{1}{2}}(p-t)v^{\frac{1}{2}}(t-q)}{\mathrm{E}_K(t)-\mathrm{E}_{\min}(k)}d\mu^*(t).$$

Definition 3 *Let* $d \geq 3$. *The operator* $H_2(k), k \in \mathrm{T}^d$ *is said to have a singular point of multiplicity* m *(resp.regular point) at the bottom* $z = \mathrm{E}_{\min}(k)$ *of the essential spectrum* $\sigma_{\text{ess}}(H_2(k))$ *if the number 1 is an eigenvalue of multiplicity* m *(resp.no eigenvalue) for the operator* $B(k, \mathrm{E}_{\min}(k))$.

Definition 4 *Let* $d = 3 \text{or} 4$. *The singular point* $z = \mathrm{E}_{\min}(k)$ *is called a virtual level (a threshold resonance) of the operator* $H_2(k)$, *if the number* 1 *is an eigenvalue for the operator* $B(k, \mathrm{E}_{\min}(k))$ *and one of the associated eigenfunction* ψ *satisfies the condition* $(v^{1/2}\psi)(0) \neq 0$.

STATEMENT OF THE RESULTS AND DISCUSSIONS

Let $d \geq 1$. We introduce a parameter $\eta_s[\mathrm{E}_K(e)] \geq 0$ as

$$\frac{1}{\eta_s[\mathrm{E}_K(e)]} = \int_{\mathrm{T}^d} \frac{\cos^2(s,q)dq}{\mathrm{E}_K(q) - \mathrm{E}_{\min}(K)}$$

for $d \geq 3$ and $\eta_s[\mathrm{E}_K(e)] := 0$, for $d = 1,2$.

The following theorems state the existence of the discrete spectrum below the essential spectrum of the operators $H_2(k)$ for all values of the quasi-momentum $k \in \mathrm{T}^d$.

Theorem 5 *Let* $d = 1 \text{or} 2 \text{and} v \neq 0$. *Then for any* $k \in \mathrm{T}^d$ *the operator* $H_2(k)$ *has an eigenvalue* $E(k)$ *below the bottom* $\mathrm{E}_{\min}(k)$ *of the essential spectrum* $\sigma_{ess}(H_2(k))$.

Definition 6 *We will order the vectors* $k^{(1)} = (k_1^{(1)}, \ldots, k_d^{(1)}) \in \mathrm{T}^d, k^{(2)} = (k_1^{(2)}, \ldots, k_d^{(2)}) \in \mathrm{T}^d$ *in following way: we say* $k^{(1)} \geq k^{(2)}$ *if* $|k_s^{(1)}| \geq |k_s^{(2)}|$ *for all* $s = 1,...,d$ *and* $k^{(1)} > k^{(2)}$, *if* $k^{(1)} \geq k^{(2)}$ *and* $|k_s^{(1)}| > |k_s^{(2)}|$ *for some* $s = 1,...,d$.

Theorem 7 *Let* $d \geq 3$ *and for some* $s \in \mathrm{Z}^d$, $k_0 \in \mathrm{T}^d$ *the inequality*

$$v(s)\int_{\Gamma*} \frac{\cos^2(s,q)dq}{\mathrm{E}_{k_0}(q) - \mathrm{E}_{\min}(k_0)} > 1$$

holds, then for all $k \in \mathrm{T}^d, |k_0| \leq |k|$ the operator $H_2(k), k \in \mathrm{T}^d$ has an eigenvalue $E(k)$. Moreover, for $k \in \mathrm{T}^d, |k_0| < |k|$ the relations $E(k_0) < E(k) < \mathrm{E}_{\min}(k)$ are hold.

The following *effect* is characteristic only for the discrete Schrödinger operators $H_2(k), k \in \mathrm{T}^d$ and describes the dependence of bound states(eigenvalues) to *the two-particle quasi-momentum* [1, 4, 5, 10].

Theorem 8 *Assume* $d \geq 3$. *Let the inequality* $H_2(0) \geq 0$ *holds and the bottom* $\mathrm{E}_{\min}(0) = 0$ *of the essential spectrum is a singular point of multiplicity* m *of* $H_2(0)$. *Then for any* $0 \neq k \in \mathrm{T}^d$ *the operator* $H_2(k)$ *has at least* m *eigenvalues* $E_1(k),...,E_m(k)$ *(counting* *multiplicities) below the bottom* $\mathrm{E}_{\min}(k)$ *of the essential spectrum* $\sigma_{ess}(H_2(k))$. *Moreover the inequalities* $E_j(k) > 0, j = 1,,,m$ *hold.*

The following theorem describes the conservation of the number of eigenvalues (counting multiplicities) of the operator $H_2(0)$ below $\mathrm{E}_{\min}(0)$.

Theorem 9 *Assume* $d \geq 1$. *Let the operator* $H_2(0), 0 \in \mathrm{T}^d$ *has* n *eigenvalues (counting multiplicities)* $E_1(0),...,E_m(0)$ *below the bottom* $\mathrm{E}_{\min}(0) = 0$. *Then for any* $0 \neq k \in T^d$ *the operator* $H_2(k)$ *has at least* n *eigenvalues (counting multiplicities) lying below* $\mathrm{E}_{\min}(k)$. *Moreover the inequalities* $E_j(k) > E_j(0), j = 1,,,m$ *hold.*

The following theorem is on the stability of the number of eigenvalues when the bottom of the essential spectrum is an regular point.

Theorem 10 *Assume* $d \geq 3$. *Let the bottom* $\mathrm{E}_{\min}(0)$ *is a regular point of the essential spectrum of* $H_2(0)$. *Then there is a* $\delta > 0-$ *neighborhood* $U_\delta(0) \subset \mathrm{T}^d$ *of* $0 \in \mathrm{T}^d$ *such that for all* $k \in U_\delta(0)$ *the number of eigenvalues of the operator* $H_2(k)$ *below the bottom* $\mathrm{E}_{\min}(k)$ *of the essential spectrum* $\sigma_{ess}(H_2(k))$ *does not change.*

Denote by $\tau(K)$ the bottom of the essential spectrum of the three-particle discrete Schrödinger operator $H_3(K), K \in \mathrm{T}^d$ and by $N(K,z)$ the number of eigenvalues below $z \leq \tau(K)$.

The following theorem described Efimov's effect [3, 4, 10, 15, 16] and *three-particle quasi-momentum phenomenon* for the discrete Schrödinger operator $H_3(K), K \in \mathrm{T}^d$ [3, 4, 10].

Theorem 11 *Let* $H_2(0) \geq 0$ *and the bottom* $\mathrm{E}_{\min}(0) = 0$ *is a singular point of the operator* $H_2(0)$.

(i) Assume $d \geq 3$. Then the essential spectrum $\sigma_{ess}(H_3(K))$ of $H_3(K), K \in \mathrm{T}^d$ satisfies the equality

$$\sigma_{ess}(H_3(K)) = [\tau(K), \mathrm{E}_{\min}(K)] \cup [\mathrm{E}_{\min}(K), \mathrm{E}_{\max}(K)],$$

where

$$\mathrm{E}_{\min}(K) = \min_{k,p \in \mathrm{T}^d} \mathrm{E}_K(k,p), \mathrm{E}_{\max}(K) = \max_{k,p \in \mathrm{T}^d} \mathrm{E}_K(k,p)$$

and

$$\tau(0) = \mathrm{E}_{\min}(0) = 0, 0 < \tau(K) < \mathrm{E}_{\min}(K).$$

(ii) Assume $d = 3$. Then the operator $H_3(0)$ has infinitely many eigenvalues below the bottom of the essential spectrum and for the number $N(0,z), z < 0$ the asymptotics

$$\lim_{z \to -0} \frac{N(0,z)}{|\log|z||} = \frac{\lambda_0}{2\pi}, \tag{2}$$

holds, where λ_0 a unique positive solution of the equation

$$\lambda = \frac{8\sinh \pi\lambda/6}{\sqrt{3}\cosh \pi\lambda/2}.$$

(iii) Assume $d = 3$. Then for some punctured $\delta > 0-$ neighborhood $U_\delta^0(0)$ of the origin and for all $K \in U_\delta^0(0)$ the number $N(K,0)$ is finite and satisfy the following asymptotics

$$\lim_{|K|\to 0} \frac{N(K,0)}{|\log|K||} = \frac{\lambda_0}{\pi}. \qquad (3)$$

These results are characteristic for the lattice system and do not have any analogue in the continuous case.

The results of Theorem **9** are in contrast to the similar results for the continuous three-particle Schrödinger operators, where the number of eigenvalues does not depend on the three-particle total momentum $K \in R^3$.

Moreover the results of Theorem **9** are also in contrast to the results of the two-particle discrete Schrödinger operators, which have finitely many eigenvalues for all $k \in \mathrm{T}^d$.

REFERENCES

[1] S. Albeverio, S. N. Lakaev,K. A. Makarov, Z. I. Muminov: Comm.Math.Phys. **262**(2006), 91--115 .

[2] Albeverio S., Hшegh-Krohn R., and Wu T. T.: Phys. Lett. A **83**, 105--109 (1971).

[3]S. Albeverio,G. F. Dell Antonio, S. N. Lakaev: J.Phys.A:Math.Theor. **40**(2007)14819-14842.

[4] Albeverio S., Lakaev S. N., and Muminov Z. I.: Ann. Henri Poincarй. **5**, 743--772 (2004).

[5] Albeverio S., Lakaev S. N., and Muminov Z. I.:J. Math. Anal. Appl. **330** (2007) 1152–1168

[6]V. Bach, W. de Siqueira Pedra, S. Lakaev: Preprint mp-arc 10-143.

[7] Faria da Veiga P. A., Ioriatti L., and O'Carroll M.: Phys. Rev. E (3) **66**, 016130, 9 pp. (2002).

[8] Graf G. M. and Schenker D.: Ann. Inst. H. Poincarй Phys. Thйor. **67**, 91--107 (1997).

[9] Klaus M. and Simon B.: Ann. Phys. **130**, 251--281 (1980).

[10] Lakaev S. N.: Funct. Anal. Appl. **27**, 166--175 (1993).

[11] Mattis D. C.: Rev. Modern Phys. **58**, 361--379 (1986).

[12] Mogilner A.: Advances in Soviet Mathematics **5**, 139--194 (1991).

[13] Sobolev A. V.: Comm. Math. Phys. **156**, 127--168 (1993).

[14] Rauch J.: J. Funct. Anal. **35**, 304--315 (1980).

[15] Tamura H.: Nagoya Math. J. **130**, 55--83(1993).

[16] Yafaev D. R.: Math. USSR-Sb. **23**, 535--559 (1974).

THE EXISTENCE AND ANALYTICITY OF BOUND STATE OF THE DISCRETE SCHRÖDINGER OPERATORS ON LATTICE

Lakaev S.N., Ulashov S.S.
Samarkand State University Samarkand, Uzbekistan,
E-mail: slakaev@mail.ru[1], sobir_s_87@bk.ru[2]

INTRODUCTION

We consider discrete Schrödinger operator $H_\mu(K),\ K\in\mathbb{T}^d,\ d\geq 3$ – two-particle quasi-momentum, corresponding to a system of two arbitrary particles on $d-$dimensional lattice $\mathbb{Z}^d,\ d\geq 3,$ interacting via zero-range pair attractive potential with interaction energy $\mu>0.$ It is shown that the upper edge of the essential spectrum is either virtual level (d=3,4) or eigenvalue ($d\geq 5$) for the operator $H_\mu(K).$ The existence of a unique eigenvalue lying above the upper edge of the essential spectrum depending on coupling constant $\mu>0$ and two-particle quasi-momentum $K\in\mathbb{T}^d$ is established.

RESULTS AND DISCUSSION

Let $\mathbb{T}^d$ be the $d-$ dimensional torus and $L^2(\mathbb{T}^d)$ be the Hilbert space of square-integrable functions on $\mathbb{T}^d$. In the momentum representation the discrete Schrödinger operator $H_\mu(K),K\in\mathbb{T}^d$ are given by the bounded self-adjoint operator on Hilbert space $L^2(\mathbb{T}^d)$ by [1], [2]:

$$H_\mu(K)=H_0(K)+\mu V.$$

The non perturbed operator $H_0(K)$ is multiplication operator on $L^2(\mathbb{T}^d)$:

$$(H_0(K)f)(q)=\mathcal{E}_K(q)f(q),\quad f\in L^2(\mathbb{T}^d),$$

$$\mathcal{E}_K(q)=(1+\gamma)d-\sum_{i=1}^{d}\sqrt{1+2\gamma\cos K^{(i)}+\gamma^2}\,\cos(p^{(i)}-p(K^{(i)})),\quad \gamma>0,\quad \gamma\neq 1,$$

where $p(K)=\left(p(K^1),...,p(K^d)\right)\in\mathbb{T}^d$ is the minimum point of $\mathcal{E}_K(q)$. The perturbation V is an integral operator of rank one:

$$(Vf)(p)=(2\pi)^{-d}\int_{\mathbb{T}^d}f(q)dq,\quad f\in L^2(\mathbb{T}^d).$$

The essential spectrum $\sigma_{ess}(H_\mu(K))$ of $H_\mu(K),K\in\mathbb{T}^d$ fills the segment $[\mathcal{E}_{\min}(K),\mathcal{E}_{\max}(K)],$ where $\mathcal{E}_{\min}(K)=\min_{p\in\mathbb{T}^d}\mathcal{E}_K(p),\qquad \mathcal{E}_{\max}(K)=\max_{p\in\mathbb{T}^d}\mathcal{E}_K(p).$

Set $$\nu(K) = (2\pi)^{-d} \int_{\mathbb{T}^d} (\mathrm{E}_{\max}(K) - \mathrm{E}_K(q))^{-1}\, dq.$$

Let $V^{1/2}$ – be positive square root of the positive operator V, and we define for any $K \in \mathbb{T}^d, d \geq 3,\ z \geq \mathrm{E}_{\max}(K)$ in the Hilbert space $L^2(\mathbb{T}^d)$ the Birman-Schwinger integral operator $G_\mu(K,z) = \mu\, V^{1/2}(z - H_0(K))^{-1}V^{1/2}$ [3].

Definition 12. *The operator* $H_\mu(K), K \in \mathbb{T}^d$ *is said to have a virtual level at the upper edge* $z = \mathrm{E}_{\max}(K)$ *of the essential spectrum* $\sigma_{ess}(H_\mu(K))$ *if the number* 1 *is a simple eigenvalue for the operator* $$G_\mu(K, \mathrm{E}_{\max}(K)) = \mu V^{\frac{1}{2}}(\mathrm{E}_{\max}(K) - H_0(K))^{-1}V^{\frac{1}{2}}$$
and associated eigenfunction ψ *satisfies the condition* $(V^{1/2}\psi)(\vec{\pi} + p(K)) \neq 0.$

For any $\mu > 0$ we define following sets:

$$M_<(\mu) = \{K \in \mathbb{T}^d : 1 - \mu\nu(K) < 0\},$$
$$M_=(\mu) = \{K \in \mathbb{T}^d : 1 - \mu\nu(K) = 0\},$$
$$M_>(\mu) = \{K \in \mathbb{T}^d : 1 - \mu\nu(K) > 0\}.$$

Theorem 1. (i) *Let* $0 < \mu < (\nu(\vec{\pi}))^{-1}$. *Then* $M_<(\mu) = \emptyset$, $M_=(\mu) = \emptyset$, $M_>(\mu) = \mathbb{T}^d$.

(ii) *Let* $\mu = (\nu(\vec{\pi}))^{-1}$. *Then* $M_<(\mu) = \emptyset$, $M_=(\mu) = \{\vec{\pi}\}$, $M_>(\mu) = \mathbb{T}^d \setminus \{\vec{\pi}\}$.

(iii) *Let* $(\nu(\vec{\pi}))^{-1} < \mu < (\nu(0))^{-1}$. *Then each of the sets* $M_<(\mu)$, $M_=(\mu)$ *and* $M_>(\mu)$ *is not empty.*

(iv) *Let* $\mu = (\nu(0))^{-1}$. *Then* $M_<(\mu) = \mathbb{T}^d \setminus \{0\}$, $M_=(\mu) = \{0\}$ *and* $M_>(\mu) = \emptyset$.

(v) *Let* $\mu > (\nu(0))^{-1}$. *Then* $M_<(\mu) = \mathbb{T}^d$, $M_=(\mu) = \emptyset$, $M_>(\mu) = \emptyset$.

Theorem 2. Let $d \geq 3$.

(i) *For any* $\mu > 0$ *and* $K \in M_<(\mu)$ *the operator* $H_\mu(K)$ *has unique eigenvalue* $E_\mu(K)$ *lying above the upper edge of the essential spectum* $\sigma_{ess}(H_\mu(K))$. *Moreover, for* $\mu \in \left((\nu(\vec{\pi}))^{-1}, (\nu(0))^{-1}\right)$, *the inequalities* $\mathrm{E}_{max}(K) < E_\mu(K) < \mathrm{E}_{max}(0)$, $K \in M_<(\mu)$ *hold and for* $\mu > (\nu(0))^{-1}$ *and* $K \in \mathbb{T}^d \setminus \{0\}$ *the relations hold* $\mathrm{E}_{\max}(K) < E_\mu(K) < E_\mu(0)$ *and* $\mathrm{E}_{\max}(0) < E_\mu(0)$.

(ii) *Let* $\mu > 0$ *and* $K \in M_=(\mu)$. *For* $d = 3,4$ *the upper edge* $\mathrm{E}_{\max}(K)$ *of the essential spectrum* $\sigma_{ess}(H_\mu(K))$ *is a virtual level of the operator* $H_\mu(K)$. *For* $d \geq 5$ *the upper edge* $\mathrm{E}_{\max}(K)$ *of the essential spectrum* $\sigma_{ess}(H_\mu(K))$ *is an eigenvalue for the operator* $H_\mu(K)$.

(iii) *For any* $\mu > 0$ *and* $K \in M_>(\mu)$ *the operator* $H_\mu(K)$ *has no eigenvalue lying above the upper edge of the essential spectrum* $\sigma_{ess}(H_\mu(K))$.

REFERENCES

[17] Albeverio S. Lakaev S.N. Makarov K.A. and Muminov Z.I. The threshold effects for the two-particle Hamiltonians on lattices. Commun. Math. Phys.**262**, 91-115 (2006).

[18] Albeverio S. Lakaev S.N. and Muminov Z.I. Schrödinger operators on lattices. The Efimov effect and discrete spectrum asymptotics. Ann. Henri Poincare. **5**, (2004), 743-772.

[19] Bach V. De Siqueira Pedra W, Lakaev S.N. On the discrete spectrum of the two-particle lattice Schrödinger operators. Preprint mp-arc 10-143.

ON THE EIGENVALUES OF THE TWO-CHANNEL MOLECULAR-RESONANCE MODEL

Latipov Sh.M.
Samarkand State university

Let $H = C \oplus L_e^2(\mathrm{T})$ be the Hilbert space consisting of the one-dimensional Hilbert space (complex plane) C and the Hilbert space $L_e^2(\mathrm{T})$ of square-integrable even functions defined on torus T.

Let $E(k),\ k \in \mathrm{T}$ act in C by

$$E(k)f_0 = \varepsilon(k)f_0 = (1-\cos k)f_0, \quad f_0 \in C,$$

and $H_\mu(k),\ k \in \mathrm{T}$ be the discrete Schrodinger operator associated to the Hamiltonian of a system of two identical particles interacting via zero-range pair potential $\mu \geq 0$.

The operator $H_\mu(k),\ k \in \mathrm{T}$ acts in Hilbert space $L_e^2(\mathrm{T})$ by the formula

$$H_\mu(k) = H_0(k) - V_\mu,$$

where

$$(H_0(k)f_1)(q) = \varepsilon_k(q)f_1(q), \quad f_1 \in L_e^2(\mathrm{T})$$

$$\varepsilon_k(q) = \varepsilon(k/2-q) + \varepsilon(k/2+q) = 2(1-\cos k/2\cos q),$$

and $V_\mu,\ \mu \geq 0$ is the integral operator:

$$(V_\mu f_1)(q) = \frac{\mu}{(2\pi)} \int_{\mathrm{T}} f_1(s)ds, \quad f_1 \in L_e^2(\mathrm{T})$$

Let $H_{\gamma\mu}(k), \quad \gamma, \mu \in [0, +\infty), \ k \in \mathrm{T}$, acts in H by

$$H_{\gamma\mu}(k)\begin{pmatrix} f_0 \\ f_1(q) \end{pmatrix} = \begin{pmatrix} E(k) f_0 + C_\gamma^* \\ C_\gamma f_0 + (H_\mu(k) f_1)(q) \end{pmatrix},$$

where, $C_\gamma^* = \gamma(f_1, \alpha_0)_{L_e^2(\mathrm{T})}$ (resp. $C_\gamma = \gamma(f_0, \alpha_0)_C$) is creation (resp. annihilation) operator.

We note that the operator $H_{0\mu}(k), \ k \in \mathrm{T}$ is the direct sum of operators $E(k)$ and $H_\mu(k)$, $k \in \mathrm{T}$ i.e., acts in H by the formula

Consequently, the operator $H_{\gamma\mu}(k), \ k \in \mathrm{T}$ has the essential spectrum $[\varepsilon_{min}(k), \varepsilon_{max}(k)]$ and embedded eigenvalue $\varepsilon(k) \in [\varepsilon_{min}(k), \varepsilon_{max}(k)]$, where

$$\varepsilon_{min}(k) = \min_{q \in \mathrm{T}} \varepsilon_k(q) = 2(1 - \cos\frac{k}{2}), \quad \varepsilon_{max}(k) = \max_{q \in \mathrm{T}} \varepsilon_k(q) = 2(1 + \cos\frac{k}{2}).$$

In accordance to Weil's theorem for the essential spectrum of $H_{\gamma\mu}(k), \ k \in \mathrm{T}$ the following equalities hold:

$$\sigma_{ess}(H_{\gamma\mu}(k)) = \sigma_{ess}(h_\mu(k)) = [\varepsilon_{min}(k), \varepsilon_{max}(k)].$$

Theorem 1. (i) Let $k \in \mathrm{T} \setminus \{0, \pi\}$. a) If $\gamma^2 \le 2\mu \cos\frac{k}{2}\left(1 - \cos\frac{k}{2}\right)$, then the operator $H_{\gamma\mu}(k)$, has a unique eigenvalue $z_{\gamma\varpi}^{(1)}(k)$ below the essential spectrum, and has non eigenvalue above the essential spectrum.

b) If $\gamma^2 > 2\mu \cos\frac{k}{2}\left(1 - \cos\frac{k}{2}\right)$, then the operator $H_{\gamma\mu}(k)$, has two eigenvalues $z_{\gamma\varpi}^{(1)}(k)$, $z_{\gamma\varpi}^{(2)}(k)$, outside the essential spectrum, satisfying the relations

$$z_{\gamma\mu}^{(1)}(k) < \varepsilon_{\min}(k) < \varepsilon_{\max}(k) < z_{\gamma\mu}^{(2)}(k).$$

(ii) Let $k = 0$. a) If $4\mu < \gamma^2$, then the operator $H_{\gamma\mu}(0)$, has two eigenvalues $z_{\gamma\mu}^{(1)}(0) < 0 = \varepsilon_{\min}(0)$, and $z_{\gamma\mu}^{(2)}(0) > 4 = \varepsilon_{\max}(0)$.

b) If $4\mu \ge \gamma^2$, then the operator $H_{\gamma\mu}(0)$, has a unique eigenvalue $z_{\gamma\mu}^{(1)}(0) < 0 = \varepsilon_{\min}(0)$.

(iii) Let $k=\pi$. Then for any $\gamma>0$, $\mu\geq 0$ the operator $H_{\gamma\mu}(\pi)$, has two eigenvalues $z_{\gamma\mu}^{(1)}(\pi)<2=\varepsilon_{\min}(\pi)$, and $z_{\gamma\mu}^{(2)}(\pi)>2=\varepsilon_{\max}(\pi)$.

REFERENCES

[1]. S. Albeverio, S. N.Lakaev, K.A.Makarov, Z.I.Muminov: Comm.Math.Phys. 262(2006), 91--115.

[2].S. Albeverio, S. N.Lakaev, T.H. Rasulov Journal of Statistical Physics, 127(2007), No. 2. 191--220.

REGULARIZATION OF THE CAUCHY PROBLEM FOR THE SYSTEM OF THE MOMENT THEORY ELASTICITY IN E^m

Makhmudov O. I., Niyozov I. E.
Samarkand State University of Uzbekistan,
iqboln@mail.ru

In this paper, we considered the problem of analytical continuation of the solution of the system equations of the moment theory of elasticity in spacious bounded domain from its values and values of its strains on part of the boundary of this domain, i.e., the Cauchy's problem.

System equation of moment theory elasticity is elliptic. Therefore the problem Cauchy for this system is ill-posed. For ill-posed problems, one does not prove the existence theorem: the existence is assumed a priori. Moreover, the solution is assumed to belong to some given subset of the function space, usually a compact one [1]. The uniqueness of the solution follows from the general Holmgren theorem [2]. On establishing uniqueness in the article studio of ill-posed problems, one comes across important questions concerning the derivation of estimates of conditional stability and the construction of regularizing operators.

Our aim is to construct an approximate solution using the Carleman function method.

Let $x=(x_1,...,x_m)$ and $y=(y_1,...,y_m)$ be points of the Euclidean space E^m, D a bounded simply connected domain in E^m, with piecewise-smooth boundary consisting of a piece Σ of the plane $y_m=0$ and a smooth surface S lying in the half-space $y_m>0$.

Suppose that vector function $U(x)=(u_1(x),...,u_m(x),v_1(x),...,v_m(x))=(u(x),v(x))$ satisfied in D the system equations moments theory elasticity [3]:

$$\begin{cases}(\mu+\alpha)\Delta u+(\lambda+\mu-\alpha)graddivu+2\alpha rotv+\rho\omega^2 u=0,\\(\nu+\beta)\Delta v+(\varepsilon+\nu-\beta)graddivv+2\alpha rotu-4\alpha v+\theta\omega^2 v=0,\end{cases} \qquad (1)$$

where $\lambda,\mu,\nu,\beta,\varepsilon,\alpha$ is coefficients which characterizing medium, satisfying the conditions $\mu>0, 3\lambda+2\mu>0, \alpha>0, \varepsilon>0, 3\varepsilon+2\nu>0, \beta>0, \theta>0, \rho>0, \omega\in R^1$.

Then system (1) maybe write in matrix from in the following way:

$$M(\partial_x)U(x)=0 \qquad (2)$$

A solution U of system (1) in the domain D is said to be regular if $U\in C^1(\overline{D})\bigcap C^2(D)$.

STATEMENT OF THE PROBLEM.

Find a regular solution U of system (1) in the domain D using its Cauchy data on the surface S:

$$U(y)=f(y), \qquad T(\partial_y,n(y))U(y)=g(y), \quad y\in S, \qquad (3)$$

where $T(\partial_y,n(y))$ is the stress operator, $n(y)=(n_1(y),\dots,n_m(y))$ is the unit outward normal vector on ∂D at a point y, $f=(f_1,\dots,f_{2m})$, $g=(g_1,\dots,g_{2m})$ are given continuous vector functions on S.

Suppose that, instead of $f(y)$ and $g(y)$, we are given their approximations $f_\delta(y)$ and $g_\delta(y)$ with accuracy δ, $0<\delta<1$ (in the metric of C) which do not necessarily belong to the class of solutions. In this paper, we construct a family of functions $U(x,f_\delta,g_\delta)=U_{\sigma\delta}(x)$ depending on the parameter σ and prove that under certain conditions and a special choice of the parameter $\sigma(\delta)$ the family $U_{\sigma\delta}(x)$ converges in the usual sense to the solution $U(x)$ of problem (1),(3), as $\delta\to 0$[4],[5].

The following formula holds [5],[6],[7],[8]:

$$U(x)=\int_{\partial D}(\Pi(y,x,\sigma)\{T(\partial_y,n)U(y)\}-\{T(\partial_y,n)\Pi(y,x,\sigma)\}^* U(y))ds_y, x\in D,$$

M

where symbol * is denote of operation transposition $\Pi(y,x,\sigma)$ Carleman matrix of problem (1),(3) depending on the two points y,x and positive numerical number parameter σ [6].

Now we write out a result that allows us to calculate $U(x)$ approximately if, instead of $U(y)$ and $T(\partial_y,n)U(y)$, their continuous approximations $f_\delta(y)$ and $g_\delta(y)$ are given on the surface S:

$$\max_S |f(y) - f_\delta(y)| + \max_S |T(\partial_y, n)U(y) - g_\delta(y)| \le \delta,\ 0 < \delta < 1.$$

We define a function $U_{\sigma\delta}(x)$ by setting

$$U_{\sigma\delta}(x) = \int_S [\Pi(y,x,\sigma)g_\delta(y) - \{T(\partial_y, n)\Pi(y,x,\sigma)\}^* f_\delta(y)]ds_y,$$

where

$$\sigma = \frac{1}{x_m^0} ln \frac{M}{\delta}, \quad x_m^0 = \max_D x_m, \quad x_m > 0.$$

Theorem. *Let $U(x)$ be a regular solution of system (1) in D satisfying condition*

$$|U(y)| + |T(\partial y, n)U(y)| \le M,\ y \in \partial D.$$

Then the following estimate is valid:

$$|U(x) - U_{\sigma\delta}(x)| \le C(x)\delta^{\frac{x_m}{x_m^0}} \left(ln \frac{M}{\delta} \right)^m, \quad x \in D.$$

where $C(x) = \int_{\partial D} \frac{ds_y}{r^m}$.

REFERENCES

[1]. M.M.Lavrent'ev. Some Ill-Posed Problems of Mathematical Physics [in Russian], Computer Center of the Siberian Division of the Russian Academy of Sciences, Novosibirck (1962) 92p.

[2]. V.D.Kupradze, T.V.Burchuladze, T.G.Gegeliya, ot.ab. Three-Dimensional Problems of the Mathematical Theory of Elasticity and ... [in Russian], Nauka, Moscow,1976.

[3]. I.G.Petrovskii. Lectures on Partial Differential Equations [in Russians],Fizmatgiz,Moscow,(1961).

[4]. Sh.Ya.Yarmukhamedov. Cauchy problem for the Laplace equation. Dokl.Acad.Nauk SSSR [Soviet Math.Dokl.],235, No.2,p.281-283.(1977).

[5]. O.I. Makhmudov and I.E. Niyozov. Regularization of a solution to the Cauchy Problem

for the System of Thermoelasticity. Contemporary Mathematics.AMS, Primary V382, 2005,74F05, 35Q72.

[6]. O.I. Makhmudov and I.E. Niyozov.Regularization of the Cauchy Problem for the system elasticity theoru in R^m. Preprint ISSN 1437-739X, Institut fur Mat-hematik, Uni Potsdam, 2005/22.

[7]. O.I. Makhmudov ,I.E. Niyozov and N.N.Tarkhanov.The Cauchy Problem of Couple-Stress Elasticity.Universidad Potsdam-Institut fur mathematic .Preprint №3. 2006.

[8].O.I. Makhmudov and I.E. Niyozov.The Cauchy problem for the Lame system in infinite domains in R^m .Journal of inverse and Ill-Posed Problems.V14.N9.2006. pp.905-924(20)

MIXED FRACTIONAL INTEGRATION OPERATORS IN MIXED HÖLDER SPACES

Mamatov T.
Samarkand State University, Mechanics-Mathematics Department
t-mamatov@samdu.uz, tmyu-04@rambler.ru

ABSTRACT

As is known, the Riemann-Liouville for one-dimensional fractional integration operator establishes an isomorphism between Hölder spaces. We study mixed Riemann-Liouville integrals of functions of two variables in mixed Hölder spaces of different orders in each variables. We consider Hölder spaces defined both by first order differences in each variable and also by the mixed second order difference, the main interest being in the evaluation of the latter for the mixed fractional integral in the case where the density of the integral belongs to the Hölder class defined by mixed differences. The obtained results extend the well known theorem of Hardy-Littlewood for one-dimensional fractional integrals to the case of mixed Hölderness.

The mapping properties of the one-dimensional fractional Riemann-Liouville operator

$$\left(I_{a+}^{\alpha} f\right)(x) = \frac{1}{\Gamma(\alpha)} \int_a^x \frac{f(t)dt}{(x-t)^{1-\alpha}}, \quad x > a \tag{1}$$

are well studied both in weighted Hölder spaces or generalized Hölder spaces. A non-weighted statement on action of the fractional integral operator from H_0^{β} into $H_0^{\beta+\alpha}$ is due to Hardy and Littlewood (see [7], Theorems 3.1 and 3.2), and it is known that the operator I_{a+}^{α} with $0<\alpha<1$ establishes an isomorphism between the Hölder spaces $H_0^{\lambda}([a,b])$ and $H_0^{\lambda+\alpha}([a,b])$ of functions vanishing at the point $x=a$, if $\lambda+\alpha<1$. The weighted results with power weights were obtained in [5], [6], see their presentation in [7], Theorems 3.3, 3.4 and 13.13). For weighted generalized Hölder spaces $H_0^{\omega}(\rho)$ of functions φ with a given dominant of continuity modulus of $\rho\varphi$, mapping properties in the case of power weight were studied in [4], [3], [8], see also their presentation in [7], Section 13.6. Different proofs were suggested in [1], [2], where the case of complex fractional orders was also considered , the shortest proof being given in [1].

In the multidimensional case statements on mapping properties in generalized Hölder spaces are known ([9]) for the Riesz fractional integrals

$$\int_{R^n} \frac{\varphi(y)dy}{|x-y|^{n-\alpha}}, \qquad x \in R^n$$

see [7], Theorem 25.5. Mixed Riemann-Liouville fractional integrals of order (α,β)

$$\left(I^{\alpha,\beta}_{a+,c+}\varphi\right)(x,y) = \frac{1}{\Gamma(\alpha)\Gamma(\beta)} \int_0^x\int_0^y \frac{\varphi(t,\tau)dtd\tau}{(x-t)^{1-\alpha}(y-\tau)^{1-\beta}}, \quad x > a,\, y > c,$$

(2)

and the corresponding mixed fractional differentials of order (α,β) in the Marchaud form

$$\left(D^{\alpha,\beta}_{a+,c+}\varphi\right)(x,y) = \frac{\varphi(x,y)}{\Gamma(1-\alpha)\Gamma(1-\beta)(x-a)^{\alpha}(y-c)^{\beta}} +$$

$$+\frac{\alpha\beta}{\Gamma(1-\alpha)\Gamma(1-\beta)} \int_a^x\int_c^y \frac{\varphi(x,y)-\varphi(t,\tau)}{(x-t)^{1+\alpha}(y-\tau)^{1+\beta}}dtd\tau, \quad x > a,\, y > c, \qquad (3)$$

were not studied either in the Hölder spaces defined by mixed differences. Meanwhile, there arise "points of interest" related to the investigation of the above mixed differences of fractional integrals (2) and mixed differences of fractional differentials (3). For operators (1.2) in Hölder spaces of mixed order there arise some questions to be answered in relation to the usage of these or those differences in the definition of Hölder spaces. Such mapping properties in Hölder spaces of mixed order were not studied. This paper is aimed to fill in this gap.

We consider the operators (2) and (3) in the rectangle $Q = \{(x,y):\ a < x < b,\ c < y < d\}$.

For a continuous function $\varphi(x,y)$ on R^2 we introduce the notation

$$\left(\overset{1,0}{\Delta}_h \varphi\right)(x,y) = \varphi(x+h,y)-\varphi(x,y), \quad \left(\overset{0,1}{\Delta}_\eta \varphi\right)(x,y) = \varphi(x,y+\eta)-\varphi(x,y),$$

$$\left(\overset{1,1}{\Delta}_{h,\eta} \varphi\right)(x,y) = \varphi(x+h,y+\eta)-\varphi(x+h,y)-\varphi(x,y+\eta)+\varphi(x,y),$$

so that

$$\varphi(x+h,y+\eta)=\left(\overset{1,1}{\Delta}_{h,\eta}\varphi\right)(x,y)+\left(\overset{1,0}{\Delta}_{h}\varphi\right)(x,y)+\left(\overset{0,1}{\Delta}_{\eta}\varphi\right)(x,y)+\varphi(x,y). \qquad (4)$$

Everywhere in the sequel by C, C_1, C_2 etc we denote positive constants which may different values in different occurences and even in the same line.

Definition 1. We say that $\varphi(x,y)\in\widetilde{H}^{\lambda,\gamma}(Q)$, where $\lambda,\gamma\in(0,1]$, if

$$\left|\left(\overset{1,0}{\Delta}_{h}\varphi\right)(x,y)\right|\le C_1|h|^{\lambda}, \quad \left|\left(\overset{0,1}{\Delta}_{\eta}\varphi\right)(x,y)\right|\le C_2|\eta|^{\gamma}, \quad \left|\left(\overset{1,1}{\Delta}_{h,\eta}\varphi\right)(x,y)\right|\le C_3|h|^{\lambda}|\eta|^{\gamma}. (5)$$

We say that $\varphi(x,y)\in\widetilde{H}_0^{\lambda,\gamma}(Q)$ if $\varphi(x,y)\in\widetilde{H}^{\lambda,\gamma}(Q)$ and $\varphi(a,y)\equiv\varphi(x,c)\equiv 0$.

This space become Banach space under the standard definition of the norm:

$$\|\varphi\|_{\widetilde{H}^{\lambda,\gamma}}=\|\varphi\|_{C(Q)}+\sup_{\substack{x,x+h\in[a,b]\\ y\in[c,d]}}\frac{\left|\left(\overset{1,0}{\Delta}_{h}\varphi\right)(x,y)\right|}{|h|^{\lambda}}+\sup_{\substack{y,y+\eta\in[c,d]\\ x\in[a,b]}}\frac{\left|\left(\overset{0,1}{\Delta}_{\eta}\varphi\right)(x,y)\right|}{|\eta|^{\gamma}}+\sup_{\substack{x,x+h\in[0,b],\\ y,y+\eta\in[0,d]}}\frac{\left|\left(\overset{1,1}{\Delta}_{h,\eta}\varphi\right)(x,y)\right|}{|h|^{\lambda}|\eta|^{\gamma}}.$$

Lemma 1. Let $\varphi(x,y)\in\widetilde{H}^{\lambda,\gamma}(Q), \quad 0\le\lambda,\gamma\le 1, \quad 0<\alpha,\beta<1$. Then for the mixed fractional integral operator (2) the representation

$$\left(I^{\alpha,\beta}_{a+,c+}\varphi\right)(x,y)=\frac{\varphi(a,c)x^{\alpha}y^{\beta}}{\Gamma(1+\alpha)\Gamma(1+\beta)}+\frac{\psi_1(x)y^{\beta}}{\Gamma(1+\beta)}+\frac{x^{\alpha}\psi_2(y)}{\Gamma(1+\alpha)}+\psi(x,y)$$

(6)

holds, where

$$\psi_1(x)=\frac{1}{\Gamma(\alpha)}\int_a^x\frac{\varphi(t,c)-\varphi(a,c)}{(x-t)^{1-\alpha}}dt, \quad \psi_2(y)=\frac{1}{\Gamma(\beta)}\int_c^y\frac{\varphi(a,\tau)-\varphi(a,c)}{(y-\tau)^{1-\beta}}d\tau,$$

$$\psi(x,y)=\frac{1}{\Gamma(\alpha)\Gamma(\beta)}\int_a^x\int_c^y\frac{\left(\Delta^{1,1}_{t,\tau}\varphi\right)(a,c)}{(x-t)^{1-\alpha}(y-\tau)^{1-\beta}}dtd\tau,$$

and

$$|\psi_1(x)|\le C_1x^{\lambda+\alpha}, \quad |\psi_2(y)|\le C_2y^{\gamma+\beta}, \quad |\psi(x,y)|\le C_3x^{\alpha+\lambda}y^{\beta+\gamma}.$$

Lemma 2. If $\varphi(x,y)\in\widetilde{H}^{\lambda,\gamma}(Q), \quad \alpha<\lambda\le 1, \quad \beta<\gamma\le 1$, then

$$\left(D^{\alpha,\beta}_{a+,c+}\varphi\right)(x,y)=\frac{\varphi(a,c)}{\Gamma(1-\alpha)\Gamma(1-\beta)(x-a)^{\alpha}(y-c)^{\beta}}+\frac{\varphi(x,y)-\varphi(a,c)}{\Gamma(1-\alpha)\Gamma(1-\beta)(x-a)^{\alpha}(y-c)^{\beta}}+$$

$$+\frac{\psi_1(x)}{\Gamma(1-\beta)(y-c)^{\beta}}+\frac{\psi_2(y)}{\Gamma(1-\alpha)(x-a)^{\alpha}}+\psi(x,y)=$$

$$=\frac{\varphi(a,c)}{\Gamma(1-\alpha)\Gamma(1-\beta)(x-a)^{\alpha}(y-c)^{\beta}}+\Psi(x,y), \tag{8}$$

where

$$\psi_1(x)=\frac{\alpha}{\Gamma(1-\alpha)}\int_a^x\frac{\varphi(t,c)-\varphi(a,c)}{(x-t)^{1+\alpha}}dt,\quad \psi_2(y)=\frac{\beta}{\Gamma(1-\beta)}\int_c^y\frac{\varphi(a,\tau)-\varphi(a,c)}{(y-\tau)^{1+\beta}}d\tau, \tag{9}$$

$$\psi(x,y)=\frac{\alpha\beta}{\Gamma(1-\alpha)\Gamma(1-\beta)}\int_a^x\int_c^y\frac{\left(\overset{1,1}{\Delta}_{t-a,\tau-c}\,\varphi\right)(a,c)}{(x-t)^{1+\alpha}(y-\tau)^{1+\beta}}dtd\tau, \tag{10}$$

and $\Psi(x,y)\in\tilde{H}^{\lambda-\alpha,\gamma-\beta}(Q)$, $\Psi(x,y)\big|_{x=a,y=c}=0$, and $\|\Psi\|_{\tilde{H}^{\lambda-\alpha,\gamma-\beta}}\le C\|\varphi\|_{\tilde{H}^{\lambda,\gamma}}$.

Theorem 1. Let $0<\lambda,\gamma<1$. Then the mixed fractional integration operator $I^{\alpha,\beta}_{a+,c+}$ isomorphically maps the space $\tilde{H}^{\lambda,\gamma}_0(Q)$ onto the space $\tilde{H}^{\lambda+\alpha,\gamma+\beta}_0(Q)$, if $\lambda+\alpha<1$ and $\gamma+\beta<1$.

REFERENCES

[1]. N.K. Karapetiants and L.D. Shankishvili. A short proof of Hardy-Littlewood-type theorem for fractional integrals in weighted Hölder spaces. *Fract. Calc. Appl. Anal.*, 2(2):177--192, 1999.

[2] N.K. Karapetiants and L.D. Shankishvili. Fractional integro-differentiation of the complex order in generalized Holder spaces $H^{\omega}_0([0,1],\rho)$. *Integral Transforms Spec. Funct.*, 13(3):199--209, 2002.

[3] Kh. M. Murdaev and S.G. Samko. Fractional integro-differentiation in the weighted generalized Hölder spaces $H^{\omega}_0(\rho)$ with the weight $\rho(x)=(x-a)^{\mu}(b-x)^{\nu})$ (Russian) given continuity modulus of continuity (Russian). Deponierted in VINITI, Moscow.

[4] Kh. M. Murdaev and S.G. Samko. Weighted estimates of the modulus of continuity for fractional integrals of function with a given continuity modulus of continuity (Russian). Deponierted in VINITI, Moscow.

[5] B.S. Rubin. Fractional integrals in Hölder spaces with weight, and operators of potential type. *Izv. Akad. Nauk Armjan. SSR Ser. Mat.*, 9(4):308--324, 1974.

[6] B.S. Rubin. Fractional integrals and Riesz potentials with radial density in spaces with power weight. *Izv. Akad. Nauk Armyan. SSR Ser. Mat.*, 21(5):488--503, 511, 1986.

[7] S.G. Samko, A.A. Kilbas, and O.I. Marichev. *Fractional Integrals and Derivatives. Theory and Applications.* London-New-York: Gordon & Breach. Sci. Publ., (Russian edition - *Fractional Integrals and Derivatives and some of their Applications*, Minsk: Nauka i Tekhnika, 1987.), 1993. 1012 pages.

[8] S.G. Samko and Kh.M. Murdaev. Weighted Zygmund estimates for fractional differentiation and integration, and their applications. *Trudy Matem. Inst. Steklov*, 180:197--198, 1987. English transl. in Proc. Steklov Inst. Math., AMS, 1989, issue 3, 233-235. [9] B.G. Vakulov. Potential type operator on a sphere in generalized Hölder classes. *Izv. Vuzov. Matematika*, (11):66--69, 1986. English transl.: Soviet Math. (Iz. VUZ) 30 (1986), no. 11, 90--94.

POINT INTERACTION BETWEEN TWO FERMIONS AND ONE PARTICLE OF A DIFFERENT NATURE ON THE THREE DIMENSIONAL LATTICE

Muminov Z.I.
Samarkand State university
zimuminov@samdu.uz

INTRODUCTION

The spectral theory of continuous and lattice three particle Schrödinger operators in R^3 shows the remarkable phenomenon known as "Efimov effect": if all hamiltonian of all the two-body subsystems are positive and if at least two of them have a zero-energy resonance, then the three-body system has an infinite number of negative eigenvalues accumulating at zero.

Since its discovery by Efimov in 1970, many works are devoted to this subject. See works of D. R. Yafaev, H. Tamura, A. V. Sobolev, Yu. N. Ovchinnikov, I. M. Sigal and see in lattice case the papers of Albeverio, S. N. Lakaev, Z. I. Muminov.

RESULT AND DISCUSSION

Let $\mathbb{T}^d$ be the d-dimensional torus. Denote by $L^2_{as}((\mathbb{T}^3)^2)$ the subspace of antisymmetric resp. symmetric functions of the Hilbert space $L^2((\mathbb{T}^3)^2)$.

In the present paper we consider a Hamiltonian H_γ of a quantum mechanical system on a lattice in $\mathbb{Z}^3$ in which three particles, two of them fermions and third one of a different nature, interact through a zero range potential. We admit a very general form for the "kinetic" part H^0_γ of the hamiltonian, which contains a parameter γ to distinguish the two fermions from the third one of a diiferent nature (in the continuum case this parameter would be the inverse of the mass).

This operator is usually associated with the following self-adjoint (bounded) operator acting on $L^2_{as}((\mathbb{T}^3)^2)$: $H_\gamma = H^0_\gamma - V$, $V = V_1 + V_2$, where H^0_γ is the multip-

lication operator by the function $E(p,q)=\gamma\varepsilon(p+q)+\varepsilon(p)+\varepsilon(q),\ p,q\in\mathbb{T}^3$:
$(H_\gamma^0 f)(p,q)=E(p,q)f(p,q),$

and $V_\alpha, \alpha=1,2,3,$ are *zero-range* interaction operators

$$(V_1 f)(p,q)=\mu\int_{\mathbb{T}^3} f(p,t)dt, \quad (V_2 f)(p,q)=\mu\int_{\mathbb{T}^3} f(t,q)dt, \quad f\in L^2_{as}((\mathbb{T}^3)^2).$$

Here $\gamma,\ \mu$, are positive real numbers and $\varepsilon(\cdot)$ is satisfied

Assumption 13. *The function $\varepsilon(\cdot)$ is conditionally negative definite function and three times differentiable function on $\mathbb{T}^3$ with a unique non-degenerate minimum at the origin.*

We prove that there is a value γ^* of the parameter such that only for $\gamma<\gamma^*$ the Efimov effect (infinite number of bound states if the two-body interaction have a resonance) is absent for the hamiltonian, while it is present for all values of $\gamma>\gamma^*$.

CONCLUSION

A similar effect to Efimov effect (infinitely many eigenvalues below a threshold) in known in Nuclear Physics, and recent years is called also Efimov effect suggesting a relation which is not proved so far.

Nuclear forces are of short range, and a mathematical model for them consists in taking them to be of zero range. In this limit not only there are infinitely many bound states for a three body system, but the spectrum is unbounded below with eigenfunctions which tend to have support in a region whose volume tends to zero ("collapse to a point" effect). This effect was discovered by Thomas and is therefore called "Thomas effect" (fall to the centre effect).

The first theoretical analysis of the Thomas effect was given by G. V. Skorniakov and K. A. Ter-Martirosian. The first rigorous proof was given by Faddeev and Minlos who gave also a precise meaning to "zero range interaction" though the theory of self-adjioint extensions.

There are striking similarities between the Efimov and Thomas effects, aside the occurrence of infinitely many bound states.

ON THE CAUCHY PROBLEM FOR THE HELMHOLTZ EQUATION

Makhmudov K.O.
Department of Mechanics and Mathematics, University of Samarkand,
University Boulevard 15, 703004 Samarkand, Uzbekistan
E-mail address: komil.84@mail.ru

INTRODUCTION

Let D be a bounded domain in $\mathbb{R}^3$ and S a closed smooth surface dividing it into two connected components: D^+ and $D^- = D$ being oriented like the boundary of D^-. In this paper we will consider the Cauchy problem for the equation

$$(\Delta + k^2)u = 0, \qquad (1)$$

where k^2 is an arbitrary real number, Δ is the Laplace operator, $u(x) = (u_1(x), u_2(x), u_3(x))$ is an unknown vector. We consider the following problem.

Problem: Let $u^0(x) = (u_1^0(x), u_2^0(x), u_3^0(x)) \in C^1(S)$, $f^0(x) = (f_1^0(x), f_2^0(x), f_3^0(x)) \in C(S)$ be given vector-functions. If is required to find (if possible) a vector-function $u(x) \in C^2(D^-) \cap C^1(\overline{D^-})$ such that

$$(\Delta + k^2)u = 0 \text{ in } D, \qquad u\big|_S = u^0, \quad \left.\frac{\partial u}{\partial n}\right|_S = f^0$$

Here, $C^s(\Omega)$ $s = 0,1,...,$ stands for the vector space of all n-vector valued function the components of which are *s* times continuously differentiable on a set $\Omega \subset \mathbb{R}^3$.

It is known that the Helmholtz equation is elliptic and it is problem has not more than one solution (see, [1]). Therefore, solvability conditions cannot be described in terms of continuous linear functions (see [2]). However, it is ill-posed, i.e. 1) not for any data there exists a solution; 2) solutions do not depend continuously on the Cauchy data on S (see Example below).

CRITERION FOR SOLVABILITY OF PROBLEM

That the Cauchy Problem for the Helmholtz equation is ill-posed, is demonsrated by a simple example similar to that of Hadamard.

Example: *Let S be a piece of the plane* $\{x_3 = 0\}$ *in* $\mathbb{R}^3$ *and*

$$u(x) = \frac{1}{m^3}\sin mx_1 \cdot \sin mx_2 \cdot sh\sqrt{2m^2 - k^2} \cdot x_3, \quad m > k, \quad m \in N$$

Each $u(x)$ *is easily verified to be a solution of the Helmholtz equation on* $\mathbb{R}^3$*. Moreover,*

$$u(x_1, x_2, 0) = 0, \qquad \frac{\partial u(x_1, x_2, 0)}{\partial x_3} = \frac{\sqrt{2m^2 - k^2}}{m^3} \cdot \sin mx_1 \cdot \sin mx_2$$

However, at each point $x = (x_1, x_2, x_3)$ *with* $x_1 \neq 0,\ x_2 \neq 0,$ *and* $x_3 > 0,$ *we have*

$$\lim_{m \to \infty} u(x) = \infty .$$

We denote by $\Phi(x, y)$ fundamental solution of the Helmholtz equation in $\mathbb{R}^3$:

$$\Phi(x, y) = -\frac{1}{4\pi} \cdot \frac{\exp(ik\,|x-y|)}{|x-y|}, \quad x \neq y .$$

From now on me will assume that D is a domain with a piecewise smooth boundary ∂D, and that the vector-functions u^0, f^0 are summable on *S*. Then we define the Green-type integral as follows:

$$F(x) = \int_S \left(u^0(y) \frac{\partial \Phi(x, y)}{\partial n_y} - f^0(y) \Phi(x, y) \right) dS_y \quad (x \in D \setminus S).$$

It is dear that F is a solution of the Helmholtz equation everymhere outside of *S*; let $F^{\pm} = F\big|_{D^{\pm}}$.

Lemma: *Let* $S \in C^2$*, and let* $u^0 \in C^1(S)$ *and* $f^0 \in C(S)$ *be summable functions on S. Then the integral* F^+ *continuously extends to* $D^+ \cap S$ *together with its first derivatives if and only if the integral* F^- *continuously extends to* $D^- \cap S$ *together with its first derivatives.*

Theorem: *Let* $S \in C^2$*, and let* $u^0 \in C^1(S)$ *and* $f^0 \in C(S)$ *be summable functions on S. Then, for Problem to be solvable, it is necessary and sufficient that the integral* F^+ *extends, as a solution of the Helmholtz equation, from* D^+ *to the domain D.*

REFERENCES

[1] Shlapunov A.A.; Tarkhanov N.N.: Bases with double orthogonality in Cauchy problem for systems with injective symbol. Proc. London Math. Soc. 3(71) (1995). 1 – 52.

[2] Tarkhanov N.N.; Solvability criterion for the ill-posed Cauchy problem for elliptic systems. Doklady Akad. Nauk SSSR 308(1989) 3, 531 – 534; English transl. in Sov. Math. Dokl. 40 (1990) 2, 341 – 345.

ON THE NUMBER OF EIGENVALUES LYING IN THE GAP OF THE CONTINUUM OF THREE-PARTICLE SCHRÖDINGER OPERATORS ON LATTICE

Muminov M. I., Omonov A.
Samarkand State University,
mmuminov@mail.ru, aomonov@mail.ru

We consider a system of three-particles, which move in the three-dimensional integer lattice Z^3 and interact through a pair zero and short-range potentials. The Hamiltonian of the

system acts in $\ell_2((Z^3)^3)$ by form

$$(H\psi)(n_1,n_2,n_3)=H_0\psi(n_1,n_2,n_3)-[\mu_1\delta_{n_1n_2}+\mu_2\delta_{n_2n_3}+\mu_3\hat{v}_3((n_1-n_2)]\psi(n_1,n_2,n_3),$$

where $H_0=\frac{1}{2m_1}\Delta\otimes I\otimes I+\frac{1}{2m_2}I\otimes\Delta\otimes I+\frac{1}{2m_3}I\otimes I\otimes\Delta$, I is identical operator, Δ is the lattice Laplacian, $m_\alpha>0$ is the mass of the particle α, $\hat{v}_3$ is even function and $\hat{v}_3\in\ell_2(Z^3)$, $\mu_\alpha>0$ and δ_{mn} is the Kronecker delta.

Applying the Fourier transform and the decomposition into the direct operator integrals we reduce the investigation of the spectral properties of the operator H to the analysis of the family of self-adjoint, bounded operators (the three-particle discrete Schrödinger operators) $H(K)$, $K\in T^3$, acting in the Hilbert space $L_2((T^3)^2)$ (where T^3 is a three-dimensional torus) by form

$$H(K)=H_0(K)-V,\quad V=\mu_1V_1+\mu_2V_1+\mu_3V_3,$$

$$(V_1f)(p,q)=\frac{1}{(2\pi)^3}\int_{T^3}f(p,s)ds,\qquad (V_2f)(p,q)=\frac{1}{(2\pi)^3}\int_{T^3}f(s,q)ds$$

$$(V_1f)(p,q)=\frac{1}{(2\pi)^{3/2}}\int_{T^3}v_3(p-s)f(s,p+q-s)ds,$$

Where $H_0(K)$ is operator multiplication by function

$$E_K(p,s)=\frac{1}{m_1}\varepsilon(p)+\frac{1}{m_2}\varepsilon(q)+\frac{1}{m_3}\varepsilon(K-p-q),\quad \varepsilon(p)=\sum_{i=1}^{3}(1-\cos p_i),$$

$$v_3(p)=\frac{1}{(2\pi)^{3/2}}\sum_{s\in Z^3}\hat{v}_3e^{i(sp)}.$$

Later on suppose that

$$v_3(p) = \sum_{n=1}^{N} \frac{\cos(np_i)}{n!}\ ,\ p \in T^3,\ N < \infty .$$

In this case $\hat{v}_3$ is even function and belongs to $\ell_2(Z^3)$.

Let

$$H_\alpha(K) = H_0(K) - \mu_\alpha V_\alpha \quad \text{and} \quad \mu_\alpha^0 = \frac{m_\beta + m_\gamma}{m_\beta m_\gamma}\Big[\frac{1}{8\pi^3}\int_{T^3}\frac{ds}{\varepsilon(s)}\Big]^{-1}.$$

For the essential spectrum $\sigma_{ess}(H(K))$ of $H(K)$ the equality [1]

$$\sigma_{ess}(H(K)) = \sigma(H_1(K)) \cup \sigma(H_2(K)) \cup \sigma(H_3(K)) \qquad (1)$$

holds.

Note that [1,2]

$$\sigma(H_0(K)) \subset \sigma(H_\alpha(K)),\ \sigma(H(K)) \cap \sup \sigma(H_0(K)) = \varnothing$$

and [3]

$$\sigma_\alpha(H_0(0)) = [0, \frac{6}{m_1} + \frac{6}{m_2} + \frac{6}{m_3}] \ \text{as}\ \mu_\alpha \le \mu_\alpha^0,\ \alpha = 1,2. \qquad (2)$$

Analyzing as [3] we get

Lemma 1. *There exist positive numbers* μ^* *and* $\tau = \tau(\mu^*)$ *such that for any* $\mu_3 > \mu^*$

$\sigma(H_3(0)) \cap (-\tau, 0) = \varnothing$ *with* $\inf \sigma(H_3(0)) < 0$.

By Lemma 1 and (1), (2) we obtain

Lemma 2. *If* $\mu_3 > \mu^*$ *and* $\mu_\alpha \le \mu_\alpha^0$, $\alpha = 1,2$, *then there exists* $\tau > 0$ *such that* $\sigma(H(0)) \cap (-\tau, 0) = \varnothing$, *i.e. the operator* $H(0)$ *such that has a gap of the essential spectrum.*

Using methods of integral equations and analyzing as [4,3] we obtain

Theorem. *Let* $\mu_3 > \mu^*$ *and* $\mu_\alpha = \mu_\alpha^0$, $\alpha = 1,2$, *then the operator* $H(0)$ *has infinitely many eigenvalues* $z_1 < z_2 < \cdots < z_n < \cdots$ *lying in the gap of the essential spectrum and* $z_n \to 0$ *as* $n \to \infty$.

REFERENCES

[1] Sergio Albeverio , Saidakhmat N. Lakaev and Zahriddin I. Muminov. Schrodinger Operators on Lattices. The Efimov Effect and Discrete Spectrum Asymptotics. Journal Annales Henri Poincare. Volume 5, 2004, 743-772.

[2] M. I. Muminov The infiniteness of the number of eigenvalues in the gap in the essential spectrum for the three-particle Schro?dinger operator on a lattice. Teoret. Mat. Fiz., 159:2 (2009), 299–317.

[3] M. I. Muminov. Formula for the number of eigenvalues of a three-particle Schro"dinger operator on a lattice Teoret. Mat. Fiz., 164:1 (2010), 46–61 4.

[4] Sobolev A. V.: The Efimov effect. Discrete spectrum asymptotics, Commun. Math. Phys. {\bf 156} (1993), 127--168.

DECOMPOSITION OF THE ARBITRARY SUBGROUP OF THE SYLOW'S p - SUBGROUP

Narzullaev U.
Samarkand Branch
of Tashkent University of Information Technologies

This article is devoted to description of the subgroup of the general linear group $GL(Z, Z/p^nZ)$ and decomposition of the arbitrary subgroup of the Sylow's p - subgroup.

The goal of this article is to descripbe of the subgroup $GL(2, Z/p^nZ)$. At first the main lemma is proved. After that the theorem about the decomposition of the arbitrary subgroup of the Sylow's p-subgroup is proved.

Let E be an elliptic curve determined over the field k. Denote by E_{p^n} the group of points of order p^n.

Let K be the field of the points E_{p^n} over k. Let $G = Gal(K/k)$. The group G is a subgroup of the group automorphisms

$$Aut(E_{p^n}) = GL(2, Z/p^nZ).$$

The study of group $GL(2, Z/p^nZ)$ is necessary for an calculation of groups local and trivial cohomologies $H^1_*(G, E_{p^n})$, definition and properties which can be found in [2] and [3]. Results about group $H^1_*(G, E_{p^n})$ can be applied to arithmetics of elliptic curves [4], in particulars, to a problem of divisibility of principal homogeneous spaces [5].

Let's recall that the order of the group $GL(2, Z/p^nZ)$ is equal to $p^{4n-3}(p^2-1)(p-1)$ where p -prime number [1].

Let π be the maximal ideal of the local ring Z/p^nZ. Denote by v the ordinal function, i.e. $v(a)=k$ means that $a\in\pi^k\setminus\pi^{k+1}$ for any rational integer a. Let's consider the set of the matrices of the form

$$\begin{pmatrix}1+\alpha & \beta\\ \gamma & 1+\delta\end{pmatrix}$$

where α,γ and δ belong to the ideal π. Denote this set of the matrices by G_p. Obviuosly that the order of the group G_p is equal to p^{4n-3}. Hence G_p is the Sylow's p-subgroup of the group $GL(2,Z/p^nZ)$.

Let the element σ belong to G_p. Denote by $\alpha(\sigma),\beta(\sigma),\gamma(\sigma),\delta(\sigma)$ the according elements of the arbitrary matrix σ. Denote by $\varepsilon(\sigma)=\alpha(\sigma)-\delta(\sigma)$.

Our first goal will be the investigation of the element $\alpha,\beta,\gamma,\delta$ and ε when σ is raised to the degree. Let's fix the element

$$\sigma=\begin{pmatrix}1+\alpha & \beta\\ \gamma & 1+\delta\end{pmatrix}$$

of the group G_p. Denote by $\alpha_k,\beta_k,\gamma_k,\delta_k$ and ε_k the according coefficients of the matrix σ^k $(k\in N)$, i.e.

$$\sigma^k=\begin{pmatrix}1+\alpha & \beta\\ \gamma & 1+\delta\end{pmatrix}^k=\begin{pmatrix}1+\alpha_k & \beta_k\\ \gamma_k & 1+\delta_k\end{pmatrix}.$$

Let's write the recurrent formulas connecting the coefficients $\alpha_k,\beta_k,\gamma_k,\delta_k$ and ε_k of the matrix σ^k.

Proposition 1. *The following relationships*

$$\alpha_{k+1}=\alpha_k+\alpha+\alpha\alpha_k+\beta\gamma_k=\alpha_k+\alpha+\alpha\alpha_k+\beta_k\gamma,$$

$$\delta_{k+1}=\gamma\beta_k+\delta+\delta_k+\delta\delta_k=\gamma_k\beta+\delta+\delta_k+\delta\delta_k,$$

$$\beta_{k+1}=\beta_k+\alpha\beta_k+\beta+\delta_k\beta=\beta+\beta\alpha_k+\beta_k+\delta\beta_k,$$

$$\gamma_{k+1}=\gamma+\gamma\alpha_k+\gamma_k+\delta\gamma_k=\gamma_k+\gamma_k\alpha+\gamma+\delta_k\gamma,$$

$$\varepsilon=\varepsilon_k+\varepsilon+\alpha\varepsilon_k+\varepsilon\delta_k$$

are true.

Lemma 1. $\gamma_k\equiv k\gamma(mod\ \ \pi^{v(\gamma)+1})$.

Lemma 2. $\alpha_k \equiv (1+\alpha)^k - 1 + \frac{k(k-1)}{2}\beta\gamma (mod \;\; \pi^{v(\beta\gamma)+1}).$

Lemma 3. $\delta_k \equiv (1+\delta)^k - 1 + \frac{k(k-1)}{2}\beta\gamma (mod \;\; \pi^{v(\beta\gamma)+1}).$

Lemma 4. $\beta_k \equiv k\beta + \frac{k(k-1)}{2}\beta(\alpha+\delta) + \frac{k(k-1)(k-2)}{2}\beta\beta\gamma (mod \;\; \pi^{v(\beta\gamma)+s+1}),$

where $s = min(v(\alpha), v(\delta), v(\beta\gamma))$.

Lemma 5. $\varepsilon_k \equiv k\varepsilon + \frac{k(k-1)}{2}\varepsilon(\alpha+\delta)(mod \;\; \pi^{v(\beta\gamma)+s+1}).$

From these lemmas we have following corollaries.

Corollary 1. Let $(k,p)=1$.Then $\beta_k \equiv k\beta(mod \;\; \pi^{v(\beta)+1}).$

Corollary 2. If $p>3$ then $\beta_p \equiv p\beta(mod \;\; \pi^{v(\beta)+2})$. When $p=2$ or $p=3$ the additional restrictions are demanded.

Corollary 3. Let $(k,p)=1$. If $v(\alpha)<v(\beta\gamma)$ then

$$\alpha_k \equiv k\alpha(mod \;\; \pi^{v(\alpha)+1}).$$

Corallary 4. Let $p>2$. If $v(\alpha)<v(\beta\gamma)$ then $\alpha_p \equiv p\alpha(mod \;\; \pi^{v(\alpha)+2})$. The followings statement is true.

Main Lemma. *Let h be one of the elements β, γ and ε; $p>3$; σ and τ be the arbitrary elements of the group $G_p, v(h(\sigma))=k, v(h(\tau))\geq k$. Then there exists $s\in Z$ such that $h(\tau\sigma^s)=0$.*

Remark 1. *In the statement of the Main Lemma instead of $\tau\sigma^s$ we can take $\sigma^s\tau$.*
Remark 2. *For the exceptional values $p=2$ and $p=3$ the statement of the Main Lemma is not true without additional assumption. So under $p=2$ and $n=2$ the matrix*

$$\sigma = \begin{pmatrix} 1+3 & 1 \\ -3 & 1+3 \end{pmatrix}$$

such that

$$\sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

and $v(\beta_p) > v(\beta)+1$. By the analogy under $p=3$ and $n=3$ the matrix

$$\sigma = \begin{pmatrix} 1+2 & 2 \\ 2 & 1 \end{pmatrix}$$

such that

$$\sigma^2 = \begin{pmatrix} 1+4 & 0 \\ 0 & 1+4 \end{pmatrix}$$

and $v(\beta_p) > v(\beta)+1$. *It is clear that under* $p=3$ *the additional conditions can be, for example:* $a) v(\beta) > 0$; $b) v(\beta) > 0$ *and* $\alpha, \gamma, \delta \in \pi^s$.

By analogy the conditions for $p=2$ can be written down. Now Let's prove the theorem about decomposition of the arbitrary subgroup G from G_p. Let $\rho \in G$ such that $k = min\{v(\beta(\rho)), v(\gamma(\rho)), v(\varepsilon(\rho))\}$.

Proposition 2. Within the conjugation we can take $k = v(\beta(\sigma))$ for some elements σ from the group G.

Proof of Proposition 2. Let $v(\gamma(\rho)) = k$ and $v(\beta(\rho)) > k$. Suppose ρ has form

$$\rho = \begin{pmatrix} 1+\alpha & \beta \\ \gamma & 1+\delta \end{pmatrix}$$

Let's take the element $w = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$. Then $\sigma = w\rho w^{-1} = \begin{pmatrix} 1+\delta & \gamma \\ \beta & 1+\alpha \end{pmatrix}$.

Thus it follows that $v(\beta(\sigma)) = k$. If $v(\varepsilon(\rho)) = k$ and $v(\gamma(\rho)) > k$ and $v(\beta(\rho)) > k$ then take the element $w_1 = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}$. Then $\sigma = w_1 \rho w_1^{-1} = \begin{pmatrix} 1+\alpha+\gamma & \beta-\gamma+(\delta-\alpha) \\ \gamma & 1+\delta-\gamma \end{pmatrix}$

thus it follows that $v(\beta(\sigma)) = k$. Thus, we have shown conjugation under that elements ρ turn into σ for that $v(\beta(\sigma)) = k$. Proposition 2 is proved.

Now denote by D the subgroup of the diagonal matrices and by G_D the subgroup of the diagonal matrices of the group G, i.e. $G_D = G \cap D$. Let $G_T = \{\rho \in G \mid \beta(\rho) = 0\}$.

Theorem. *There exist the elements* σ *and* τ *of the group* G *such that*

$$G = G_\sigma G_D G_\tau$$

where $\sigma \in G, \tau \in G_T$ *and* G_σ, G_τ *are the cyclic subgroup generated by elements* σ *and* τ *respectively.*

Proposition 3. *There exist the elements* d *of the group* G_D *such that* $G_D = G_d G_\Lambda$, *where* G_d *is the cyclic subgroup generated by the element* d.

REFERENCES

[1] Kargapolov, M.I. and Merzlyakov, *Yu. I.* Foundation of the group's theory. M., 1982.

[2] Bashmakov, M. I. and Narzullaev, *U.* About one cohomological construction. Rings and matrix group. Ordjonikidze, 1984, p. 11-19.

[3] Serre J.-*P.* Sur les groupes de congruence des varietes abeliennes. Izv. Akad. Nauk SSSR, ser. mat., 1964, **28**, 252 1, 3-20.

[4] Bashmakov M.I.. Cohomologies of abelian varietes over the number field. Uspekhi Mat. Nauk (USSSR), 1972, **27**, 252 6, 25-66.

[5] Bashmakov M.I..About devisibility of the main homogeneous spaces over abelian varietes. Izv. Akad. Nauk SSSR, ser. mat., 1964, **28**, 252 3, 661-664.

SOLVING ONE PROBLEM WITH UNKNOWN BOUNDARY

Nizamova N., Suyarshayev M.
Faculty of Mechanics and Mathematics, Samarkand State University, Uzbekistan
nizamova-n-n@rambler.ru

INTRODUCTION

We discuss the problem about consecutive movement of viscous liquid in rounded cylinder pipe taking into account radial component of speed. Impacts of this component on the movement character of interface boundary are studied. It is important to take into account these impacts in consistent pumping-over, cementing of well, pumping of liquids into well, hydro break etc. It is assumed that at initial moment displaced liquid is in the state of rest or moves under the known law. When pumping of displacing liquid is started it is squeezed in the liquid, which was in the pipe initially, and displaces it. The character of movement changes and becomes two-phased. Due to the fact that the movement of liquid is studied in vertical pipe, it is assumed as symmetric.

RESULT AND DISCUSSION

Navier-Stokes's set of equation of movements and continuity of incompressible viscous liquid in isothermal process at corresponding boundary condition comes to the system of ordinary differential equations in order to identify position of points, located on the surface of liquid boundary. Supposing that variation of longitudinal velocity V_z in the direction alongside the radius takes place significantly faster compared with the velocity in the direction of pipe axis we consider term $\frac{\partial^2 V_z}{\partial z^2}$ as negligible if compared with $\frac{\partial^2 V_z}{\partial r^2}$. Taking into account the fact the velocities are constant during the each period of time, we discard the t derivatives in the equations. Moreover, we assume that the liquids are not mixed and there is no rotary motion. Then the set of equation of movements and continuity of incompressible viscous liquid in isothermal process will look like as following:

$$\frac{\partial V_{ri}}{\partial t}+V_{ri}\frac{\partial V_{ri}}{\partial r}+V_{ri}\frac{\partial V_{ri}}{\partial z}=-\frac{1}{p_i}\frac{\partial P_i}{\partial r}+\gamma_i\left(\frac{\partial^2 V_{ri}}{\partial r^2}+\frac{\partial V_{ri}}{\partial z^2}+\frac{1}{r}\frac{\partial V_{ri}}{\partial r}-\frac{V_{ri}}{r^2}\right);$$

$$\frac{\partial V_{zi}}{\partial t}+V_{ri}\frac{\partial V_{zi}}{\partial r}+V_{zi}\frac{\partial V_{zi}}{\partial z}=F_z-\frac{1}{p_i}\frac{\partial P_i}{\partial z}+\gamma_i\left(\frac{\partial^2 V_{zi}}{\partial r^2}+\frac{1}{r}\frac{\partial V_{zi}}{\partial r}\right);\quad \frac{\partial V_{ri}}{\partial r}+\frac{\partial V_{zi}}{\partial z}+\frac{V_{ri}}{r}=0, \quad (1)$$

where, i=1 – related to the movement of displacing liquid; i=2 – related to the movement of displaced liquid;

In order to solve given problem, non-dimensional quantities, small parameter $\varepsilon = R/l$ - ratio of the radius with the pipe length and stream function are introduced; then the solution of equations' set is checked as a series on ε degrees. Inserting the steam function into the equations' set, taking into account additional conditions, equating coefficients at equal degrees and for identification of x_r, y_r - positions of points, located on the surface of liquid boundary we will have following task:

$$\frac{dx}{d\tau}=a_{11}\left(x_{\tilde{a}}^3-2x_{\tilde{a}}+\frac{1}{\tilde{o}_{\tilde{a}}}\right)+\varepsilon\left[b_{11}\left(x_{\tilde{a}}^3-2\tilde{o}_{\tilde{a}}+\frac{1}{\tilde{o}_{\tilde{a}}}\right)-Ga_i a_{11}^2\left(\frac{x_{\tilde{a}}^7}{36}-\frac{x_{\tilde{a}}^5}{6}+\frac{x_{\tilde{a}}^3}{4}+\frac{x_{\tilde{a}}}{9}\right)\right]; \quad (2)$$

$$\frac{dy_{\tilde{a}}}{d\tau}=-4a_{11}\left(x_{\tilde{a}}^2-1\right)y_{\tilde{a}}+\frac{a_{12}\beta_1\left(x_{\tilde{a}}^2-1\right)}{4}-\frac{x_{\tilde{a}}^2-1}{4}+\varepsilon\left[-4b_{11}\left(x_{\tilde{a}}^2-1\right)y_{\tilde{a}}+\frac{b_{12}\left(x_{\tilde{a}}^2-1\right)}{4}-\right.$$

$$\left.6a_1\left(\frac{2}{9}x_{\tilde{a}}^6-x_{\tilde{a}}^4+x_{\tilde{a}}^2+\frac{2}{9}\right)y_{\tilde{a}}-Ga_i a_{12}\left(\frac{\beta_1}{72}x_{\tilde{a}}^6-\frac{\beta_1}{16}x_{\tilde{a}}^4-\frac{\beta_1}{8}x_{\tilde{a}}^2-\frac{25}{144}\right)+a_{11}\left(\frac{x_{\tilde{a}}^6}{72}-\frac{x_{\tilde{a}}^4}{16}+\frac{x_{\tilde{a}}^2}{8}+\frac{25}{144}\right)\right]$$

where y_0- is a distance from origin of coordinates to the initial position of boundary surface; x_0 - is a radius of points, located on initial boundary of liquid.

CONCLUSION

With the goal of illustration of applied method the calculations are done on computer and the liquid boundaries depending on time are identified using numerical method of Runge-Kutta. If the velocity radial component is not taken into account then the solving of problem is simplified; and the set of equations is replaced by one equation, thereby in the set (2) $\frac{dx_r}{dt}=0$, hence $x_r = const = x_0$. Then x_0 - is defined randomly and changes within limit from 0 to 1 and its values coincide with existent data positions x. Comparison of conducted calculations of equations' set with ignoring velocity radial component and with taking into account velocity radial components shows, that the impact of velocity radial component on time dependency of liquid boundaries is significant.

REFERENCE

[1] Abbasov A.A., Suyarshaev M.M. Solving the task of consecutive movement of liquid, Problems of computational and applied mathematics, Issue №2, Tashkent,1970.

LOCALIZATION OF SPECTRAL EXPANSIONS

Rakhimov A.A.
Institute for Mathematical research
Universiti Putra Malaysia
abdumalik2004@mail.ru

INTRODUCTION

Present paper is an introduction of the localization and summability problems of spectral expansions connected with partial differential operators of mathematic physics and it is given some theorems regarding these problems recently obtained.

Investigation of various vibration processes in real space requires investigation of multiple Fourier series. Specification of multidimensional case is in existing of various methods of summation such series. At the beginning of XX century, it is fixed connection between theory of multiple Fourier series and theory of partial differential equations. More specifically it was fixed that spherical partial sums of multiple trigonometric Fourier series coincides with spectral expansions connected with Laplace operator on torus. This approach became a starting point of the development spectral theory of the differential operators.

Proving of applicableness of the method of separation of variables for the solution of the problems of mathematic physics. Hundred years this part of mathematical science has been developing and being powerful instrument for the solution of the problems.

Classical analysis deals with the smooth or piecewise smooth functions. But many phenomena in nature require for its description either "bad" functions or even they can not be described by regular functions. Therefore, one has to deal with distributions that describe only integral characteristics of phenomena.

Application of the modern methods of mathematical physics in the spaces of distributions, leads to the convergence and sumability problems of spectral expansions of distributions. The convergence and sumability problems of spectral expansions of distributions, associated with partial differential operators, connected with the development of mathematical tools for modern physics.

Especially simple and important example is Fourier series of Dirac's delta function, partial sum of which is well known Dirichlet's kernel. From the classic theory of trigonometric series it is known that Dirichlet's kernel is not uniformly approximation of delta function. So spectral expansions of Dirac's delta function is not convergent in any compact set out of the support of the distribution. But arithmetic means of the partial sum of Fourier series of Dirac's delta function coincides with Fejer's kernel and in one dimensional case it uniformly convergent to zero in any compact set where delta function is equal to zero. In multidimensional case the problem become more complicated.

RESULT / DISCUSSION

One can study convergence and summability problems of spectral expansions of distributions in classical means in the domain where they coincide with regular functions. But singularities of the distribution still will be essential for convergence problems even at regular points as it was mentioned above in case of Delta function.

For spectral expansions one can apply Riesz's method os sumability or other regular methods (for instant Chezaro method). Spectral decompositions of distributions also can be studied in topology of the spaces of distributions.

Development of theory of distributions and their application in mathematical physics required study of spectral expansions of distributions. For instance, in second volume of the book R. Edvadce [13] it is introduced Fourier series of distributions and proved general properties of Fourier-Schwarz series. More specific questions regarding pointwise convergence and summability of these series studies in G.Walter [18]. Limit values of distribution expansions considered in S.Lojasiewicz [16]-[17]. V.M. Gorbachuk and M.L. Gorbachuk [8] studied trigonometric series and generalized periodic functions. M.L Gorbochuk and I. G. Izvekov studied localization principles for the linear methods of summation of Fourier series of generalized functions. A.O. Poteho in [10]-[11] studied some problems of convergence of Fourier series in generalized sense. Francisco J. Vieli in [15] studied localization of the Fourier integral of the distribution with negative smoothness.

Spectral expansions of the distributions with negative smoothness for the first time studied in [1]-[2] by Sh.A. Alimov. In these works, it was obtained sharp conditions of localization in Hilbert spaces of distributions expansions connected with Laplace operator. A new methods developed in these two works by Sh. Alimov gave new stimulus for the more deep research spectral expansions of distributions connected with partial differential operators. In the present work, we consider spectral expansions connected with more general differential operators.

Let $H^{-l}(\Omega), l > 0$ denotes S.L. Sobolev's spaces. Let $E_\lambda^s f(x)$ - Riesz's means of the spectral expansions connected with partial differential operator of an arbitrary order.

Let remind the determination of the problem of localization for $E_\lambda^s f(x)$. Let f infinite differential able in neighborhood of a point x_0 function. What is the influence of smoothness (or non-smoothness) of function f in some other point for the convergence of $E_\lambda^s f(x)$ in small neighborhood of the point x_0.

Main results here is following two theorem (see in [1]- [4]).

Theorem 1. Let $f \in H^{-l}(\Omega) \cap \Sigma'(\Omega)$, $l > 0$. If $s \geq \frac{N-1}{2} + l$, then uniformly on each compact K from $\Omega\backslash\mathrm{supp}\, f$ it is true that

$$\lim_{\lambda \to \infty} E_\lambda^s f(x) = 0.$$

Theorem 2. Let $l > 0$ and x_0 an arbitrary point of the domain If $s < \frac{N-1}{2} + l$, then there exists a distribution $f \in H^{-l}(\Omega) \cap \Sigma'(\Omega)$, such, that f is equal zero in some neighborhood and

Note, that theorem 1.2 characterizes sharpness of the condition $s \geq \frac{N-1}{2} + l$, in theorem 1.1.

CONCLUSION

Necessary and sufficient conditions for regularized series and spectral expansions can be obtained in the Sobolev spaces with negative smoothenss.

REFERENCES

Sh.A Alimov, A.A. Rakhimov. // On the localization spectral expansions in a closed domain. J. Differential equations, V 33, N 1., 1997, pp. 80-82. IF 0.312

Sh.A Alimov, A.A. Rakhimov. // On the localization spectral expansions of distributions. J. Differential equations, V 32, N 6., 1996, pp. 798-802. IF 0.312

Abdumalik Rakhimov, Anvarjon Ahmedov, Hishamuddin Zainuddin, Localization Principle of the Spectral Expansions of Distributions Connected with Schodinger Operator, Australian Journal of Basic and Applied Sciences, 5(5): 1-4, 2011. (ISI).

Rakhimov A.A., Kamran Zakariya and Nazir Alikhan, // Uniform convergence spectral expansions connected with Schrodinger's operator of continuous function in a closed domain. J. The Nucleus, 2010, V.47, No 4, p.261-265.

INVESTIGATIONS OF THE SPECTRUM OF A OPERATOR MATRIX

Rasulov T.H.
Faculty of physics and mathematics, Bukhara State University
rth@mail.ru

Muminov M.I.
Faculty of mathematics and mechanics, Samarkand State University
mmuminov@mail.ru

Hasanov M.
Faculty of arts and sciences, Dogus University
hasanov61@yahoo.com

In the present note a operator matrix A associated to a system describing four particles in interaction, without conservation of the number of particles, is considered. We describe the essential spectrum of A by the spectrum of the channel operators and obtain the Hunziker-van Winter-Zhislin (HWZ) theorem for the operator A.

The well-known methods for the investigation of the location of essential spectra of Schroedinger operators are Weyl criterion for the one particle problem and the HWZ theorem for multiparticle problems, the modern proof of which is based on the Ruelle-Simon partition of unity. The theorem on the location of the essential spectrum of multi-particle Hamiltonians was named the HWZ theorem by Cycon et al (1987) to the honor of Hunziker (1966), van Winter (1964) and Zhislin (1960).

Let T^{ν} be the ν-dimensional torus, the cube $(-\pi,\pi]^{\nu}$ with appropriately identified sides, C be the field of complex numbers, $(T^{\nu})^{n}$, $n=1,2,3$ be the Cartesian nth power of T^{ν}

and $L_2((T^\nu)^n)$, $n=1,2,3$ be the Hilbert space of square-integrable (complex) functions defined on $(T^\nu)^n$, $n=1,2,3$.

Set $H_0 = C,\ H_k = L_2((T^\nu)^k),\ k=1,2,3,\ H^{(n,m)} = \bigoplus\limits_{i=n}^{m} H_i,\ 0 \le n < m \le 3.$

Let us consider a operator matrix A associated to a system describing four particles in interaction, without conservation of the number of particles, acts in the Hilbert space $H^{(0,3)}$ as

$$A = \begin{pmatrix} A_{00} & A_{01} & 0 & 0 \\ A_{10} & A_{11} & A_{12} & 0 \\ 0 & A_{21} & A_{22} & A_{23} \\ 0 & 0 & A_{32} & A_{33} \end{pmatrix},$$

where the components $A_{ij} : H_j \to H_i$, $i,j = 0,1,2,3$ are defined by the rule

$$(A_{00} f_0)_0 = w_0 f_0, \quad (A_{01} f_1)_0 = \int\limits_{T^\nu} v_1(s) f_1(s) ds, \quad (A_{10} f_0)_1(p) = v_1(p) f_0,$$

$$(A_{11} f_1)_1(p) = w_1(p) f_1(p), \quad (A_{12} f_2)_1(p) = \int\limits_{T^\nu} v_2(s) f_2(p,s) ds,$$

$$(A_{21} f_1)_2(p,q) = v_2(q) f_1(p), \quad A_{22} = A_{22}^0 - V_{21} - V_{22},$$

$$(A_{22}^0 f_2)_2(p,q) = w_2(p,q) f_2(p,q), \quad (V_{21} f_2)_2(p,q) = v_{21}(p) \int\limits_{T^\nu} v_{21}(s) f_2(s,q) ds,$$

$$(V_{22} f_2)_2(p,q) = v_{22}(q) \int\limits_{T^\nu} v_{22}(s) f_2(p,s) ds, \quad (A_{23} f_2)_3(p,q,t) = \int\limits_{T^\nu} v_3(s) f_3(p,q,s) ds,$$

$$(A_{32} f_2)_3(p,q,t) = v_3(t) f_2(p,q), \quad (A_{33} f_3)_3(p,q,t) = w_3(p,q,t) f_3(p,q,t).$$

Here $f_i \in H_i, i = 0,1,2,3,$ w_0 is a real number, $v_i(\cdot),\ i=1,2,3,\ v_{2i}(\cdot),\ i=1,2,\ w_1(\cdot)$ are real-valued continuous functions on T^ν and $w_2(\cdot,\cdot)$ resp. $w_3(\cdot,\cdot,\cdot)$ is a real-valued continuous function on $(T^\nu)^2$ resp. $(T^\nu)^3$.

To formulate the main results of the present note we introduce the following channel operators $A_n, n=1,3$ resp. A_2 acting in $H^{(2,3)}$ resp. $H^{(1,3)}$ by the following formula

$$A_1 = \begin{pmatrix} A_{22}^0 - V_{21} & A_{23} \\ A_{32} & A_{33} \end{pmatrix}, \quad A_2 = \begin{pmatrix} A_{11} & A_{12} & 0 \\ A_{21} & A_{22}^0 - V_{21} & A_{23} \\ 0 & A_{32} & A_{33} \end{pmatrix}, \quad A_3 = \begin{pmatrix} A_{22}^0 & A_{23} \\ A_{32} & A_{33} \end{pmatrix}.$$

The essential spectrum of the operator A can be precisely described as well as in the following

Theorem 1. *The essential spectrum $\sigma_{ess}(A)$ of the operator A is the union of spectra of channel operators A_1, A_2 and A_3.*

Let $\sigma(A_n)$, $n=1,2,3$ be the spectrum of the operator A_n, $n=1,2,3$.

The following theorem shows that the least element of the essential spectrum of A belongs to the spectrum of channel operator A_1 or A_2 and it is analogues of HWZ theorem for A.

Theorem 2. *The following equality*

$$\min \sigma_{ess}(A) = \min\{\min \sigma(A_1), \min \sigma(A_2)\}$$

holds.

REFERENCES

Cycon H. L., Froese R., Kirsch W., Simon B. (1987) Schroedinger operators with applications to quantum mechanics and global geometry. Springer, Berlin-Heidelberg-New York, 400 p.

Hunziker W. (1966) On the spectra of Schroedinger multiparticle Hamiltonians. Helv. Phys. Acta. 39, 451-462.

Van Winter C. (1964) Theory of finite systems of particles. I. Mat. Fys. Skr. Danske Vid. Selsk. 1, 1-60.

Zhislin G. M. (1960) Investigations of the spectrum of the Schroedinger operator for a many body system. Trudy Moskov. Mat. Obshch. 9, 81-128.

UNIFORM DISTRIBUTIONS OF LATTICE POINTS

Ruzimuradov H.
Samarkand state university
rxx05@mail.ru

Let be R^n - a space of n - measurable vectors. We will define product of vectors $X=(x_1, x_2, ..., x_n)$ and $Y=(y_1, y_2, ..., y_n)$ as a vector $XY=(x_1y_1, x_2y_2, ..., x_ny_n)$. We will put $NmX=|x_1x_2\cdots x_n|$. It is obvious that $Nm(XY)=Nm(X)Nm(Y)$. The space R^n with the multiplication of vectors defined above becomes a commutative ring with unit $1=(1,...,1)$. The zero divider in this ring satisfies to a condition $NmX=0$. For $T=(t_1,t_2,...,t_n)$ with $NmT\neq 0$ we will define $T^{-1}=(t_1^{-1},t_2^{-1},...,t_n^{-1})$.

Let now $M\subset R^n$ be a compact set, $V(M)$- its volume, $M+X$ - shift M on a vector $X\in R^n$, TM - a set, turning out multiplication of each point of a set M to a vector $T\in R^n$, $NmT\neq 0$. Thus $V(TM)=|NmT|V(M)$. We will notice that multiplication M to a vector T can be considered as non-homogeneous stretching of a set M, accordingly in t_j - time along j-th axis of co-ordinates $(1 \leq j \leq n)$. A homogeneous stretching $T = (t, t, ..., t)$ body M in t- time we will designate through tM, $t > 0$.

Let $K^n = [-1/2; 1/2]^n$ be an unit cube in R^n with the centre in the origin and edges parallel to axes of co-ordinates, then

$$TK^n = \prod_{j=1}^{n} \left[-\frac{|t_j|}{2}; \frac{|t_j|}{2} \right] \tag{1}$$

is parallelepiped with the centre at the origin, edges parallel to axes of co-ordinates and volume is equal to $|NmT|$.

Let now $L \subset R^n$ be a lattice with a determinant $d(L)$; $F(L) = R^n / L$ - the fundamental domain of the lattice L, thus $V(F(L)) = d(L)$. $L + X$ - lattice shift on a vector $X \in R^n$, TL - a lattice, turning out multiplication of each point of a lattice L to a vector T, $NmT \neq 0$. Thus $d(TL) = |NmT| d(L)$.

For any discrete set $D \subset R^n$ we will put

$$N(M, D) = Card\{M \cup D\} \tag{2}$$

In particular, $N(M, L)$ is equal to the number of points of lattice L lying in set M, thus

$$N(M + X, L) = N(M, L - X), \quad N(TM, L) = N(TM, T^{-1}L) \tag{3}$$

These equalities are proved as equality of two sets.

Let's put by definition

$$N(M, L) = \frac{V(M)}{d(L)} + R(M, L) \tag{4}$$

It is the formula of calculation of the number of points of lattice L lying in set the M. The remainder $R(M + X, L)$ in the formula (4) is periodic function of X with a lattice L of the periods.

Consider the following quantity

$$r(M, L) = \sup_{x \in F(L)} | R(M + X, L). \tag{5}$$

We are interested of estimation of the quantity (5). In paper [6] the following result is obtained:

Theorem 1. *Let K^2 be an unit square in R^2 with the centre at the origin and edges parallel to axes of co-ordinates, $L \subset R^2$ be unimodular lattice with $\mu > 0$,, i.e. $L \subset R^2$*

is an admissible lattice. Then there is an infinite sequence of positive numbers $v_1, v_2, ..., v_r, ...$ *with a condition* $2 \le \frac{v_{r+1}}{v_r} \le c$ *(here –c is some constant) for a polygon* TK^2, *where* $T = (u, v_r)$, $u > \mu$ *and v any number, the estimate*

$$r(TK^2, L) \le \frac{10\sqrt{2}}{\mu^2} t^2$$

holds.

Let - L be an admissible lattice in R^n, K^n - unit cube in R^n with the centre at the origin, edges parallel to axes of co-ordinates.

$$T = (t_1, t_2, ..., t_n) \in R^n, \ NmT \ne 0.$$

Then TK^n is a non-homogeneous stretching of cube K^n accordingly in t_j - time along axis of co-ordinates. Clearly that TK^n is a parallelepiped with the centre at the origin and edges parallel to axes of co-ordinates and its volume is equal to $|NmT| = |t_1 t_2 ... t_n|$.

Let's introduce into consideration following limited sets in K^n:

$$M_{T,Z}(L) = T^{-1}(L - Z) \cap K^n \qquad (6)$$

i.e., the lattice L moves on a vector $-Z$, the shifted lattice $L - Z$ is multiplied by vector $T^{-1} = (t_1^{-1}, t_2^{-1}, ..., t_n^{-1})$ and the points of lattice $T^{-1}(L - Z)$ got in cub K^n are considered.

For quantity of points of set $M_{T,Z}(L)$ following equalities hold:

$$N(M_{T,Z}(L)) = N(K^n, T^{-1}(L - Z)) = N(TK^n + Z, L). \qquad (7)$$

We are interested uniform distribution of points $X \in M_{T,Z}(L)$ at $NmT \to \infty$. As a uniform measure the so-called discrepancy serves. Let $A_S \subset K^n$ is a set consisting of $s > 1$ points. We will put

$$\Delta(A_s) = \sup |g(A_s, X)| \qquad (8)$$

where

$$g(A_s, X) = N(XK^n, A_s) - sNmX, \ \ X \in R^n, \ NmX \ne 0 \qquad (9)$$

The quantity $\Delta(A_s)$ is called as an extreme discrepancy. If $\Delta(A_s)/s \to 0$ at $s \to \infty$, the points of set A_s will in regular intervals fill the unit cube. Thus about sets $A_s + Z^n$ periodically continued on R^n by means of a lattice Z^n speak as about in regular intervals distributed on $\mathrm{mod} 1$.

The main problem of the theory of uniform distributions consists in construction of sets with the least extreme discrepancy.

Theorem 2. *At all* $T \in R^n$ *with* $NmT \neq 0$ *the estimate*

$$\Delta(M_{T,Z}(L)) < 2r(TK^N + Z, L) \qquad (10)$$

holds.

Corollary 2.1. *For a class of polygons* $TK^2 + Z$ *defined by the theorem 1 the estimate*

$$\Delta(M_{T,Z}(L)) < O(1)$$

holds.

The theorem 2 shows that the points of the set $M_{T,Z}(L)$ at big enough $|NmT|$ rather in regular intervals fill unit cubic K^n.This circumstance allows to use points of a lattice L as points of integration for the approximated calculation of n-dimensional integrals. So the estimate (10) in a combination to inequalities Koksma-Glavka boards (see [5], chapter 2.5) directly conducts to following result.

Corollary 2.2. *Let* $f(x)$ *be the function defined in square* K^2 *with the limited variation* $V(F)$ *(in Hard and Krauze's sense). Then at* $|NmX| \to \infty$ *the estimate*

$$\left| \int_{K^2} f(x)dx - \sum_{\omega \in M_{T,z}(L)} f(\omega) \right| \leq C_L V(f) \frac{\ln(N(TK^n + Z, L))}{N(TK^n + Z, L)}.$$

holds.

REFERENCES

[1] Skriganov, M. On integer points in polygons//Prepublication de l'institut Fourier. No 204, 1992, 10 p.

[2] Kratzel, E. Lattice points//VEB Deutscher Verlag der Wissenschaften//Berlin. 1988, 320 p.

[3] Ruzimuradov H.H. Fundamental rectangles of applicable nets. Analytical number theory RAN, 1993, v.204. p.83-89.

[4] L.A. Moshchevitina. On Integer Points in Polygons. Mat.zametki, Vol. 65. No 4. pp 629-632. April. 1999.

[5] Keypers L. Niderreytel K., Uniform distributions by mod1. Moscow, 1978.

[6] Ruzimuradov H.H. On the circulation problem of numbers of points of nets in rectangles. Uzb Math J. № 4, 2008,pp. 116-124.

ESTIMATES FOR OSCILLATORY INTEGRALS WITH SOME MODEL PHASES

Safarov A.R.
Samarkand state university
safarov-akbar@mail.ru

ABSTRACT

We consider uniform estimates for oscillatory integrals with model phases in the present note. We obtain generalization of V.N.Chubarikov's theorem and estimates for Fourer transform of measures, concentrated on some hypersurfaces of special form.

Many problems in harmonic analysis, mathematical physics and analytic numbers theory are reduced to the investigation of oscillatory integrals [2].

In article [1] by V.N.Chubarikov estimates for oscillatory integrals with polynomial phases are obtained. In that article [1] the trigonometric integrals are considered with special amplitude function $a=\chi_{[0,1]^r}(x)$. We consider analogical oscillatory integrals with smooth amplitude functions and with some model phases.

STATEMENT OF RESULTS

Let

$$J(\lambda,\Phi,a)=\int_{R^r} e^{i\lambda\Phi(x,s)}a(x)dx, \qquad (1)$$

where Φ is the phase, the amplitude function $a(x)$ is a smooth function from the space $C_0^\infty(U)$ and U is a neighborhood of the point $0\subset R^r$ and λ is a large real parameter. By $C_0^\infty(U)$ we mean a class of smooth functions concentrated in U.

Let $\Phi(x,s)$ be a phase function defined by

$$\Phi(x,s)=\sum_{\substack{0\le t_1,t_2,\dots,t_r\le M\\ t_i\ne t_j,\ i\ne j}} s_{t_1t_2\dots t_r}x_1^{t_1}x_2^{t_2}\dots x_r^{t_r}, \qquad (2)$$

where $s_{00\dots0}=0$ and $\{s_{t_1t_2\dots t_r}\}$- are real numbers.

Theorem 1. Let U be a bounded neighborhood of the origin. Assume the phase function of the oscillatory integral (1) has the form (2).

Then the following estimate

$$|J|\le\frac{c(\ln|\lambda s|)^{r-1}}{|\lambda s|^{1/M}}$$

holds, where $|s|=\sum\limits_{\substack{0\le t_1,t_2,\dots,t_r\le M\\ t_i\ne t_j}}|s_{t_1t_2\dots t_r}|$.

This theorem proved by V.N.Chubarikov in the note [1] for the case $a=\chi_{[0,1]^r}(x)$.

Assume $\{n_1, n_2, ..., n_r\}$ are positive integer numbers. Denote by M the number defined by $M = \max\{n_1, n_2, ..., n_r\}$ and also $l = \#\{j : n_j = M \geq 2\}$, where $\# A$ is a cardinality of the finite set A. Consider the phase function

$$\Phi(x,s) = \sum_{\substack{0 \leq t_j \leq n_j \\ 1 \leq j \leq r}} s_{t_1 t_2 ... t_r} x_1^{t_1} x_2^{t_2} ... x_r^{t_r} . \tag{3}$$

Theorem 2. Assume the phase function of the oscillatory integral (1) has the form (3). Then there exists a neighborhood U of the origin such that for any smooth amplitude function $a \in C_0^\infty(U)$ the following estimate

$$|J| \leq \frac{c(\ln|\lambda s|)^{l-1} \|a\|_{C^1}}{|\lambda s|^{1/M}}$$

holds, where $|s| = \sum_{\substack{0 \leq t_j \leq n_j \\ 1 \leq j \leq r}} |s_{t_1 t_2 ... t_r}|$.

Corollary 1. Assume

$$\Phi(x,s) = s_{M t_2 ... t_n} x_1^M x_2^{t_2} ... x_r^{t_r} + \sum_{\substack{0 \leq t_1, t_2, ..., t_r \leq M-1 \\ t_i \neq t_j}} s_{t_1 t_2 ... t_r} x_1^{t_1} x_2^{t_2} ... x_r^{t_r} .$$

Then the following estimate

$$|J| \leq \frac{c}{\lambda^{1/M} \left(\sum |s|\right)^{1/M}}$$

holds, where $\sum |s| = |s_{M t_2 ... t_{rr}}| + \sum_{\substack{0 \leq t_1, t_2, ..., t_r \leq M-1 \\ t_i \neq t_j}} |s_{t_1 t_2 ... t_r}|$.

Now we consider the phase function of the form:

$$\Phi(x,s) = s_0 x_1^{n_1} x_2^{n_2} ... x_r^{n_r} b(x_1, ..., x_r, s) + s_1 x_1 + s_2 x_2 + ... + s_r x_r , \tag{4}$$

where $\{n_1, n_2, ..., n_r\}$ are non-negative integer numbers, $b(x_1, ..., x_r, s)$ is a smooth function with $b(0,0,...,0) = 1$.

Theorem 3. Assume the phase function of the oscillatory integral (1) has the form (4). Then there exists a neighborhood $U \times V \subset R^r \times R^{r+1}$ of the origin such that for any smooth amplitude function $a \in C_0^\infty(U \times V)$ the following estimate

$$|J| \leq \frac{c\|a\|_{C^2} (\ln|\lambda s|)^{l-1}}{|\lambda s|^{1/M}}$$

holds, where $|s| = \sum_{i=0}^{r} |s_i|$ and $\|a\|_{C^2} = \max_{\substack{|\alpha| \leq 2 \\ x \in U}} |D^\alpha a(x,s)|$.

Suppose $S \subset R^{r+1}$ is a smooth hypersurface given as the graph of the function

$$x_{r+1} = x_1^{n_1} x_2^{n_2} \dots x_r^{n_r} b(x), \tag{5}$$

where $b(x)$ is a smooth function with $b(0,0,\dots,0) = 1$, $M = \max\{n_1, n_2, \dots, n_r\}$ and $l = \#\{j : n_j = M \geq 2\}$.

Consider the smooth Borel measure given by

$$d\mu = \psi \, ds ,$$

where $\psi \in C_0^\infty(R^{r+1})$ and ds is an induced Lebesgue measure on the surface S.

We denote by $d\hat{\mu}(\xi)$ the Fourier transform of the measure $d\mu$.

Corollary 2. Suppose S is the smooth hypersurface given as the graph (5). Then there exists a neighborhood $U \in R^{r+1}$ of the origin such that for any $\psi \in C_0^\infty(U)$ the following estimate

$$|d\hat{\mu}(\xi)| \leq \frac{\|\psi\|_{C^2} \left(\log(|\xi| + 2)\right)^{l-1}}{(|\xi| + 2)^{1/M}}$$

holds.

REFERENCES

[1] V.N.Chubarikov Math. Notes, 20 (1976), no. 1, 61-68.

[2] Stein E.M. Harmonic analysis , Princeton Univ. press Princeton , 1993.

ON THE CONTINATION OF THE SOLUTIONS OF A GENERALIZED CAUCHY-RIEMANN SYSTEM IN R^n

Sattorov E., Ermamatova M.
Samarkand State University
Sattorov-e@rambler.ru

INTRODUCTION

The problem of describing the function given on a part of the boundary of a domain which can be analytic continued into the domain has been thoroughly studied. The first result was obtained by V.A. Fock and F.M.Kyni (Fock, 1959) in the one-dimensional case. A generalization of the theorem of Fock-Kyni was obtained in a series of the papers for holomorphic function in several variables (Aizenberg, 1991). The problem of continuation of function given on a part of the boundary of a domain to this domain as a solution of the Helmholtz equation, a equation theory elasttisity, i.e, as harmonic function has been considered in (Yarmukhamedov, 1977, 1997, 2004).

RESULT AND DISCUSSION

In this paper, we consider the problem of describing the vector-function given on a part of the boundary of a n–dimensional domain can be continued to this domain as a solution of the generalized Cauchy-Riemann system of equations. Our investigation is based on the analog of the generalized Cauchy integral formula for the generalized system of Cauchy-Riemann and a jimp formula for the limiting values of the generalized Cauchy – type integral (Obolashvili, 1974). Continuation for the solution of the generalized Cauchy-Riemann system equations to the domain by its values on a part of the boundary is based on constructing the Carleman matrix for the generalized system of Cauchy-Riemann equation. The notation of Carleman finction was introduced by M.M.Lavrent'ev (Lavrent'ev, 1962). By using the continuation formula we found necessary and sufficient for the extendibility of functions given an a part of a boundary to the domain as a solution of the generalized Cauchy-Riemann system. We prove the Fock-Kuni theorem for this one.

REFERENCE

[1] Fock V.F., Kuni F.V. Dokl. Acad. Nauk SSSR 127, №6, p.1195-1198 (1959)

[2] Aizenberg L.A, Kytmanov A.M. Mat. Sb. №4, p.490-507 (1991)

[3] Yarmukhamedov Sh. Dokl. Acad. Nauk SSSR 235, №2, p.281-283 (1977)

[4] Yarmukhamedov Sh. Dokl. Acad. Nauk SSSR 357, №3, p.320-323 (1997)

[5] Yarmukhamedov Sh. Sibirsk. Math. Zh. 45, №3, p.702-719 (2004)

[6] Obolashvili E.I. Note Sic. Acad. GSSR 73, №1 (1974)

[7] Lavrent'ev M.M. Some ill-posed problems of mathematical physics, Computer Center of the Sibirian Divison of the Russion Academy of Sciences, Novosibirsk, 92 p. (1962)

GENERAL APPROACH TO THE POWER GEOMETRY

Soleev A.
Samarkand State University, Samarkand, Uzbekistan
asoleev@yandex.ru

The aim of this lecture is to explain basic ideas; general statements of the Power Geometry are in [1,2,3,4] (in the case d=2). Power Geometry is a new calculus developing the differential calculus and aimed at the nonlinear problems. The algorithms of Power Geometry are based on the study of nonlinear problems not in the original coordinates, but in the logarithms of these coordinates. Then to many properties and relations, which are nonlinear in the original coordinates, some linear relations can be put in correspondence. They allow to simplify equations, to resolve their singularities,to isolate their first approximations, and to find either their solutions or the asymptotic of the solutions. Algorithms of Power Geometry are applicable to equations of various types: algebraic, ordinary differential and partial differential, and also to system of such equations. There are many nonlinear problems which may be solved by these algorithms (and by them only). The effectiveness of the algorithms was demonstrated on some complicated problems from various fields of science (Robotics, Celestial Mechanics, Hydrodynamics, Thermodynamics). Power Geometry is based upon the three concepts: the Newton polyhedron, the power transformation and the logarithmic transformation.

Let in the plane $\mathbb{R}^2$ with Cartesian coordinates q_1, q_2 we have a finite set D of points $Q_j = (q_{1j}, q_{2j}),\ j = 1, \dots, m$. Let $\overline{\mathbb{R}}^2$ denote the dual plane such that for $P = (p_1, p_2) \in \overline{\mathbb{R}}^2$ and $Q_j = (q_{1j}, q_{2j}) \in \mathbb{R}^2$ there is the scalar product $\langle P, Q \rangle = p_1 q_1 + p_2 q_2$. Let Dp denote such subset of D on which $\langle P, Q \rangle$ has the maximal value c, that is

$$\langle P, Q_j \rangle = c \ \text{ for } \ Q_j \in D_P \ \text{ and}$$

$$\langle P, Q_j \rangle < c \ \text{ for } \ Q_j \in D \setminus D_P. \tag{1}$$

Problem 1. For the given set D and for each $P \neq 0$, find the corresponding subset Dp.

The problem can be solved as follows. Let M denote the convex hull of the set D. The boundary ∂M of the polygon M consists of vertexes $\Gamma_k^{(0)}$ and edges $\Gamma_k^{(1)}$. In the dual plane $\overline{\mathbb{R}}^2$ to each *face* $\Gamma_k^{(d)}$ there corresponds its own *normal cone* $U_k^{(d)}$. The $U_k^{(1)}$ is a ray orthogonal to the edge $\Gamma_k^{(1)}$ and $U_k^{(0)}$ is a sector bounded by two rays $U_k^{(1)}$ and $U_{k+1}^{(1)}$. For $D = \{(3,0), (0,3), (1,1), (1,2)\}$ the polygon M and normal cones $U_k^{(d)}$ are shown in Figs. 1 and 2 correspnding. We denote $D_k^{(d)} = \Gamma_k^{(d)} \cap D$. If $P \in U_j^{(d)}$ then $D_P = D_j^{(d)}$. See detailes in [3]. **2.** Let us consider the polynomial

$$f(x_1, x_2) = \sum_{j=1}^{m} f_j X^{Q_j},$$

where $X^Q = x_1^{q_1} x_2^{q_2}$ and $D = \{Q_1, \dots, Q_m\}$. Let $D_k^{(d)}$ be a boudary subset of the set D, as describted above. The sum

$$\hat{f}_k^{(d)}(X) = \sum f_j X^{Q_j} \ \text{ for } Q_j \in D_k^{(d)} \tag{2}$$

is called the *truncation* of $f(X)$ (or its first approximation).

Now we consider a curve of the form

$$x_i = b_i \tau^{p_i} (1 + o(1)), \ \ i = 1, 2 \tag{3}$$

where $\tau \to \infty$. On such a curve a monomial

$$X^Q = B^Q \tau^{\langle P, Q \rangle} (1 + o(1))$$

where $B = (b_1, b_2)$. If $P \in U_k^{(d)}$, then on a curve (3)

$$f(X) = \hat{f}_k^{(d)}(B)\tau^c + \tau^c o(1),$$

where c is from (1). That is, the truncation (2) is the first approximation of the polynomial $f(X)$ on curves (3) with $P \in U_k^{(d)}$.

If a curve (3) is a solution of the equatiuon $f(X) = 0$ then $\hat{f}_k^{(d)}(B) = 0$. That is, the first approximation $x_i = b_i \tau^{P_i}$, $i = 1,2$ of the solution (3) is a solution of the corresponding truncated equation $\hat{f}_k^{(d)}(X) = 0$. This approach allows to find solutions of the equation $f = 0$ with any degree of accuracity (see [1]).

Now we consider a differential polynomial

$$g(x_1, x_2, \ldots, x_n) = \sum g_R X^R \tag{4}$$

where $X^R = x_1^{r_1} \ldots x_n^{r_n}$ and

$$x_2 = y,\ x_{2+l} = d^l y / dx_1^l,\ l = 1, \ldots, n-2. \tag{5}$$

To each monomial X^R we put in correspondence the point $Q = (q_1, q_2) = Q(R)$ of the plane IR^2 with

$$q_1 = r_1 - \sum_{l=1}^{n-2} l r_{2+l},\ q_2 = r_2 + \ldots + r_n. \tag{6}$$

After substitution

$$x_1 = \tau^{p_1},\ y = b\tau^{p_2} \tag{7}$$

we have $X^R = const\, \tau^{\langle P,Q \rangle}$. Now the set $D = D(g)$ is the set of all $Q(R)$ with $g_R \neq 0$. Using boundary subsets $D_j^{(d)}$ of the D we define *truncations*

$$\hat{g}_k^{(d)}(X) = \sum g_R X^R \ \ for\ \ R : Q(R) \subset D_k^{(d)}$$

of the differential polynomial (4).

Theorem 1. *Let the differential equation* $g(X) = 0$ *have a solution of the form*

$$x_1 = \tau^{p_1},\ y = \tau^{p_2}(b + \sum_{i=1}^{\infty} b_i \tau^{s_i}) \tag{8}$$

where $\tau \to \infty$ and $0 > s_i > s_{i+1}$. If $P \in U_k^{(d)}$ then the first approximation (7) of the solution (8) is a solution of the corresponding truncated equation $\hat{f}_k^{(d)}(X) = 0$.

REFERENCES

[1] A.D. Bruno: Power Geometry in Algebraik and Differential Equations, Elsevier, 2000.

[2] A.D. Bruno and A. Soleev: Local uniformization of branches of a space curve, and Newton polyhedra. St. Peterburg Math. J. 3:1 (1992) 53-82.

[3] A.D. Bruno and A. Soleev: First approximations of algebraic equations. Doklady Akad. Nauk 335:3 (1994) 277-278 (Russian) = Russian Ac. Sci. Doklady. Mathem. 48:3 (1994) (English).

[4] A.D. Bruno: First approximations of differential equations, Ibid. 335:4 (1994) 413-416 (Russian) = Ibid. 48:4 (1994) (English).

[5] A.Soleev and A.Baratov. The Method of Newton Polyhedron and Special Positions of some Mechanisms. Proceedings of The Fourths International Workshop On Computer Algebra in Scientific Computing (CASC-2001), Konstanz (Germany) 22-26 September 2001, P.491-498.

[6] A.Soleev. Newton Polyhedra for Investigation of Complex Bifurcations of Periodic Solutions in some System of ODE. Proceedings of The Fifths International Workshop on Computer Algebra in Scientific Computing (CASC-2002), Germany, Muneck, 22-26 September, 2002, p.123-131.

AUTOMATION OF OUTPUT OF MATHEMATICAL MODEL OF MOVEMENT OF A CAR AND DEFORMATION OF ITS TIRES

Turaev Kh., Mamatkobilov A., Urunbayev E.
Samarkan state university, Samarkand,Uzbekistan
turayev_x@rambler.ru

Let us consider movement of the car at the following conditions: the basket of the car is a solid body fixed on four linear springs with linear dampers; the weight center of the front dependent suspension with manual wheels is situated at the equal distance from all the wheels; the wheels are dynamically balanced, their middle platitude are always parallel; the axis of pivot pins is placed at the angle to vertical in longitudinal and transversal platitudes; the axis of the front suspension, connected to the basket, can move around the longitudinal axis of the car; the front wheels are tightly connected with each other and can be turned around kingpins simultaneously at the angle of ϑ_1 и ϑ_2; back wheels are also tightly connected with each other, transversely tilted, and the angel of breakdown of the wheels can be same.

Let us enter the following indication : (x,y,z) – coordination of the car's weight center; θ is the angle of the wheel's turn around vertical axis; ϑ_1 и ϑ_2 are angles of front wheel's turn around kingpins. Positive direction of countdown for angles θ, ϑ_1 и $\vartheta_{2\,\mathrm{I}}$ is the wheel's turn to the right; ψ is the angle of the turn of the axis of the front suspension together with the wheels around transverse axis of the car. Positive value of ψ angle is the rise of the left wheel; γ_0 is the angle of the transverse decline of the kingpin. It is considered to be positive if the upper end of the kingpin is turned inward the gauge line; β_0 angle of transverse decline of the back wheels. It is considered to be positive if the upper part of the wheel is turned inwardly; m is the car's weight; m_1 – weight of the car' back part without wheels and front suspension; m_{2i} – weight of the i wheel; m_3– weight of the front suspension of the car; l – distance from weight center of the front suspension to the kingpin's center; l_3 – distance from kingpin's center to the wheel's center; l_1 - distance from the car's weight center to its from axis; l_2 – distance from the car's weight center to its back axis; $L=l_1+l_2$ – the car's base; $L_1=l+l_3$ – distance from weight center of the front suspension to the car's wheel center; A_i–moment of inertia of i-wheel with a hub and brake drum relatively to its diameter; B – moment of inertia of the front suspension relatively to its axis perpendicular to it n lying through the weight center (central moment of inertia of the front suspension); Ci-axis moment of inertia of i-wheel; D – moment of inertia of the car's back part without front suspension and wheels relatively the axis that is lying thorough the weight center; J_1– moment of car's inertia without front wheels relatively vertical axis lying through the car's weight center; J_2 – moment of inertia of the front wheels relatively the kingpin's axis; J_3– moment of inertia of the front suspension with wheels relatively the car's longitudinal axis; K_2 –angular stiffness of the front suspension according to ψ coordinate; K_1–angular stiffness of steering control system; h_{2i} – coefficient of viscous friction according to ψ coordinate; h_{1i}-coefficient of viscous friction in steering control; $K_2^{/}$–angular stiffness of axis device; C_{pc} –radial stiffness of a tire; L_{pc}–distance from the suspension's weight center to spring; $h_{ш}$–inner resistance of a tire; h_a –inner resistance of shock absorber h_{pc} -inner resistance of springs; h_c-coefficient of viscous friction of axis device; α_i – kinematical parameter of i-tire; β_i coefficient of tire's elasticity; γ_i -coefficient of angular stiffness of side i-tire; a_i -coefficient of tire's elasticity related to side deformation of i-tire; ρ_i coefficient of i-tire's elasticity related to angle of the wheels tilt; r_i-radius of i-wheel's rolling; ξ_I -side deformation of i-tire; φ_i-angular deformation of i-tire; χ_i-is angle of tilt of i-wheel; η_i -longitudinal deformation of i-tire; Δ_i– angle of rotation of i-wheel ($i=\overline{1,4}$) around its axis.

Let us assume that at car' minor deviation from movement with speed V along Oy there is no sliding of tires on the road. In this case, $\dot{\Delta}_i = \frac{V}{r_i}$, where r_i is radius of the wheels' rolling.

Equations of basket's movement on m tire wheels are written in following ways

$$\frac{d}{dt}\frac{\partial T}{\partial \dot{q}_j} - \frac{\partial T}{\partial q_j} = Q_j + R_j \quad (j=\overline{1,11}) \quad (1)$$

where we define summed forces influencing the system $Q_j(q,\dot{q},t)$, R_j- summed forces, conditioned by deformation of pneumatics

$$R_j(\xi,\varphi,\chi,\eta) = \sum_{i=1}^{4} \Big[\left(P_i \cos\theta_i - F_i \sin\theta_i\right)\frac{\partial y_i}{\partial q_j} + \left(P_i \sin\theta_i + F_i \cos\theta_i\right)\frac{\partial x_i}{\partial q_j} +$$

$$+(N_i+F_{zi})\frac{\partial z_i}{\partial q_j}+M_{\theta i}\frac{\partial \theta_i}{\partial q_j}+M_{\chi i}\frac{\partial \chi_i}{\partial q_j}+(r_iP_i+M_i)\frac{\partial \Delta_i}{\partial q_j}\Bigg]\ (j=\overline{1,\ 11}), \qquad (2)$$

Here forces F_i, P_i and momenta $M_{\theta i}, M_{\chi i}, M_i$ are defined by the expression

$$F_{xi}=a_i\xi_i+h_{1i}\dot{\xi}_i+\sigma_iN_i\chi_i+h_{2i}\dot{\chi}_i,\ M_{\chi i}=-\sigma_iN_i\xi_i-h_{3i}\dot{\xi}_i-\rho_iN_i\chi_i-h_{4i}\dot{\chi}_i,$$

$$M_{\theta i}=b_i\varphi_i^{'}+h_{si}\dot{\varphi}_i^{'},\ P_i=k_{\tau i}\eta_i,\ M_i=\mu_i\ N_i\eta_i \qquad (i=\overline{1,4}),$$

$q_1=x,\ q_2=y,\ q_3=z,\ q_4=\theta,\ q_5=\psi,\ q_6=\vartheta_1,\ q_7=\vartheta_2,\ q_8=\Delta_1,\ q_9=\Delta_2,\ q_{10}=\Delta_3,\ q_{11}=\Delta_4$
are summed coordinates of the system.

Putting the numbers for $T,\ Q_j, R_j$ in equation (2), we will find dynamic equations of curvilinear movement of the car with consideration of elasticity and deformation of the tires (all calculations are done in Maple system)

$$m\dot{u}_1+\alpha_5\ddot{\theta}-(h_{21}+h_{22})\dot{\psi}-2\sigma N\psi+\gamma_0\sigma N(\vartheta_1+\vartheta_2)+\gamma_0h_{21}\dot{\vartheta}_1+\gamma_0h_{22}\dot{\vartheta}_2-$$
$$-a_{12}\xi_1-a_{12}\xi_2-h_{11}\dot{\xi}_1-h_{12}\dot{\xi}_2-a_{34}\xi_3-h_{13}\dot{\xi}_3-a_{34}\xi_4-h_{14}\dot{\xi}_4=0,$$

$$m\dot{u}_2+\beta_5\ddot{\theta}-m_{21}\ell_3\ddot{\vartheta}_1+m_{22}\ell_3\ddot{\vartheta}_2-K_{\tau_1}\eta_1-K_{\tau_2}\eta_2-K_{\tau_3}\eta_3-K_{\tau_4}\eta_4=0,$$

$$m\dot{u}_3+\gamma_5\ddot{\psi}+\gamma_0m_{21}\ell_3\ddot{\vartheta}_1-\gamma_0m_{22}\ell_3\ddot{\vartheta}_2-K_{r_1}r_1-h_{61}\dot{r}_1-K_{r_2}\dot{r}_2-$$
$$-h_{62}\dot{r}_2-K_{r_3}r_3-h_{63}\dot{r}_3-K_{r_4}r_4-h_{64}\dot{r}_4=N_1+N_2+N_3+N_4,$$

$$n_5\ddot{\theta}+n_6\dot{u}_1+n_7\dot{u}_2+A_1\ddot{\vartheta}_1+A_2\ddot{\vartheta}_2-\beta C_3\ddot{\Delta}_3+\beta C_4\ddot{\Delta}_4+2\sigma N\ell_1\psi+(\ell_1h_{21}+\ell_1h_{22})\dot{\psi}-$$
$$-\gamma_0\sigma N\ell_1\vartheta_1-\gamma_0h_{21}\ell_1\dot{\vartheta}_1-\gamma_0\sigma N\ell_1\vartheta_2-\gamma_0h_{22}\ell_1\dot{\vartheta}_2+a\ell_1\xi_1+h_{11}\ell_1\dot{\xi}_1+a\ell_1\xi_2+$$
$$+h_{12}\ell_1\dot{\xi}_2-a_3\ell_2\xi_3-h_{13}\ell_2\dot{\xi}_3-a_4\ell_2\xi_4-h_{14}\ell_2\dot{\xi}_4-b_1\varphi_1-h_{51}\dot{\varphi}_1-b_2\varphi_2-h_{52}\dot{\varphi}_2-$$
$$-b_3\varphi_3-h_{53}\dot{\varphi}_3-b_4\varphi_4-h_{54}\dot{\varphi}_4+K_{\tau_2}\ell\eta_2-K_{\tau_1}\ell\eta_1+K_{\tau_4}L\eta_4-K_{\tau_3}L\eta_3=0,$$

$$m_5\ddot{\psi}+m_6\dot{u}_3+\gamma_0m_7\ddot{\vartheta}_1+\gamma_0m_8\ddot{\vartheta}_2+(a_1r_1+\sigma_1N_1)\xi_1+(a_2r_2+\sigma_2N_2)\xi_2+(a_3r_3+\sigma_3N_3)\xi_3+$$
$$+(a_4r_4+\sigma_4N_4)\xi_4-(\sigma_1r_1-\rho_1)N_1\psi+\gamma_0(\sigma_1r_1-\rho_1)N_1\vartheta_1-(\sigma_2r_2-\rho_2)N_2\psi+$$
$$+\gamma_0(\sigma_2r_2-\rho_2)N_2\vartheta_2-(\sigma_3r_3-\rho_3)N_3\beta-(\sigma_4r_4-\rho_4)N_4\beta+(r_1h_{11}+h_{31})\dot{\xi}_1+(r_2h_{12}+h_{32})\dot{\xi}_2+$$
$$+(r_3h_{13}+h_{33})\dot{\xi}_3+(r_4h_{14}+h_{34})\dot{\xi}_4-(h_{21}-h_{41})\dot{\psi}+\gamma_0(h_{21}-h_{41})\dot{v}_1-(h_{22}-h_{42})\dot{\psi}+$$
$$+\gamma_0(h_{22}-h_{42})\dot{v}_2=0,$$

$$\ell_8\ddot{v}_1+\ell_5\dot{u}_2+\ell_6\dot{u}_3+A\ell\ddot{\theta}+\ell_7\ddot{\psi}-\beta_0C_1\ddot{\Delta}_1+\beta_0C_2\ddot{\Delta}_2-$$
$$-b_1\varphi_1-h_{51}\dot{\varphi}_1+(\beta_0\mu_1N_1-\gamma_0K_{\tau_1}e_1)\eta_1=0, \qquad (3)$$

$$g_5\dot{u}_2-\gamma_0g_5\dot{u}_3+A2\ddot{\theta}+\gamma_0g_6\ddot{\psi}+g_7\ddot{v}_1-b_2\varphi_2-h_{52}\dot{\varphi}_2+(\beta_0\mu_2N_2-\gamma_0K_{\tau_2}e_2)\eta_2=0,$$

$$C_1(-\beta_0\ddot{v}_1+\ddot{\Delta}_1)+(r_1K_{\tau_1}-\mu_1N_1)\eta_1=0,\quad C_2(\beta_0\ddot{v}_1+\ddot{\Delta}_2)+(r_2K_{\tau_2}-\mu_2N_2)\eta_2=0,$$

$$C_3(-\beta\ddot{\theta}+\ddot{\Delta}_3)+(r_3K_{\tau_3}-\mu_3N_3)\eta_3=0,\ C_4(\beta\ddot{\theta}+\ddot{\Delta}_4)+(r_4K_{\tau_4}-\mu_4N_4)\eta_4=0,$$

where α_5 , β_5, γ_5, $n_i\,(i=\overline{5,7})$, $m_i\,(i=\overline{5,8})$, $l_i\,(i=\overline{5,8})$, g_5 are functions of time and expressed through constructive parameters of the system.

When composing equations of movement of systems with rolling, we shoud take into consideration kinematical connections, imposed on the wheels rolling [2,3]. In the case of curvilinear movement the kinematical equations, expressing conditions of rolling wheels without sliding effect, at longitudinal and transversal deformation look as follows:

$$\dot{x}_1\sin(\theta+\vartheta_1+\varphi_1)-\dot{y}_1\cos(\theta+\vartheta_1+\varphi_1)+\dot{\xi}_1=0,$$
$$\dot{\theta}+\dot{\vartheta}_1+\dot{\varphi}_1-(\dot{x}_1\cos(\theta+\vartheta_1+\varphi_1)+\dot{y}_1\sin(\theta+\vartheta_1+\varphi_1))(\alpha_1\xi_1-\beta_1\varphi_1-\gamma_1\chi_1)=0,$$
$$r_1\dot{\Delta}_1+\dot{\eta}_1+(\dot{x}_1\cos(\theta+\vartheta_1)+\dot{y}_1\sin(\theta+\vartheta_1))[1+\lambda_1\eta_1-\nu_1(r_{01}-r_1)]=0,$$

$$\dot{x}_2\sin(\theta+\vartheta_2+\varphi_2)-\dot{y}_2\cos(\theta+\vartheta_2+\varphi_2)+\dot{\xi}_2=0,$$
$$\dot{\theta}+\dot{\vartheta}_2+\dot{\varphi}_2-(\dot{x}_2\cos(\theta+\vartheta_2+\varphi_2)+\dot{y}_2\sin(\theta+\vartheta_2+\varphi_2))(\alpha_2\xi_2-\beta_2\varphi_2-\gamma_2\chi_2)=0,$$
$$r_2\dot{\Delta}_2+\dot{\eta}_2+(\dot{x}_2\cos(\theta+\vartheta_2)+\dot{y}_2\sin(\theta+\vartheta_2))[1+\lambda_2\eta_2-\nu_2(r_{02}-r_2)]=0,$$
$$\dot{x}_3\sin(\theta+\varphi_3)-\dot{y}_3\cos(\theta+\varphi_3)+\dot{\xi}_3=0,$$
$$\dot{\theta}+\dot{\varphi}_3-(\dot{x}_3\cos(\theta+\varphi_3)+\dot{y}_3\sin(\theta+\varphi_3))(\alpha_3\xi_3-\beta_3\varphi_3-\gamma_3\chi_3)=0,$$
$$r_3\dot{\Delta}_3+\dot{\eta}_3+(\dot{x}_3\cos\theta+\dot{y}_3\sin\theta)[1+\lambda_3\eta_3-\nu_3(r_{03}-r_3)]=0,\ (3)$$

$$\dot{x}_4\sin(\theta+\varphi_4)-\dot{y}_4\cos(\theta+\varphi_4)+\dot{\xi}_4=0,$$
$$\dot{\theta}+\dot{\varphi}_4-(\dot{x}_4\cos(\theta+\varphi_4)+\dot{y}_4\sin(\theta+\varphi_4))(\alpha_4\xi_4-\beta_4\varphi_4-\gamma_4\chi_4)=0,$$
$$r_4\dot{\Delta}_4+\dot{\eta}_4+(\dot{x}_4\cos\theta+\dot{y}_4\sin\theta)[1+\lambda_4\eta_4-\nu_4(r_{04}-r_4)]=0, \qquad (4)$$

The systems of equations (3), (4) represent mathematical model of curvilinear movement of a car with considerations of elasticity and deformation of the tires (longitudinal, transversal, angular deformation of the tires and tilt of the wheels), as well as non potential forces in the tires' material.

Equations (3) and (4) describe movement of depicting point in 34-phase space

$$(x,\ y,\ z,\ \dot{x},\ \dot{y},\ \dot{z},\ \theta,\ \dot{\theta},\ \vartheta_1,\ \dot{\vartheta}_1,\ \vartheta_2,\ \dot{\vartheta}_2,\ \psi,\ \dot{\psi},\ \Delta_1,\ \Delta_2,\ \Delta_3,\ \Delta_4,\ \dot{\Delta}_1,\ \dot{\Delta}_2,\ \dot{\Delta}_3,\ \dot{\Delta}_4,$$
$$\xi_1,\ \xi_2,\ \xi_3,\ \xi_4,\ \varphi_1,\ \varphi_2,\ \varphi_3,\ \varphi_4,\ \eta_1,\ \eta_2,\ \eta_3,\ \eta_4).$$

Static movement of the system is depicted in this space by the state of balance.

REFERENCE

[1] Turaev X., Urunbaev E. Software of "Automation of research on sustainability of wheeled transport machines". Certificate of official registration of the program. State Patent Office, № DGU 00305, 28.02.2000.

[2] Turaev X., Fufaev, Mussarskiy Theory of movement of rolling systems. Tashkent, Fan 1987 . p. 158.

[3] Turaev X. Modeling and research of the dynamics of wheeled transport machines with deformation of tires. Tashkent, Fan 1995 p 168.

ON THE CAUCHY PROBLEM FOR FIRST – ORDER ELLIPTIC SYSTEMS

Tursunov F. R, Malikov Z.
Samarkand state university
Dilorom_yunusova@mail.ru

In this note we present a formula for reconstruction of the solutions for the system of first – order differential equations of elliptic type for a special class of domains by their values on the boundary.

We denote by $x=(x_1,\dots,x_m)$ and $y=(y_1,\dots,y_m)$ points of real Euclidean space $R^m,\ m\geq 3,$ and $x^T=(x_1,\dots,x_m)^T$ the transposed column vector of x. We set

$$y'=(y_1,\dots,y_{m-1}),\qquad x'=(x_1,\dots,x_{m-1}),\qquad r=|y-x|,$$

$$\alpha=|y'-x'|,\qquad \alpha^2=s,\quad \alpha_1^2=y_1^2+\dots+y_{m-1}^2,\quad \alpha_0^2=x_1^2+\dots+x_{m-1}^2$$

$$w=i\sqrt{u^2+\alpha^2}+\beta,\qquad u\geq 0,\qquad w_0=i\tau\alpha+\beta,$$

$$\partial/\partial x=(\partial/\partial x_1,\dots,\partial/\partial x_m)^T,\qquad u(x)=(u_1(x),\dots,u_n(n))^T,$$

$$u^0=(1,\dots,1)\in R^n$$

Moreover, $E(x)=\{x_1,\dots,x_m\}$ is a diagonal matrix, and w_m is the area of the unit sphere in R^m.

$A_{l\times n}(x)$ is denoted the class of matrices $D(x^T)$ with constant complex coefficients satisfying the condition

$$D*(x^T)D(x^T)=E(|x|^2u^0),$$

Where $D^*(x^T)$ is the Hermitian conjugate of the matrix $D(x^T)$.

Let's the boundary of domain G_p of hyper lane $y_m=0$ and smooth noncom pact surface S lying on the layer

$$0<y_m\leq h,\ h=\frac{\pi}{\rho},\qquad \rho>0.$$

We consider the system of differential equations

$$D\left(\frac{\partial}{\partial x}\right)u(x)=0, \tag{1}$$

where the characteristic matrix $D(x^T)\in A_{l\times n}(x)$.

If $u(x)\in C^1(G_\rho)\bigcap G(\overline{G}_\rho)$ then we have the integral formula proved by [1]

$$u(x)=\int_{\partial G_\rho} M(y,x)u(y)ds_y,\quad x\in G_\rho \tag{2}$$

where

$$M(y,x)=\left(E\left(\frac{\overline{C_m}}{r^{m-2}}u^0\right)D*\left(\frac{\partial}{\partial x}\right)\right)D(t)^T,\ \overline{C_m}=\frac{1}{(m-2)\omega_m}; \tag{3}$$

$t=(t_1,,...,t_m)$ and the t_i are the direction cosines of the outer normal at the point $y\in\partial G_\rho$. Formula (2) is preserved if in (3) an arbitrary harmonic function is added to f$\overline{C}_m/r^{m-2}$. We set

$$N_\sigma(y,x)=\left(E\left(\Phi_\sigma(y,x)u^0\right)D*(\partial/\partial x)\right)D(t^T),$$

$$\Phi_\sigma(y,x)=\frac{\varphi_\sigma(y,x)}{C_m K(x_m)},\ C_m=\begin{cases}(-1)^k\cdot 2^{-k}(2k-1)!\pi/\overline{C}_m, & m=2k+1\\ (-1)^{k-1}(k-1)!/\overline{C}_m, & m=2k,\end{cases}$$

where $\varphi_\sigma(y,x)$ is defined by the following formula:

$$\varphi_\sigma(y,x)=\begin{cases}\dfrac{\partial^{k-1}}{\partial s^{k-1}}\displaystyle\int_0^\infty \operatorname{Im}\frac{K(w)}{w-x_m}\frac{du}{\sqrt{u^2+\alpha^2}}, & x\neq y,\quad for\ m=2k+1,\quad k\geq 1,\\ \dfrac{\partial^{k-2}}{\partial s^{k-2}}\operatorname{Im}\dfrac{K(w_0)}{\alpha(w_0-x_m)}, & x\neq y,\quad for\ m=2k,\quad k\geq 2,\end{cases}$$

$$K(w)=\frac{\exp\left[\sigma w-bchi\rho_1\left(w-\frac{h}{2}\right)-b_1 chi\rho_0\left(w-\frac{h}{2}\right)\right]}{(w-x_m+2h)^q}$$

$$w=i\sqrt{u^2+\alpha^2}+y_m,\quad 0<\rho_1<\rho,\quad 0<\rho_0<\rho,$$

$$0<x_m<h,\quad b>0,\quad b_1>b_0\left(\cos\rho\frac{h}{2}\right)^{-1}$$

$K(w)$ is an entire function, real for real w, which satisfies the condition $K(u) \neq 0$, and

$$\sup_{v \geq 1} \left| v^p K^p(w) \right| = M(p,u) < +\infty, \quad p = 0,1,\ldots,m, \quad w = u + iv$$

for $-\infty < u < +\infty$ (see [2]).

In this note we consider the Cauchy problem in the formulation of by M. M. Lavrent'ev [3]. Suppose $u(x) \in C^1(G_\rho) \bigcap C(\overline{G}_\rho)$ and $u(x)$ satisfies (1), and let $u_\delta(x)$ be a continuous approximation to $u(x)$ on S, i.e.,

$$\max_S \left| u(x) - u_\delta(x) \right| < \delta, \quad 0 < \delta < 1.$$

We want to construct $u(x)$ in G_ρ.

Theorem 1. Suppose $u(x) \in C^1(G_\rho) \bigcap C(\overline{G}_\rho)$ is a solution of (1) and

$$\left| u(x) \right| \leq M, \quad y \in T = \partial G_\rho / S,$$

where M is a given positive number. Set

$$u_\sigma(x) = \int_S N_\sigma(y,x) u(y) dS_y, \quad x \in G_\rho,$$

Then

$$\left| u(x) - u_\sigma(x) \right| \leq M C(x) \tilde{C}(\sigma) \exp(-\sigma \gamma^\rho), \; x \in G_\rho.$$

where

$$C(x) = C_\rho \int_{\partial G_\rho} \frac{ds}{r^{m-1}}, \quad \tilde{C}(\sigma) = C = \begin{cases} \sigma^m & for \quad m = 2k+1, \; k \geq 1, \\ \sigma^{m-1} & for \quad m = 2k, \qquad k \geq 2. \end{cases}$$

Corollary. The limit equality

$$\lim_{\sigma \to \infty} u_{\sigma(x) = u(x)}, \quad x \in G_\rho,$$

is satisfied uniformly in each compact set in G_ρ.

REFERENCES.

[1] N. N. Tarhanov, Some Questions of Multidimensional Complex Analysis (A.M.Kytmanov, editor), Inst. Fiz. Sibirsk. Otdel. Akad. Nauk SSSR, Krasnoyarsk, 1980, pp. 147 – 160. (Russian).

[2] Sh. Yarmukhamedov, Dokl. Akad. Nauk. SSSR 204 (1972), 799 – 802; English transl. in Soviet Math. Dokl. 13 (1972).

[3] M. M. Lavrent'ev, On some ill-posed problems of mathematical physics, Izdat. Sibirsk. Otdel. Akad. Nauk SSSR, Novosibirsk, 1962; English transl., Springer-Verlag, Berlin, 1967.

[4] Sh. Yarmukhamedov, Dokl. Akad. Nauk. SSSR 306 (1989), 795 – 798; English transl. in Soviet Math. Dokl. 39 (1989).

FRACTIONAL INTEGRO-DIFFERENTIATION BY CHEN-HADAMARD

Yakhshiboev M.U., Yakhshiboev A.M.
Samarkand Branch of Tashkent University of Information Technologies,
yahshiboev@rambler.ru

Definition 1. Let us fix an arbitrary point $c \in R_+^1$, and for R_+^1 function $\varphi(x)$ the following integral

$$\Im_c^\alpha \varphi(x) := \frac{1}{\Gamma(\alpha)} \begin{cases} \int\limits_c^x \frac{\varphi(t)}{(\ln\frac{x}{t})^{1-\alpha}} \frac{dt}{t}, x > c, \\ \int\limits_x^c \frac{\varphi(t)}{(\ln\frac{t}{x})^{1-\alpha}} \frac{dt}{t}, x < c, \end{cases} \tag{1}$$

is referred as the *Chen-Hadamard fractional integral* of order α , where $\alpha > 0$.

By introducing the following functions

$$P_{c+}\varphi := \varphi_{c+}(x) = \begin{cases} \varphi(x), & x > c, \\ 0, & x < c, \end{cases} \qquad P_{c-}\varphi := \varphi_{c-}(x) = \begin{cases} 0, & x > c, \\ \varphi(x), & x < c, \end{cases}$$

we can write the fractional integral (1) in the form

$$\Im_c^\alpha \varphi(x) = \Im_+^\alpha \varphi_{c+}(x) + \Im_-^\alpha \varphi_{c-}(x)$$

where $\Im_+^\alpha$ and $\Im_-^\alpha$ are the Riemann-Liouville fractional integral

$$(\Im_+^\alpha \varphi)(x) := \frac{1}{\Gamma(\alpha)} \int\limits_0^x \frac{\varphi(t)}{(\ln\frac{x}{t})^{1-\alpha}} \frac{dt}{t}, \; 0 < x < \infty,$$

$$(\Im_-^\alpha \varphi)(x) := \frac{1}{\Gamma(\alpha)} \int\limits_x^\infty \frac{\varphi(t)}{(\ln\frac{t}{x})^{1-\alpha}} \frac{dt}{t}, \; 0 < x < \infty.$$

Definition *2*. Let us fix an arbitrary point $c \in R_+^1$, and for R_+^1 function, $f(x)$ the following expression

$$(D_c^\alpha f)(x) := (\Im_c^\alpha)^{-1} f(x) = \frac{1}{\Gamma(1-\alpha)} \begin{cases} x\dfrac{d}{dx}\displaystyle\int_c^x f(t)(\ln\frac{x}{t})^{-\alpha}\frac{dt}{t}, \; x > c, \\ -x\dfrac{d}{dx}\displaystyle\int_x^c f(t)(\ln\frac{t}{x})^{-\alpha}\frac{dt}{t}, \; x < c, \end{cases} \qquad (2)$$

is referred as the *Chen-Hadamard fractional derivative* of order α, with $(0 < \alpha < 1)$. In the case $\alpha \geq 1$ it is necessary to use the relations (18.57) from [1].

Using the expression for fractional derivatives in the form of Marchaud (see (18.58), [1], p. 253), from (2) we obtain

$$(D_c^\alpha f)(x) = \frac{f(x)}{\Gamma(1-\alpha)\left|\ln\frac{x}{c}\right|^\alpha} + \frac{\alpha}{\Gamma(1-\alpha)}\int_{\min(x,c)}^{\max(x,c)} \frac{f(x)-f(t)}{\left|\ln\frac{x}{t}\right|^{1+\alpha}}\frac{dt}{t}. \qquad (3)$$

in the case $0 < \alpha < 1$. Transformation of (2) into (3) is possible in case of rather "nice" functions, see e.g. [1, §18.3]. If we denote the right-hand side of (3) as $(D_c^\alpha f)(x)$, then for rather "nice" functions we have

$$\left(D_c^\alpha f\right)(x) = \left(D_c^\alpha f\right)(x).$$

The right-hand side $D_c^\alpha f$ can also be written as

$$(D_c^\alpha f)(x) = \frac{\alpha}{\Gamma(1-\alpha)}\int_0^1 \frac{f(x) - f_{c+}(x\cdot t) - f_{c-}(x\cdot t^{-1})}{|\ln t|^{1+\alpha}}\frac{dt}{t} =$$
$$= (D_+^\alpha f_{c+})(x) + (D_-^\alpha f_{c-})(x),$$

where $D_\pm^\alpha$ are the Marchaud fractional derivatives

$$(D_+^\alpha f)(x) = \frac{\alpha}{\Gamma(1-\alpha)}\int_0^1 \frac{f(x) - f(x\cdot t)}{|\ln t|^{1+\alpha}}\frac{dt}{t},$$
$$(D_-^\alpha f)(x) = \frac{\alpha}{\Gamma(1-\alpha)}\int_0^1 \frac{f(x) - f(x\cdot t^{-1})}{|\ln t|^{1+\alpha}}\frac{dt}{t}.$$

Since the Chen construction can be applied to functions with an arbitrary growth when $x \to \infty$ or $x \to 0$, this construction is more convenient when applied to such functions than the integro-differentiation by Hadamard [1] itself. As usual, the fractional derivative is to be treated as a certain limit. To this end, several types of different "truncation" of the Chen-Hadamard fractional derivative are introduced, denoted by.

$$D_{c,\rho}^{\alpha} f, \quad \overset{*}{D}{}_{c,\rho}^{\alpha} f, \quad D_{c,\bar{\rho}}^{\alpha} f,$$

where $\frac{1}{\sqrt{e}} < \rho < 1, \quad \bar{\rho} = \left|\ln\frac{x}{c}\right| \ln\frac{1}{\rho}, \quad 0 < \alpha < 1$. The following inversion theorem is valid, where $I_c^{\alpha}\varphi$ stands for the corresponding Chen-Hadamard fractional integral.

Theorem. Let $f = \mathfrak{I}_c^{\alpha}\varphi, \quad \varphi \in L_p\left(R_+^1, \frac{dx}{x}\right)$ (or $\varphi \in L_p^{loc}\left(R_+^1, \frac{dx}{x}\right)$), $1 \le p < \infty, \quad 0 < \alpha < 1, \quad c > 0$. Then

$$\lim_{\rho \to 1} D_{c,\rho}^{\alpha} f = \varphi, \tag{4}$$

$$\lim_{\rho \to 1} \overset{*}{D}{}_{c,\rho}^{\alpha} f = \lim_{\rho \to 1} D_{c,\bar{\rho}}^{\alpha} f = \varphi, \tag{5}$$

The limits in (4)-(5) can be understood both in $L_p\left(R_+^1, \frac{dx}{x}\right)$ or $L_p^{loc}\left(R_+^1, \frac{dx}{x}\right)$, correspondingly, $1 \le p < \infty$, except for the case $p = 1$ in (5), or almost everywhere.

REFERENCES

[1] Samko S.G., Kilbas A., Marichev O. Fractional Integrals and Derivatives. Theory and Applications. London-New-York: Gordon & Beeach. Sci. Pubi., (Russian edition-Fractional Integrals and Derivatives and some of their Applications, Minsk; Nauka I Tekhnika, 1987.), 1993. 1012 pages.

TRIVIALITY OF HOMOLOGY SIMPLICIAL'S SCHEMES UNIMODULUAR'S REPERS OVER RINGS OF ARITHMETIC TYPE

Zaynalov B.R.
Samarkand state university
brzaynalov@mail.ru

In the given work the first non-trivial group of a homology simplicial's schemes unimoduluar's reper for rings of arithmetic type which give a solution of a stabilization problem in algebraic K - theories in that specific case.

Let F - a global field [1] and S - nonempty finite of points of the field F, containing all infinite points. Through $A = A_S$ we will designate subring F, consisting of the x elements which do not have poles out of S, i.e., $V(x) \ge 0$ at $V \notin S$. It is known that [4] if A – Dedekind's ring, a field which coincides with F. The ring of this type is called as to be an arithmetic.

If is A - any commutative ring and I - an ideal in A. Following Wasserstein [2], $SL_2(I,I)$ we will consider a subgroup $SL_2(A)$ consisting of matrixes of a kind

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}, \quad \text{where } b,c \in I, \quad a,d \equiv 1(\mathrm{mod}\, I^2)$$

Through $E_2(I,I)$ we will designate a subgroup $SL_2(I,I)$ generated by elementary matrixes

$$\begin{pmatrix} 1 & b \\ 0 & 1 \end{pmatrix}, \quad \begin{pmatrix} 1 & 0 \\ c & 1 \end{pmatrix}, \text{ where } b,c \in I .$$

Then from deep results of Wasserstein [2] and Bassa [7], Serra [8], follows justice of the following theorem:

Theorem 1. Let A-Dedekind's ring of arithmetic type with infinite group of units, i.e. $card\ S > 1$Then

a) $E_2(A) = SL_2(A)$;

b) For any I group the $E_2(I,I)$ normal in $SL_2(I,I)$, the factor group $SL_2(I,I)/E_2(I,I)$ is finite cyclic group of an order $r(I)$. The number $r(I)$ depends only on exponents $V(I)$ for the points $V \notin S$ dividing number m of roots from units in F .In particular, $SL_2(I,I) = E_2(I,I)$ if I mutually prime with m.

Now we will give the main theorem of the given work.

Theorem 2. If A - Dedekind's ring of arithmetic type with infinite group of units, $H_0\left(GL_n(A), \tilde{H}_{n-2}\left(Um\left(A^n\right)\right)\right) = 0$ for any n.

Let's notice that the theorem means that the group $GL_n(A)$ operates by the left multiplication on simplicial the scheme of unimoduluar repers $Um\left(A^n\right)$ and induces action $GL_n(A)$ on homologyes this scheme, in particular, on $\tilde{H}_{n-2}\left(Um\left(A^n\right)\right)$. (Suslin [5] and Nesterenko [6] see [3].)

Now we will dive the proof of the theorem 2. The statement is trivial at $n < 2$. We will consider further a case $n = 2$. Then according to the theorem [4], group $\tilde{H}_0\left(Um\left(A^2\right)\right)$ it is generated by standard cycles $[V_0, V_1]$, where $V_i \in A^2$ unimoduluar columns.

The sign ~ means that two cycles is translated against each other by action $GL_2(A)$. We will find $\alpha \in GL_2(A)$ such that $\alpha V_0 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ Also we will write $\alpha V_1 = \begin{pmatrix} a \\ b \end{pmatrix}$.Then

$$[V_0, V_1] \sim \begin{bmatrix} 1 & a \\ 0 & b \end{bmatrix} .$$

Cycles of last type we will denote by $< a,b >$ in particular takes place $aA + bA = A$. We will note some relation on these cycles. Operating with a kind matrix $\begin{pmatrix} 1 & x \\ 0 & 1 \end{pmatrix}$ We see that $< a,b > \sim < a + xb, b >$ for any $x \in A$.Let's use also a following relations: $< a,b > = \begin{bmatrix} 1 & a \\ 0 & b \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & a \\ 1 & b \end{bmatrix}$.The first cycle is homologous to zero, because $\begin{pmatrix} 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \end{pmatrix} \in Um(A^2)$. And therefore, $< a,b > = \begin{bmatrix} 0 & a \\ 1 & b \end{bmatrix} \sim \begin{bmatrix} 1 & b \\ 0 & a \end{bmatrix} = < b,a >$.Thus, have obtained $< a,b > \sim < b,a >$, as, hence $< a,b > \sim < a, b + a >$, and cycles $< a,b >$ are replaced on equivalent if to (a,b) to apply an elementary matrix. Sink $E_2(A) = SL_2(A)$ is transitive on unimoduluars lines, then $< a,b > \sim < 1,1 > = 0$.

Thus, in this case we have proved in that specific case more than subtle statement:

Theorem 3. In the statement of the theorem 2 at $n = 2$

$$H_0\left(GL_2(A), \tilde{H}_0\left(Um\left(A^2\right)\right)\right) = 0 \text{ I.e. } \tilde{H}_0\left(Um\left(A^2\right)\right) = 0.$$

The proof of the theorem 2 of general case I hope to publish in the further works.

REFERENCES.

[1] Kassels J., Frelih And. The algebraic theory of numbers. Moscow (Russia), the World, 1969. P.483.

[2] Wasserstein L. About group over Dedekind's rings of arithmetic type. Matem. The collection, 1972,89, №2. p.313-322.

[3] Zaynalov B.R.Dejstvie of full linear group on simplicial schemes unimoduluar repers over commutative rings.Vestnik SamGU, 2009, 5 (57), p.3-5.

[4] Zaynalov B.R.Predatsiklichnost over some types of rings. In book. The international conference on algebra of memory of Shirshov of A.I.collection of theses. Barnaul (Russia), 1991, p.42.

[5] Suslin A.A. Homology, characteristic classes and the theory of Milnora. Works the Floor-mat. Inst. ANSSSR, т 165, 1984, p.188-203.

[6] Nesterenko JU.P.Homological stabilization over rings with a great number units. №5031-84, Dep. On July, 12th 1984. VINITI, p.11.

[7] Bass H., Milnot Y., Serre J.P. Solution of the congruence subgroup problem for and. Publs. Math. Inst. Tantes etudes scient. 1967, №33.

[8] Serre J. Le probleme des groups de congruence pour. Ann. Math., 1970,92, №3, the river p.489-527.