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KO’PHADLAR ALGEBRASI

«Algebra va sonlar nazariyasi» fanidan amaliy mashg’ulotlar o’tkazish uchun uslubiy tavsiyalar

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(Uslubiy qo‘llanma)

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Ushbu uslubiy qo'llanma «Algebra va sonlar nazariyasi» fani bo'yicha «5460100 – matematika» ta'lim yo'nalishi bakalavr talabalari va «5A460100 – Matematik mantiq, Algebra va sonlar nazariyasi» mutaxassisligi magistrantlari uchun mo'ljallangan bo'lib, unda shu fanning namunaviy o'quv dasturidan kelib chiqib, ko'phadlar xalqasi, ko'phadlarning EKUB va EKUKi, ko'phadlarning ildizlari, keltirilmaydigan ko'paytuvchilarga yoyish, rasional kasrlar, bir necha o'zgaruvchili ko'phadlar mavzulariga oid qisqacha nazariy ma'lumotlar, bu usullarning taqbiqiga oid namunaviy misollar yechimlari, mustaqil ish topshiriqlari va boshqa tarqatma materiallar keltirilgan. Uslubiy qo'llanma talabalarga shu fanni yanada chuqurroq o'zlashtirishga yaqindan yordam beradi degan umiddamiz.

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KO'PHADLAR ALGEBRASI

Kalit so'zlar va ifodalar: *bir o'zgaruvchili ko'phad; ko'phadning koeffisiyentlari; ko'phadning darajasi; ko'phadning bosh koeffisiyenti; ko'phadning bosh hadi; ko'phadning ozod hadi; nol ko'phad; ko'phadlar yig'indisi va ko'paytmasi; bir o'zgaruvchili ko'phadlar xalqasi;qoldiqli bo'lish; bo'linma; qoldiq; ko'phadning bo'luvchisi; ko'phadning karralisi; ko'phadlarning umumiy bo'luvchi; ko'phadlarning eng kata umumiy bo'luvchisi (EKUB); Yevklid algoritmi; ko'phadlar EKUBini chiziqli tasvirlash; ko'phadlarning umumiy karralisi; ko'phadlarning eng kichik umumiy karralisi (EKUK); Yevklid ketma-ketligi; ko'phadning qiymati; Bezu teoremasi; ko'phadning ildizi;karali ildiz; tub ildiz; Viyet formulalari; Lagranjning interpolyasion formulasi; Nyutonning interpolyasion formulasi; Teylor formulasi; keltirimaydigan ko'phad; unitar ko'phad; berilgan maydon ustida ko'phadning kanonik yoyilmasi; ko'phadning keltirilmaydigan* $k-$*karrali ko'paytuvchisi; algebraning asosiy teoremasi; Gauss teoremasi; algebraik yopiq maydon; Eyzenshteyn alomati; ko'phadning rasional ildizlari to'g'risidagi birinchi teorema; ko'phadning rasional ildizlari to'g'risidagi ikkinchi teorema; rasional kasr yoki kasr-rasional funksiya; berilgan maydon ustida rasional kasrlar maydoni yoki kasr-rasional funksiyalar maydoni; qisqarmaydigan, to'g'ri va sodda rasional kasrlar; Lagranj formulasi; hadning darajasi; ko'phadning barcha o'zgaruvchilari bo'yicha darajasi; bir jinsli ko'phad yoki forma; ko'phadning bir o'zgaruvchisi bo'yicha darajasi; leksikografisk yoki lug'atiy tartiblash; ko'phadning yuqori hadi; simmetrik ko'phad; elementar (yoki asosiy) simmetrik ko'phad; monogen ko'phad; darajali yig'indilar; Nyuton formulalari.*

§ 1. Ko'phadlar xalqasi. Ko'phadlarning EKUB va EKUKi

P maydon ustidagi b*ir* x *o'zgaruvchili ko'phad deb* quyidagi ko'rinishdagi ifodaga aytiladi

$$a_0x^n + a_1x^{n-1} + \ldots + a_{n-1}x + a_n, \qquad (1)$$

bu yerda $a_0, a_1, \cdots, a_{n-1}, a_n - P$ maydonning elementlaridan iborat bo'lib, ular (1) *ko'phadning koeffisiyentlari deyiladi*. Agar $a_0 \neq 0$, bo'lsa, n ko'phadning *darajasi* deb yuritiladi, $a_0 -$ *bosh koeffisiyent deb,* $a_n -$*ozod had deb,* $a_0x^n -$ (1) ko'phadning bosh *hadi* deb yuritiladi.

Barcha koeffisiyentlari nolga teng bo'lgan ko'phad nol ko'phad deyiladi va 0 bilan belgilanadi. Nol ko'phadning darajasi aniqlanmagan.

Ikkita ko'phadning o'zgaruvchining bir xil darajalari oldidagi koeffisiyentlari teng bo'lsa, ular teng ko'phadlar deyiladi.

$$f(x)=a_0x^n+a_1x^{n-1}+\cdots+a_{n-1}x+a_n=\sum_{i=0}^{n}a_ix^{n-i},$$

$$g(x)=b_0x^m+b_1x^{m-1}+\cdots+a_{m-1}x+b_m=\sum_{k=0}^{m}b_kx^{m-k}.$$

ko'phadlar berilgan bo'lsin.

$f(x)$ va $g(x)$ ko'phadlarning ko'paytmasi deb

$f(x)g(x)=c_0x^{n+m}+c_1x^{n+m-1}+\cdots+c_{n+m-1}x+c_{n+m}=\sum_{j=0}^{n+m}c_jx^{n+m-j}$, ko'phadga aytiladi, bu yerda

$$c_j=\sum_{i+k=j}a_ib_k= =a_0b_j+a_1b_{j-1}+a_2b_{j-2}+\cdots+a_{j-1}b_1+a_jb_o,\quad j=0,1,\ldots,n+m.$$

Agar $n\geq m$ bo'lsa, $f(x)$ va $g(x)$ *ko'phadlarning yig'indisi deb*

$$f(x)+g(x)=(a_n+b_m)+(a_{n-1}+b_{m-1})x+\cdots+(a_{n-m}+b_m)x^m+a_{n-m-1}x^{m+1}+\cdots+a_0x^n.$$

ko'phadga aytiladi. P maydon ustida x o'zgaruvchili barcha ko'phadlar to'plami yuqorida keltirilgan ko'phadlarni qo'shish va ko'paytirish amallariga nisbatan birlik elementli kommutativ xalqa tashkil qiladi. Bu xalqa $P[x]$ bilan belgilanadi va P maydon ustida bir o'zgaruvchili *ko'phadlar xalqasi deb yuritiladi*. Bu xalqaning nol elementi vazifasini Ox^0, nol ko'phad bajaradi, birlik element esa ex^0 ko'phaddan iborat bo'lib, bu yerda e element P maydonning birlik elementidan iborat. $f(x)=a_0x^0+a_1x^1+\cdots+a_nx^n$ $-$ko'phadga qarama-qarshi ko'phad deb $(-f(x))=(-a_0)\,x^0+(-a_1)\,x^1+\cdots+(-a_n)\,x^n$. ko'phadga aytiladi.

Agari $f(x)\in P[x],\ \ 0\neq g(x)\in P[x],$ bo'lsa, $f(x)$ ni $g(x)$ ga qoldiqli bo'lish deb quyidagi munosabatga aytiladi:

$$f(x)=g(x)q(x)+r(x),$$

bu yerda $q(x)$ va $r(x)-$ P, maydon ustidagi ko'phadlar bo'lib, $r(x)$ ning darajasi $g(x)$ ning darajasidan kichik bo'ladi yoki $r(x)=0$ bo'ladi. Bu tasvirlash yagonadir. $q(x)$ ko'phad bo'linma deb, $r(x)-$ esa $f(x)$ ni $g(x)$ bo'lgandagi qoldiq deb yuritiladi.

$r(x)=0$ bo'lsa, $f(x)$ ko'phad $g(x)$ ko'phadga bo'linadi deyiladi va $g(x)\mid f(x)$ (yoki $f(x)\vdots\, g(x)$) ko'rinishda yoziladi, bu holda $g(x)$ ko'phad $f(x)$ *ko'phadning* bo'luvchisi, $f(x)$ *esa* $g(x)$ *ko'phadning karralisi* deb yuritiladi.

Agar $P[x],\ \ k>1,$ dan olingan $f_1(x),f_2(x),\ldots,f_k(x)$ ko'phadlardan har biri $\varphi(x),$ ko'phadga bo'linsa, u holda $\varphi(x)$ ko'phad $f_1(x),f_2(x),\ldots,f_k(x).$ *ko'phadlarning umumiy bo'luvchisi deyiladi.*

$P[x],\ \ k>1,$ dan olingan $f_1(x),f_2(x),\ldots,f_k(x)$ ko'phadlarning *eng katta umumiy bo'luvchisi* (EKUB) berilgan ko'phadlarning barcha umumiy bo'luvchilariga qoldiqsiz bo'linadigan umumiy bo'luvchiga aytiladi. Bir vaqtda nolga teng bo'lmagan ixtiyoriy ko'phadlar uchun EKUB mavjud bo'lib, u noldan farqli o'zgarmas son ko'paytmasi aniqligida yagona ravishda aniqlangan. Barcha eng katta umumiy bo'luvchilar orasidan bosh koeffisiyenti 1 ga teng bo'lgan ko'phad tanlab olinadi. $f_1(x), f_2(x), \ldots, f_k(x)$ ko'phadlarning EKUBi $(f_1(x), f_2(x), \ldots, f_k(x))$. bilan belgilanadi

Agar $g(x)$ ko'phad $f(x)$ ning bo'luvchisi bo'lsa, u holda $(f(x), g(x)) = b_0^{-1}\, g(x)$, bo'ladi, bu yerda $b_0 -$ $g(x)$ ko'phadning bosh koeffisiyenti. Agar $f(x)$ ko'phad $g(x)$ ko'phadga bo'linmasa, u holda $f(x)$ va $g(x)$ ko'phadlarnig EKUBi $f(x)$ va $g(x)$ ko'phadlar uchun *Yevklid algoritmi*dagi oxirgi noldan farqli ko'phadga teng bo'lib, uni bosh koeffisiyentiga bo'lib olinadi. Yevklid algoritmi $f(x)$ va $g(x)$ ko'phadlar uchun quyidagicha ketma-ket bo'lish jarayonidan iborat: dastlab $f(x)$ ko'phad $g(x)$ga qoldiqli bo'linadi va $r_1(x)$ qoldiq hosil qilinadi; so'ngra $g(x)$ ko'phad $r_1(x)$ga qoldiqli bo'linadi va $r_2(x)$ qoldiq hosil qilinadi; agar $r_2(x) \neq 0$, bo'lsa, $r_1(x)$ ko'phad $r_2(x)$ ga bo'linadi va hokazo bu jarayon qoldiqda nol hosil bo'lguncha davom ettiriladi. Oxirgi noldan farqli $r_k(x)$ qoldiq $f(x)$ va $g(x)$ ko'phadlarning EKUBidan iborat bo'ladi .

Uchta va uchtadan ko'p ko'phadlarning EKUBini topish quyidagi tenglikka asosan ikkita ko'phadning EKUBini topishga keltiriladi:

$$(f_1(x), f_2(x), \ldots, f_k(x)) = ((f_1(x), f_2(x), \ldots, f_{k-1}(x)), \quad f_k(x)), \quad k \geq 3.$$

Agar $(f_1(x), f_2(x), \ldots, f_k(x)) = d(x)$, bo'lsa, u holda $P[x]$ xalqada shunday $g_i(x), \quad i = \overline{1,k}$, ko'phadlar mavjudki, ular uchun

$$d(x) = \sum_{i=1}^{k} f_i(x) g_i(x) = f_1(x) g_1(x) + \ldots + f_k(x) g_k(x). \qquad (2)$$

tenglik o'rinli bo'ladi. (2) tenglik *ko'phadlar EKUBining chiziqli tasviri deyiladi.*

Agar bir necha ko'phadlarning EKUBi birga teng bo'lsa, ular o'zaro tub deyiladi. $P[x]$ xalqadan olingan $f_1(x), f_2(x), \ldots, f_k(x)$ ko'phadlar uchun $P[x]$ xalqada shunday $g_i(x), \quad i = \overline{1,k}$, ko'phadlar mavjud bo'lib,

$$f_1(x) g_1(x) + f_2(x) g_2(x) + \ldots + f_k(x) g_k(x) = 1.$$

tenglik o'rinli bo'lgandagina o'zaro tub bo'ladi.

Agar $h(x) \in P[x]$ ko'phad $P[x]$ dan olingan nol bo'lmagan $f_1(x), f_2(x), \ldots, f_k(x)$ ko'phadlarning har biriga qoldiqsiz bo'linsa, $h(x)$ ko'phad $f_1(x), f_2(x), \ldots, f_k(x)$ *ko'phadlarning umumiy karralisi deyiladi.* $P[x]$ dan olingan nol bo'lmagan $f_1(x), f_2(x), \ldots, f_k(x), \quad k > 1$, ko'phadlarning *eng kichik karralisi (EKUK)* deb, ularning shunday ummiy karralisiga aytiladiki, u boshqa ixtiyoriy umumiy karrali ko'phadning bo'luvchisi bo'ladi. Odatda barcha EKUKlar orasidan bosh koeffisiyenti 1ga teng bo'lgan ko'phad EKUK sifatida olinadi. $f_1(x), f_2(x), \ldots, f_k(x)$ ko'phadlarning EKUKi $[f_1(x), f_2(x), \ldots, f_k(x)]$. bilan belgilanadi. Ikkita ko'phadning EKUKi quyidagi formula bilan topiladi:

$$[f(x), g(x)] = \frac{f(x)\, g(x)}{a_0\, b_0\ (f(x), g(x))},$$

bu yerda $a_0, b_0 -$ mos ravishda $f(x)$ va $g(x)$ ko'phadlarning bosh koeffisiyentlari.

Uchta va undan ortiq ko'phadlarning EKUKini topish quyidagi tenglikka asosan ikkita ko'phadning EKUKini topishga keltiriladi:

$$[f_1(x), f_2(x), \ldots, f_k(x)] = [[f_1(x), f_2(x), \ldots, f_{k-1}(x)], \quad f_k(x)], \quad k \geq 3.$$

1-m i s o l. $\mathbf{Q}(x)$ xalqada $f(x) = 2x^4 + x^3 + x^2 - x - 3$ ko'phadni $g(x) = x^3 + 2x^2 - 1$. ko'phadga bo'lgandagi $q(x)$ bo'linma va $r(x)$ qoldiqni toping.

Yechish. Qoldiqli bo'lish algoritmiga asosan:

$$\begin{array}{l|l} 2x^4 + x^3 + x^2 - x - 3 & \underline{x^3 + 2x^2 - 1} \\ \underline{2x^4 + 4x^3 - 2x} & 2x - 3 \\ \quad -3x^3 + x^2 + x - 3 & \\ \quad \underline{-3x^3 - 6x^2 + 3} & \\ \qquad 7x^2 + x - 6 & \end{array}$$

bu yerdan $f(x) = g(x)\, q(x) + r(x)$ ga asosan $q(x) = 2x - 3; \quad r(x) = 7x^2 + x - 6.$ ni hosil qilamiz. ■

2-m i s o l. $\mathbf{Z}_7[x]$ xalqada $f(x) = 2x^4 + x^3 + x^2 + 6x + 4$ ko'phadni $g(x) = x^3 + 2x^2 + 6$. ko'phadga bo'lgandagi bo'linma $q(x)$ va qoldiq $r(x)$ ni toping.

Yechish. Qoldiqli bo'lish sxemasi bo'yicha hosil qilamiz :

$$\begin{array}{l|l} 2x^4 + x^3 + x^2 + 6x + 4 & \underline{x^3 + 2x^2 + 6} \\ \underline{2x^4 + 4x^3 + 5x} & 2x + 4 \\ \quad 4x^3 + x^2 + x + 4 & \\ \quad \underline{4x^3 + x^2 + 3} & \\ \qquad\quad x + 1 & \end{array}$$

bu yerdan $f(x) = g(x)\, q(x) + r(x)$ ga asosan $q(x) = 2x + 4; \quad r(x) = x + 1.$ •

3-m i s o l. $\mathbf{Q}[x]$ xalqada $f(x) = 2x^4 + x^3 + x^2 - x - 3$ va $g(x) = x^3 + 2x^2 - 1$. ko'phadlarning EKUBini toping

Yechish. Bu ko'phadlar uchun Yevklid algoritmi quyidagicha bo'ladi:

$$\begin{array}{l|l} 2x^4+x^3+x^2-x-3 & x^3+2x^2-1 \\ \underline{2x^4+4x^3-2x} & 2x-3 \\ \quad -3x^3+x^2+x-3 & \\ \quad \underline{-3x^3-6x^2+3} & \end{array}$$

$$\begin{array}{l|l} x^3+2x^2-1 & 7x^2+x-6 \\ \underline{x^3+\frac{1}{7}x^2-\frac{6}{7}x} & \frac{1}{7}x+\frac{13}{49} \\ \quad \frac{13}{7}x^2+\frac{6}{7}x-1 & \\ \quad \underline{\frac{13}{7}x^2+\frac{13}{49}x-\frac{78}{49}} & \end{array}$$

$$\begin{array}{l|l} 7x^2+x-6 & \frac{29}{49}x+\frac{29}{49} \\ \underline{7x^2+7x} & \frac{343}{29}x-\frac{294}{29} \\ \quad -6x-6 & \\ \quad \underline{-6x-6} & \\ \qquad 0 & \end{array}$$

Demak, $f(x)$ va $g(x)$ ko’phadlar uchun *Yevklid algoritmi* ketma-ketligi quyidagi ko’rinishda bo’ladi:

$$f(x)=g(x)(2x-3)+r_1(x) \qquad (r_1(x)=7x^2+x-6),$$

$$g(x)=r_1(x)\left(\frac{1}{7}x+\frac{13}{49}\right)+r_2(x) \qquad \left(r_2(x)=\frac{29}{49}x+\frac{29}{49}\right),$$

$$r_1(x)=r_2(x)\left(\frac{343}{29}x-\frac{294}{29}\right).$$

Bu yerdan $f(x)$ va $g(x)$ ko’phadlarning EKUBi $r_2(x)$ ga tengligi kelib chiqadi. Endi $r_2(x)$ ni uning bosh koeffisiyenti $\frac{29}{49}$ ga bo’lib, $(f(x),\ g(x))=x+1.$ ni hosil qilamiz. ■

4-m i s o l. Yevklid algoritmidan foydalanib $Q(x)$ xalqada $f(x)=x^4+2x^3-x^2-4x-2,$ va $g(x)=x^4+x^3-x^2-2x-2.$ ko’phadlar uchun $(f(x),g(x))=f(x)\varphi(x)+g(x)\psi(x):$ tenglikni qanoatlantiradigan $\varphi(x)$ va $\psi(x),$ ko’phadalrni toping.

Yechish. $f(x)$ va $g(x)$ ko'phadlar uchun Yevklid algoritmi quyidagi ko'rinishda bo'ladi:

$$f(x)=g(x)\cdot 1+(x^3-2x)$$
$$g(x)=(x^3-2x)(x+1)+(x^2-2)$$
$$x^3-2x=(x^2-x)\cdot x.$$

Demak, $(f(x),g(x))=x^2-2$.

Yevklid algoritmining ikkinchi tengligidan quyidagini hosil qilamiz:

$$x^2-2=g(x)-(x^3-2x)(x+1).$$

Bu tenglikda (x^3-2x) ning o'rniga uning birinchi tenglikdan topilgan qiymatini qo'yib:

$$x^2-2=g(x)-(f(x)-g(x))(x+1)=g(x)-(x+1)f(x)+(x+1)g(x)$$

ni hosil qilamiz. Shunday qilib,

$$(f(x),g(x))=x^2-2=-(x+1)f(x)+(x+2)g(x), \text{ ya'ni}$$
$$\varphi(x)=-(x+1), \quad \psi(x)=x+2. \blacksquare$$

Agar $f(x)$ va $g(x)$ ko'phadlarning darajalari noldan katta bo'lsa, u holda quyidagi tenglikda

$$f(x)\varphi(x)+g(x)\psi(x)=(f(x),g(x)) \qquad (3)$$

$\varphi(x)$ va $\psi(x)$ ko'phadlarni shunday tanlab olish mumkinki, $\varphi(x)$ ning darajasi $g(x)$ ning darajasidan, $\psi(x)$ niki esa $-$ $f(x)$ nikidan kichik bo'ladi.

Amaliyotda $\varphi(x)$ va $\psi(x)$ ko'phadlarni topish uchun $f(x)$ va $g(x)$ ko'phadlarning o'rniga $f_1(x)=\dfrac{f(x)}{(f(x),g(x))}$, $g_1(x)=\dfrac{f(x)}{(f(x),g(x))}$ ko'phadlarni olish qulaydir. Bu holda dastlab $\varphi(x)$ va $\psi(x)$ ko'phadlar shunday tanlanadiki,

$$1=f_1(x)\varphi(x)+g_1(x)\psi(x). \qquad (4)$$

tenglik o'rinli bo'lsin. Bu quyidagicha amalga oshiriladi. Agar $f_1(x)$ yoki $g_1(x)-$ ko'phad nolinchi darajali bo'lsa, u holda $\varphi(x)$ va $\psi(x)$ ko'phadlar osongina tanlanadi: masalan, $g_1(x)=a\neq 0$ bo'lganda $\varphi(x)=0, \psi(x)=\dfrac{1}{a}$ deb olish mumkin. Agar $f_1(x)$ va $g_1(x)$ larning darajalari musbat bo'lsa, u holda $\varphi(x)$ va $\psi(x)$ ko'phadlarni aniqmas koeffisiyentlar orqali ifodalanadi (bu holda $\varphi(x)$ ning darajasi $g_1(x)$ darajasidan, $\psi(x)$ ning darajasi esa $f_1(x)$ ning darajasidan kichik hisoblanadi) va (4) tenglikning chap va o'ng tarafidagi x ning bir xil darajalari oldidagi koeffisiyentlar tenglashtiriladi (yoki x o'zgaruvchiga har xil qiymatlar berilib, $f_1(x)\varphi(x)+g_1(x)\psi(x)$); ko'phadning qiymati 1 ga tenglashtiriladi, natijada $\varphi(x)$ va $\psi(x)$ ko'phadlarning koeffisiyentlariga nisbatan chiziqli tenglamalar sistemasi hosil bo'ladi). Agar (4) tenglikning ikkala tomonini $(f(x), g(x))$ ga ko'paytirsak, u holda (3) tenglikni hosil qilamiz ((4) tenglikdagi $\varphi(x)$ va $\psi(x)$ ko'phadliar bilan). $\varphi(x)$ va $\psi(x)$ larni bunday topishda $\varphi(x)$ ning darajasi $g(x)$ va

$(f(x),\ g(x))$ lar darajalari ayirmasidan kichik, $\psi(x)$ ning darajasi esa $f(x)$ va ($f(x),\ g(x))$ lar darajalari ayirmasidan kichik bo'ladi. (3) tenglikka kirgan $\varphi(x)$ va $\psi(x)$ ko'phadlarning darajalariga qo'yilgan bunday shartlarda ular bir qiymatli ravishda aniqlanadi.

5 - m i s o l . Aniqmas koeffisiyentlar usuli bilan

$f(x)=3x^5-4x^4-3x^3-3x^2+4x-1,\ \ g(x)=3x^5+5x^4+x^3-x^2-3x+1.$

ko'phadlar uchun $Q[x]$ xalqada

$f(x)\varphi(x)+g(x)\psi(x)=(f(x),g(x))$

tenglikni qanoatlantiradigan $\varphi(x)$ va $\psi(x)$ ko'phadlar topilsin.

Yechish. Yevklid algoritmidan foydalanib,

$$(f(x),g(x))=3x^3+2x^2+2x-1.$$

ekanligini topamiz.

U holda $f_1(x)=x^2-2x+1,\quad g_1(x)=x^2+x-1.$

$f_1(x)$ va $g_1(x)$ ko'phadlar uchun (4) tenglikni aniqmas koeffisiyentli $\varphi(x)$ va $\psi(x)$ ko'phadlar orqali yozamiz:

$$1=(x^2-2x+1)(ax+b)+(x^2+x-1)(a_1x+b_1).$$

x ning bir xil darajalari oldidagi koeffisiyentlarni tenglashtirib quyidagi tenglamalar sistemani hosil qilamiz:

$$\begin{cases} a+a_1=0\\ b-2a+b_1+a_1=0\\ -2b+a+b_1-a_1=0\\ b-b_1=0.\end{cases}$$

Bu sistemani yechib: $a=3,\quad a_1=-3,\quad b=5,\quad b_1=4,$ larni topamiz, demak, $(f(x),g(x))=f(x)(3x+5)+g(x)(-3x+4).$ ■

6 - M i s o l. Aniqmas koeffisiyentlar usuli bilan $Q[x]$ xalqada shunday $\varphi(x)$ va $\psi(x)$ ko'phadlar topilsinki,

$$f(x)\varphi(x)+g(x)\psi(x)=(f(x),g(x))$$

tenglik o'rinli bo'lsin, bu yerda

$f(x)=2x^4-4x^3-3x^2+7x-2,\ \ g(x)=6x^3+4x^2-5x+1.$

Yechish. Yevklid algoritmidan foydalanib, $(f(x),g(x))=2x^2+2x-1.$ ekanligini topamiz. U holda $f_1(x)=x^2-3x+2,\quad g_1(x)=3x-1.$

(4) tenglikni quyidagicha tuzib olamiz:

$$1=(x^2-3x+2)a+(3x-1)(bx+c).$$

x ga ketma-ket 0, 1, 2 qiymatlarni berib, Ushbu tenglamalar sistemasini hosil qilamiz:

$$\begin{cases} 2a - c = 1 \\ 2b + 2c = 1 \\ 10b + 5c = 1. \end{cases}$$

Bu sistemani yechib: $a = 0{,}9$; $a_1 = -0{,}3$; $c = 0{,}8$ larni topamiz, demak $(f(x), g(x)) = f(x) \cdot 0{,}9 + g(x)(-0{,}3x + 0{,}8)$. ■

$(f(x), g(x))$ dan farqli bo'lgan $h(x)$, ko'phadni

$$h(x) = f(x)\varphi_1(x) + g(x)\psi_1(x), \qquad (5)$$

ko'rinishda tasvirlash uchun $h(x)$ning $(f(x), g(x))$ga qoldiqsiz bo'linishi zarur va yetarlidir, bu yerda $\varphi_1(x), \psi_1(x) -$lar qandaydir ko'phadlar. Agar bu holda $h(x)$ ning darajasi $f(x)$ va $g(x)$ larning darajalari yig'indisidan kichik bo'lsa, unda $\varphi_1(x)$ и $\psi_1(x)$ ko'phadlarni shunday tanlash mumkinki, $\varphi_1(x)$ ning darajasi $g(x)$, ning darajasidan kichik, $\psi_1(x) -$ niki esa $f(x)$ ning darajasidan kichik bo'ladi. Bu holda $\varphi_1(x)$ и $\psi_1(x)$ larni topish uchun, ularni aniqmas koeffisiyentlar orqali ifodalab, (5) tenglikning chap va o'ng tarafidagi x ning bir xil darajalari oldidagi koeffisiyentlar tenglashtiriladi ($\varphi_1(x)$ и $\psi_1(x)$ lar yagona ravishda topilmasligi ham mumkin). Agar $h(x)$ ning darajasi $f(x)$ va $g(x)$ lar darajalari yig'indisidan katta yoki teng bo'lsa va $h(x)$=$(f(x), g(x))$ $m(x)$, bo'lsa, u holda dastlab $(f(x), g(x))$ ni (3) ko'rinishda tasvirlab, so'ngra $\varphi_1(x) = \varphi(x)m(x)$, $\psi_1(x) = \psi(x)m(x)$. deb olish kerak.

7 - M i s o l. $\boldsymbol{Q}[x]$ xalqada eng kichik darajali shunday $\varphi(x)$ va $\psi(x)$ ko'phadlar topilsinki,

$$(x^3 - x^2 - x + 1)\varphi(x) + (x^2 + x - 2)\psi(x) = x^4 - 1.$$

tenglik o'rinli bo'lsin.

Yechish. Yevklid algoritmidan foydalanib, $(f(x), g(x)) = x - 1$. ni topamiz. U holda $x^4 - 1 = (f(x), g(x)) \cdot m(x)$, bo'ladi, bu yerda $m(x) = x^3 + x^2 + x + 1$. $h(x) = x^4 - 1$ ning darajasi $f(x)$ va $g(x)$ lar darajalari yig'indisidan kichik bo'lganligi uchun quyidagi tenglikni yozib olamiz

$$x^4 - 1 = (x^3 - x^2 - x + 1)(ax + b) + (x^2 + x - 2)(a_1x^2 + b_1x + c_1).$$

x ning bir xil darajalari oldidagi koeffisiyentlarni tenglashtirib,

$$\begin{cases} a + a_1 = 1 \\ -a + b + a_1 + b_1 = 0 \\ -a - b - 2a_1 + b_1 + c_1 = 0 \\ a - b - 2b_1 + c_1 = 0 \\ b - 2c_1 = -1. \end{cases}$$

sistemani hosil qilamiz. Bu sistema cheksiz ko'p yechimga ega. Ulardan ixtiyoriylari (5) tenglikni qanoatlantiruvchi $\varphi_1(x)$ и $\psi_1(x)$ ko'phadlarning koeffisiyentlarini beradi. Masalan,

$$\varphi_1(x) = x + \frac{1}{3}, \quad \psi_1(x) = \frac{2}{3}(x+1).$$ deb olish mumkin■

$f(x)$ va $g(x)$ ko'phadlar EKUBini (3) ko'rinishda tasvirlash quyidagi masalani yechishda qo'llaniladi.

$\frac{m(\alpha)}{g(\alpha)}$, ko'rinishdagi ifoda berligan bo'lsin, bu yerda $m(x)$ va $g(x)-$ lar rasional koeffisiyentli ko'phadlar, $\alpha-$ esa $g(x)$ bilan o'zaro tub bo'lgan rasional koeffisiyentli $f(x)$ ko'phadning ildizi. $\frac{m(\alpha)}{g(\alpha)} = N(\alpha)$. tenglikni qanoatlantiradigan rasional koeffisiyentli $N(x)$ ko'phadni topish talab qilinadi. Bu masala kasr maxrajidagi irrasionallikni yo'qotish masalasi ham deyiladi.

Bu masalani yechish uchun $(f(x), g(x)) = 1$ tenglikni (3) ko'rinishda tasvirlab olamiz va $N(x)$ sifatida $m(x)v(x)$ ko'phadni olamiz (yoki bu ko'phadni $f(x)$ ga bo'lganda hosil bo'ladigan qoldiqni olamiz).

8 - M i s o l. Quyidagi ifodaning maxrajidagi irrasionallik yo'qotilsin

$$\frac{10\sqrt[3]{2} - 10}{\sqrt[3]{4} - 2\sqrt[3]{2} + 2}.$$

Yechish. Bu yerda

$$\alpha = \sqrt[3]{2}, \quad m(x) = 10x - 10, \quad g(x) = x^2 - 2x + 2, \quad f(x) = x^3 - 2.$$

Yevklid algoritmidan foydalanib $(f(x), g(x)) = 1$. ni topamiz. So'ngra EKUB ning chiziqli ifodasini topamiz:

$$1 = f(x)(-0{,}1x - 0{,}1) + g(x)(0{,}1x^2 + 0{,}3x + 0{,}4), \text{ ya'ni, } \psi(x) = 0{,}1x^2 + 0{,}3x + 0{,}4.$$

Bu yerdan $m(x)\psi(x) = x^3 + 2x^2 + x - 4 = (x^3 - 2)\cdot 1 + 2x^2 + x - 2.$

$N(x) = 2x^2 + x - 2$ deb olamiz va $\frac{10\sqrt[3]{2} - 10}{\sqrt[3]{4} - 2\sqrt[3]{2} + 2} = 2\sqrt[3]{4} + \sqrt[3]{2} - 2.$ ni hosil qilamiz.■

M A SH Q L A R

1. $\boldsymbol{R}[x]$ xalqada

a) $x^4 - 4x^3 + 5x^2 + x - 1$ ko'phadni $x^2 - 2x - 3$ ko'phadga;

b) $5x^4 - x^2 + 6$ ko'phadni $x^2 + 3x + 2$; ko'phadga;

c) $2x^2 - 3x + 1$ *ni* ko'phadni $x^3 + 4$; ko'phadga

d) $2x^4 - 3x^3 + 4x^2 - 5x + 6$ на ko'phadni $x^2 - 3x + 1$. ko'phadga bo'lishdan hosil bo'lgan $q(x)$ bo'linmani va $r(x)$ qoldiqni toping:

2. a, p va q larning qanday qiymatlarida $\boldsymbol{Q}[x]$ xalqada quyidagi ko'phadlar $x^2 + ax + 1$ ko'phadga qolddiqsiz bo'linadi:

a) $x^4 + q$; b) $x^4 - 21x + q$; c) $x^4 + px + q$; d) $x^4 - 7x^2 + q$.

3. $\boldsymbol{Z_3}[x], \boldsymbol{Z_5}[x]$ va $\boldsymbol{Q}[x]$ xalqalarda

a) x^5+x^2-x-1 *ni* ko'phadni x^3-2x+1; ko'phadga;

b) $2x^4+x^2+2x$ *ni* ko'phadni x^2-2. ko'phadga bo'lishdan hosil bo'lgan $q(x)$ bo'linmani va $r(x)$ qoldiqni toping

4. $(f(x),g(x))=(\alpha f(x),\beta g(x)),\ 0\neq\alpha,\beta\in P$. tenglikni isbotlang.

5. $\boldsymbol{R}[x]$ xalqada $f(x)$ ва $g(x)$ ko'phadlarning EKUBi va EKUK ini toping:

a) $f(x)=x^4+x^3-3x^2-4x-1,\quad g(x)=x^3+x^2-x-1;$

b) $f(x)=x^5+x^4-x^3-2x-1,\quad g(x)=3x^4+2x^3+x^2+2x-2;$

c) $f(x)=x^6-7x^4+8x^3-7x+7,\quad g(x)=3x^5-7x^3+3x^2-7;$

d) $f(x)=x^4-4x^3+1,\quad g(x)=x^3-3x^2+1;$

e) $f(x)=x^4+2x^3+2x+2,\quad g(x)=x^3+3x+2.$

6. Yevklid algoritmidan foydalanib, $\boldsymbol{Q}[x]$ xalqada $f(x)$ va $g(x)$ ko'phadlar uchun shunday $\varphi(x)$ va $\psi(x)$, ko'phadlarni topingki,

$$(f(x),g(x))=f(x)\,\varphi(x)+g(x)\,\psi(x):$$ tenglik o'rinli bo'lsin.

a) $f(x)=x^4+2x^3-x^2-4x-2,\quad g(x)=x^4+x^3-x^2-2x-2;$

b) $f(x)=x^5+3x^4+x^3+x^2+3x+1,\quad g(x)=x^4+2x^3+x+2;$

c) $f(x)=4x^4-2x^3-16x^2+5x+9,\quad g(x)=2x^3-x^2-5x+4;$

d) $f(x)=3x^3-2x^2+x+2,\quad g(x)=x^2-x+1;$

e) $f(x)=x^4-x^3-4x^2+4x+1,\quad g(x)=x^2-x-1;$

f) $f(x)=x^5-5x^4-2x^3+12x^2-2x+12,\quad g(x)=x^3-5x^2-3x+17.$

7. Agar $f(x)$ va $g(x)-$ lar mos ravishda n ва m, darajali o'zaro tub ko'phadlar bo'lsa, u holda $f(x)\,\varphi(x)+g(x)\,\psi(x)=1$, tenglikni qanoatlantiradigan shunday darajasi $m-1$ dan katta bo'lmagan $\varphi(x)$ va darajasi $n-1$, dan katta bo'lmagan $\psi(x)$ faqat bir juft ko'phadlarni topish mumkinligini isbotlang.

8. n- darajali $f(x)$ va m - darajali $g(x)$ ko'phadlarning EKUBi k - darajali $d(x)-$ ko'phad bo'lsin. Darajasi $m-k-1$ dan oshmaydgan $\varphi(x)$ va darajasi $n-k-1$, dan oshmaydigan $\psi(x)$ ko'phadlarni shunday tanlab olish mumkinki (yagona ravishda),

$$f(x)\,\varphi(x)+g(x)\,\psi(x)=d(x).$$

tenglik o'rinli bo'lishini isbotlang.

9. Aniqmas koeffisiyentlar usuli bilan $\boldsymbol{Q}[x]$ xalqada shunday $\varphi(x)$ va $\psi(x)$ ko'phadlarni tanlab olingki, $f(x)\,\varphi(x)+g(x)\,\psi(x)=1:$ tenglik o'rinli bo'lsin:

a) $f(x)=x^3,\qquad g(x)=(1-x)^2;$

b) $f(x)=x^4,\qquad g(x)=x^3-3x^2+4;$

c) $f(x)=x^3+3x+3,\quad g(x)=x^2-x-2;$

d) $f(x)=x^4-4x^3+1,\quad g(x)=x^3-3x^2+1.$

10. $\boldsymbol{Q}[x]$ xalqada

a) $(x-1)^2$ ga bo'lganda 1 qoldiq qoladigan va na $(x+1)^2$; ga bo'lganda 5 qoldiq qoladigan;

b) $(x-1)^2$ ga bo'lganda $2x$ va $(x-2)^3$;ga bo'lganda esa $3x$ qoldiq qoladigan;

c) $(x-1)^2$ ga bo'lganda $1-2x$ va $(x+1)^2$. ga bo'lganda $1+2x$ qoldiq qoladigan eng kichik darajali ko'phadni toping.

11. Quyidagi kasrlarning maxrajidagi irrasionallikni yo'qoting:

a) $\dfrac{1}{1+\sqrt[3]{2}+2\sqrt[3]{4}}$; b) $\dfrac{7}{1-\sqrt[4]{2}+\sqrt{2}}$; c) $\dfrac{1}{\sqrt[3]{2}+\sqrt{2}-1}$.

12. Eng kichik darajali $\varphi(x)$ ва $\psi(x)$ ko'phadlarni shunday tanlab olingki, quyidagi tengliklar o'rinli bo'lsin:

a) $(x^4-2x^3-4x^2+6x+1)\varphi(x)+(x^3-5x-3)\psi(x)=x^4$;

b) $(x^4+2x^3+x+1)\varphi(x)+(x^4+x^3-2x^2+2x-1)\psi(x)=x^3-2x.$

13 *. $x^m\,\varphi(x)+(1-x)^n\,\psi(x)=1.$tenglikni qanoatlantiradigan $\varphi(x)$ ва $\psi(x)$ ko'phadlarni toping.

14. $\boldsymbol{Z_3}\,[x], \boldsymbol{Z_5}\,[x]$ va $\boldsymbol{Q}\,[x]$ xalqalarda $f(x)$ va $g(x)$ ko'phadlarning EKUBi va EKUKlarini toping:

a) $f(x)=x^3+x^2+2x+2,\qquad g(x)=x^2+x+1;$

b) $f(x)=x^4+x^3-x^2+1,\qquad g(x)=x^3-2x-1;$

c) $f(x)=x^4-2x^2+2x-2,\qquad g(x)=x^3-x^2+1.$

15. $\boldsymbol{Z_2}\,[x]$ xalqada $f(x)$ va $g(x)$ ko'phadlarning EKUBini va $(f(x)g(x))=f(x)\,\varphi(x)+g(x)\,\psi(x)$ tenglikni qanoatlantiradigan $\varphi(x)$ ва $\psi(x)$ ko'phadlarni toping:

a) $f(x)=x^5+x^4+1,\qquad g(x)=x^4+x^2+1;$

b) $f(x)=x^5+x^3+x+1,\qquad g(x)=x^4+1;$

s) $f(x)=x^5+x+1,\qquad g(x)=x^4+x^3+1;$

d) $f(x)=x^5+x^3+x,\qquad g(x)=x^4+x+1.$

§ 2. Ko'phadlarning ildizlari

$f(x)=a_0x^n+a_1x^{n-1}+\cdots+a_{n-1}x+a_n\in P[x], \alpha\in P.$ bo'lsin. $f(x)$ *ko'phadning* $x=\alpha$ dagi qiymati deb, P maydonning $a_0\alpha^n+a_1\alpha^{n-1}+\cdots+a_{n-1}\alpha+a_n$ elementiga aytiladi va $f(\alpha)$. bilan belgilanadi.

$f(x)$ ko'phadni $x-\alpha$ ga bo'lgandagi qoldiqn $f(\alpha)$ ga teng (*Bezu teorema*).

Agar

$$f(x)=(x-\alpha)g(x)+f(\alpha),\quad g(x)=b_0x^{n-1}+b_1x^{n-2}+\cdots+b_{n-2}x+b_{n-1},$$

bo'lsa, $g(x)$ ko'phadning koeffisiyentlarini va $f(\alpha)$ning qiymatini Gorner sxemasi yordamida topish qulay bo'ladi:

	a_0	a_1	$\dots$	a_{n-1}	a_n
α	b_0	b_1	$\dots$	b_{n-1}	$f(\alpha)$

Bu yerda
$b_0 = a_0, \quad b_1 = \alpha b_1 + a_1, \dots, b_k = \alpha\, b_{k-1} + a_k, \dots, b_{n-1} = \alpha\, b_{n-2} + a_{n-1},\ f(\alpha) = \alpha\, b_{n-1} + a_n.$

Agar $f(\alpha)=0$ bo'lsa, $\alpha \in P$ element $f(x) \in P[x]$, *ko'phadning ildizi* deyiladi. Bezu teoremasidan $x-\alpha$ chiziqli ko'phad $f(x)$ ko'phadning bo'luvchisi bo'lganda va faqat shu holdagina α soni $f(x)$ ko'phadning ildizi ekanligi kelib chiqadi.

Agar $(x-\alpha)^k$, $k \in N$, ko'phad $f(x)$, ning bo'luvchisi bo'lsa, lekin $(x-\alpha)^{k+1}$ ko'phad esa $f(x)$ ning bo'luvchisi bo'lmasa, u holda α soni $f(x)$ ko'phadning *k-karrali ildizi deyiladi.* $k=1$ bo'lgan holda ildiz *sodda ildiz* deyiladi.

$f(x) = a, a \in P$ ko'phad o'zgarmas ko'phad deyiladi. Har qanday o'zgarmas bo'lmagan $f(x) \in P[x]$ ko'phad P, maydonning shunday kengaytmasi mavjudki, bu kengaytmada $f(x)$ ko'phadning barcha ildizlari yotadi, boshqacha qilib aytganda bu kengaytmada $f(x)$ ko'phad chiziqli ko'paytuvchilarga ajraladi, ya'ni $f(x) = a_0(x-\alpha_1)(x-\alpha_2)\cdots(x-\alpha_n)$, bu yerda $a_0 -$ $f(x)$ ko'phadning bosh koeffisiyenti.

$f(x) = a_0x^n + a_1x^{n-1} + \dots + a_n$, $a_0 \neq 0$, ko'phadning ildizlari $\alpha_1, \alpha_2, \dots, \alpha_n$ uning koeffisiyentlari bilan quyidagi *Viyet formulalari orqali bog'langan:*

$$-\frac{a_1}{a_0} = \alpha_1 + \alpha_2 + \cdots + \alpha_n = \sum_{i=1}^{n} \alpha_i,$$

$$\frac{a_2}{a_0} = \alpha_1\alpha_2 + \alpha_1\alpha_3 + \cdots + \alpha_1\alpha_n + \alpha_2\alpha_3 + \cdots + \alpha_{n-1}\alpha_n = \sum_{i_1<i_2}^{n} \alpha_{i_1}\,\alpha_{i_2},$$

$$\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots$$

$$\frac{a_k}{a_0} = (-1)^k \sum_{i_1<i_2<\cdots<i_k} \alpha_{i_1}\,\alpha_{i_2}\cdots\alpha_{i_k},$$

$$-\frac{a_n}{a_0} = (-1)^n\,\alpha_1\,\alpha_2\cdots\alpha_n.$$

$f(x) = a_0x^n + a_1x^{n-1} + \cdots + a_{n-1}x + a_n \in P[x]$, $a_0 \neq 0$, $n \geq 1$ - ko'phadning hosilasi deb,

$f'(x) = na_0x^{n-1} + (n-1)a_1x^{n-2} + \cdots + 2a_{n-2}x + a_{n-1}$ ko'phadga aytiladi.

Birinchi hosiladan yana bir marta olingan hosila $f(x)$ ko'phadning ikkinchi tartibli hosilasi deyiladi va $f''(x)$ orqali belgilanadi. O'zgarmas ko'phadning hosilasi ta'rifga binoan nol ko'phadga teng deb hisoblanadi. *Agar* $f(x) \in P[x]$ *ko'phad o'zining birinchi tartibli hosilasi bilan o'zaro tub bo'lsa, u* P, *maydonning o'zida ham va uning har qanday kengaytmasida ham ildizga ega bo'lmaydi.*

Nol xarakteristikali maydon ustidagi ko'phadlar uchun quyidagi tasdiq o'rinli: ko'phadning k–karrali ildizi uning hosilasining $(k-1)-$karrali ildizi bo'ladi.

$f(x)=a_0(x-\alpha_1)^{k_1}(x-\alpha_2)^{k_2}\cdots(x-\alpha_s)^{k_s}\in P[x],$ va $i\neq j,$ $k_i>0.$ bo'lganda $char\,P=0,\ \alpha_i\neq\alpha_j$ bo'lsin. U holda

$$(f(x),f'(x))=(x-\alpha_1)^{k_1-1}(x-\alpha_2)^{k_2-1}\cdots(x-\alpha_s)^{k_s-1}$$

va $f_1(x)=\dfrac{f(x)}{(f(x),f'(x))}=a_0(x-\alpha_1)(x-\alpha_2)\cdots(x-\alpha_s)$ ko'phadlar $f(x)$ ko'phad bilan bir xil ildizlarga ega bo'ladi va bu ildizlarning barchasi sodda ildizlardan iborat bo'ladi.

Agar $\alpha_1,\alpha_2,\cdots,\alpha_n$ lar $P,$ maydonning har xil elementlari bo'lsa, $\beta_1,\beta_2,\cdots,\beta_n-$ lar esa $P,$ maydonning ixtiyoriy elementlari bo'lsa, u holda $P[x]$ xalqada $f(\alpha_i)=\beta_i,$ $i=\overline{0,n}.$ tenglikni qanoatlantiradigan darajasi n dan oshmaydigan faqat va faqat bitta $f(x)$ ko'phad mavjud bo'ladi. Bu ko'phad quyidagi formula orqali beriladi:

$$f(x)=\sum_{i=0}^{n}\beta_i\frac{(x-\alpha_0)\cdots(x-\alpha_{i-1})(x-\alpha_{i+1})\cdots(x-\alpha_n)}{(\alpha_i-\alpha_0)\cdots(\alpha_i-\alpha_{i-1})(\alpha_i-\alpha_{i+1})\cdots(\alpha_1-\alpha_n)}.\qquad (*)$$

(*) formula Lagranjning interpolyasion formulasi deb yuritiladi.

$f(x)$ ko'phadni (talab qilingan xossalari bilan) *Nyutonnig interpolyasion formulasi* orqali ham hosil qilish mumkin:

$$f(x)=\lambda_0+\lambda_1(x-\alpha_0)+\lambda_2(x-\alpha_0)(x-\alpha_1)+\cdots$$
$$\cdots+\lambda_n(x-\alpha_0)(x-\alpha_1)(x-\alpha_2)\cdots(x-\alpha_{n-1}),$$

bu yerda $\lambda_0,\lambda_1,\ldots,\lambda_n$ koeffisiyentlar x ning o'rniga ketma-ket $\alpha_1,\alpha_2,\cdots,\alpha_n$ qiymatlarni quyish bilan aniqlanadi.

1 - m i s o l. $C\,[x]$ xalqada $f(x)=x^4+(2+i)x^2+(1+2i)x-i$ ko'phadni $x+i.$ ga bo'lganda hosil bo'ladigan $q(x)$ bo'linma va $r(x)$ qoldiqni toping.

Yechish. Gorner sxemasini tuzamiz:

	1	0	$2+i$	$1+2i$	$-i$
$-i$	1	$-i$	$1+i$	$2+i$	$1-3i$

Demak, $q(x)=x^3-ix^2+(1+i)x+2+i,\quad r(x)=1-3i$. ■

2 - M i s o l. Gorner sxemasidan foydalanib, $f(x)=x^4-2x^3+x^2+x+1.$ ko'phad uchun $f(-3),$ ni hisoblang.

Yechish. Gorner sxemasini tuzamiz:

	1	-2	1	1	1
-3	1	-5	16	-47	42

Demak, $f(-3)=42.$■

Har qanday $f(x)=a_0x^4+a_1x^{n-1}+\cdots+a_{n-1}x+a_n$ ko'phad va ixtiyoriy α son uchun $f(x)$ ning $x-\alpha$: ayirmaning darajalari orqali yoyilmasini yozish mumkin:

$$f(x)=b_0(x-\alpha)^n+b_1(x-\alpha)^{n-1}+\cdots+b_{n-1}(x-\alpha)+b_n.$$

Bu yoyilmaning koeffisiyentalirini topish uchun dastlab $f(x)$ ni $x-\alpha.$ga qoldiqli bo'lish kerak. Natijada qoldiqda $b_n,$ hosil bo'ladi, bo'linma esa qandaydir

$q(x)$. ko'phad bo'ladi. So'ngra $q(x)$ bo'linma $x-\alpha$; ga bo'linadi, qoldiqda b_{n-1}, bo'linma esa- $q_1(x)$. bo'ladi. Keyin $q_1(x)$ ni $x-\alpha$, ga bo'lamiz va b_{n-2} qoldiqni hosil qilamiz va hakoza shu kabi davom etib yoyilmaning barcha koeffisiyentlarini hosil qilamiz.

Bu yoyilmaning koefffisiyentlarini Gorner sxemasi yordamida hisoblash juda qulay bo'lib, u barcha hisoblashlarni bita jadvalga birlashtiradi..

3-m i s o l. $f(x)=x^4+2x^3-x-1$ ko'phad $x+2$. ning darajalari bo'yicha yoying.

Yechish. Gorner sxemasini tuzamiz, uning birinchi satriga $f(x)$ ko'phadning koeffisiyentlarini yozib chiqamiz. Ikkinchi satrida $f(x)$ ni $x+2$, ga bo'linganda hosil bo'ladigan $q(x)$ bo'linmaninng koeffisiyentlarini va b_4 qoldiqni yozamiz, uchinchi satriga $q(x)$ ni $x+2$ ga bo'lingandagi $q_1(x)$ bo'linmaning koeffisiyentlarini va b_3 qoldiqni yozamiz va hakoza shu yo'sinda davom etamiz :

	1	2	0	-1	-1
-2	1	0	0	-1	1
-2	1	-2	4	-9	
-2	1	-4	12		
-2	1	-6			
-2	1	.			

Demak, $f(x)=(x+2)^4-6(x+2)^3+12(x+2)^2-9(x+2)+1$. ■

4-m i s o l. Agar $f(x)=x^4-5x^3-3x^2+9$. bo'lsa, Gorner sxemasi yordamida $f(x+3)$, ko'phadni x ning darajalari bo'yicha yoying.

Yechish. $f(x+3)$ ko'phadni $x$ning darajalari bo'yicha yoyish uchun dastlab $f(x)$ni $x-3$, ning darajalari bo'yicha yoyamiz, so'ngra bu yoyilmada x ni $x+3$. ga almashtiramiz.

Gorner sxemasini tuzamiz:

	1	-5	-3	0	9
3	1	-2	-9	-27	-72
3	1	1	-6	-45	
3	1	4	6		
3	1	7			
3	1				

Demak, $f(x)=(x-3)^4+7(x-3)^3+6(x-3)^2-45(x-3)-72$.

Bu yerdan esa $f(x-3)=x^4+7x^3+6x^2-45x-72$. ■

5-m i s o l. Gorner sxemasi yordamida $f(x)=x^4-7x^3+9x^2+8x+16$. ko'phadning 4 ga teng bo'lgan ildizining necha karrali ekanligini aniqlang.

Yechish. α soni $f(x)$ ko'phadning necha karrali ildizi bo'lishini tekshirish uchun Gorner sxemasidan quyidagicha foydalanish mumkin. Avval $f(x)$ ko'phad

$x-\alpha$, ga bo'linadi, agar qoldiq nolga teng bo'lsa, hosil qilingan bo'linma yana $x-\alpha$ ga bo'linadi va hakoza bu jarayon qoldiq noldan farqli bo'lguncha davom ettiriladi.

Shunga asosan Gorner sxemasini tuzamiz:

	1	-7	9	8	16
4	1	-3	-3	-4	0
4	1	1	1	0	
4	1	5	21	.	

Demak, $\alpha = 4-$ soni berilgan ko'phadning ikki karrali ildizi ekan. ■

Ko'phadni $x-\alpha$ ning darajalari bo'yicha yoyilmasi maxraji chiziqli ikki hadning darajasidan iborat bo'lgan rasional kasrni sodda kasrlarga yoyishda ham ishlatilishi mumkin.

6-m i s o l. Gorner sxemasidan foydalanib $\dfrac{x^3+x-1}{(x+2)^5}$. kasrni sodda kasrlarga yoying.

Yechish. $f(x) = x^3 + x - 1$ ko'phadni $x-(-2) = x+2$: ayirmaning darajalari bo'yicha yoyamiz:

	1	0	1	-1
-2	1	-2	5	-11
-2	1	-4	13	
-2	1	-6		
-2	1	.		

Demak, $f(x) = (x+2)^3 - 6(x+2) + 13(x+2) - 11.$

Bu yerdan izlangan yoyilmani hosil qilamiz

$$\frac{x^3+x-1}{(x+2)^2} = \frac{1}{(x+2)^2} - \frac{6}{(x+2)^3} + \frac{13}{(x+2)^4} - \frac{11}{(x+2)^5}. \ ■$$

$f(x) = b_0(x-\alpha)^n + b_1(x-\alpha)^{n-1} + \cdots + b_{n-1}(x-\alpha) + b_n$ ko'phadning

$x-\alpha$ ning darajalari bo'yicha yoyilmasi orqali bu ko'phadning ixtiyoriy tartibli hosilalarining α dagi qiymatlarini topish mumkin, ya'ni:

$f^{(k)}(\alpha) = k! \quad b_{n-k}.$ (* *)

7-m i s o l. $f(x) = x^4 + 2x^3 - x - 1$ ko'phad hosilalarining $x=-2.$dagi qiymatlarini hisoblang.

Yechish. 3-misoldagi Gorner sxemasi $f(x) = x^4 + 2x^3 - x - 1$ ko'phad hosilalarining $x=-2$: qiymatlarini hisoblashga imkon beradi:

$f'(-2) = -9, \quad f''(-2) = 12 \cdot 2! = 24,$

$f'''(-2) = -6 \cdot 3! = -36, \quad f^{('''')}(-2) = 1 \cdot 4! = 24$. ■

(* *) munosabatdan $f(x)$ ko'phadning $x-\alpha$: ning darajalari bo'yicha yoyish uchun *Teylor formulasi* kelib chiqadi:

$$f(x)=f(\alpha)+\frac{f'(\alpha)}{1!}(x-\alpha)+\frac{f''(\alpha)}{2!}(x-\alpha)^2+\cdots+\frac{f^{(n)}(\alpha)}{n!}(x-\alpha)^n.$$

8-m i s o l. $f(x)$ ko'phad $x-2$ va $x-3$ larga bo'linganda mos ravishda 5 va 7 qoldiqlar hosil bo'ladi. $f(x)$ ni $(x-2)(x-3)$ ga bo'lganda hosil bo'ladigan qoldiqni toping.

Yechish. $f(x)$ ni $(x-2)(x-3)$ larga bo'lganda $r(x)$ qoldiq $ax+b$, ko'rinishda bo'ladi, ya'ni $f(x)=(x-2)(x-3)q(x)+(ax+b)$.

$x=2$ va $x=3$ (2 va 3 $(x-2)$ va $(x-3)$ larning ildizlari) larniyuqoridagi tenglikka qo'yib a va b: larni topish uchun quyidagi chiziqli tenglamalar sistemasini hosil qilamiz:

$$\left.\begin{array}{l} f(2)=2a+b=5 \\ g(3)=3a+b=7 \end{array}\right\}$$

Bu sistemadan $a=2,\quad b=1$. Demak, $f(x)$ ni $(x-2)(x-3)$ ga bo'lganda hosil bo'ladigan qoldiq $2x+1$. ga teng. ■

9-m i s o l. Qoldiqli bo'lish algoritmini qo'llamasdan $f(x)=x^{128}+x^{64}+x^{32}+x^8+x^4+x^2+x+1$ ko'phadni x^2-1. ga bo'lganda hosil bo'ladigan qoldiqni toping.

Yechish. $f(x)$ ni x^2-1 ga bo'lganda hosil bo'ladigan $r(x)$ qoldiq $ax+b$. ko'rinishda bo'ladi. $x=1$ va $x=-1$ (1 va –1 x^2-1 ning ildizlari) larda $f(x)$ ko'phadning qiymatlarini hisoblab, a va b:larni topish uchun quyidagi tenglamalar sistemasini hosil qilamiz:

$$\left.\begin{array}{l} f(1)=8=a+b \\ f(-1)=6=-a+b \end{array}\right\}.$$

Bu sistemaning yechimlari $a=1,\quad b=7$. Demak, $f(x)$ ni x^2-1ga bo'lgandagi qoldiq $x+7$ ga teng. ■

10-m i s o l. Qanday p,q,r larda $f(x)=x^3+px^2+qx+r$ ko'phad $(x-1)^3$ ga $\boldsymbol{R}[x]$ xalqada qoldiqsiz bo'linadi.

Yechish. $f(x)$ ning $(x-1)^3$ ga qoldiqsiz bo'linishi $x=1$ning $f(x)$ ko'phadning 3-karrali ildizi ekanligiga teng kuchli. Demak, $x=1$ soni $f(x)$, $f'(x)$ va $f''(x)$. larning ildizlari bo'ladi. $f'(x)$ va $f''(x)$: $f'(x)=3x^2+2px+q,\quad f''(x)=6x+2p.$ $x=1$ ni $f(x),\ f'(x)$ ва $f''(x)$ larga qo'yib, p,q va $r$larni topish quyidagi uchta qiziq-li tenglamalar sistemasini hosil qilamiz:

$$\left.\begin{array}{l} f(1)=1+p+q+2=0 \\ f'(1)=3+2p+q=0 \\ f''(1)=6+2p=0 \end{array}\right\}.$$

Bu sistemadan: $p=-3,\quad q=3,\quad r=-1$. ■

11-m i s o l. 1 soni $f(x)=x^{2n}-nx^{n+1}+nx^{n-1}-1$. ko'phadning uch karrali ildizi ekanligini isbotlang.

Yechish. $x=1$ ning $f(x)$ ko'phadning uch karrali ildizi ekanligi $x=1$ ning $f(x),\ \ f'(x)$ ва $f''(x)$ ko'phadlarlarning ildizi ekanligiga teng kuchli.

$f'(x)$ va $f''(x)$: larni topamiz:

$f'(x)=2nx^{2n-1}-n(n+1)x^{n}+n(n-1)x^{n-2}$,

$f''(x)=2n(2n-1)x^{2n-1}-n^{2}(n+1)x^{n-1}+n(n-1)(n-2)x^{n-3}$.

U holda

$$f(1)=1-n+n-1=0,$$

$$f'(1)=2n-n(n+1)+n(n-1)=2n-n^{2}-n+n^{2}-n=0,$$

$$f''(1)=2n(2n-1)-n^{2}(n+1)+n(n-1)(n-2)=$$

$$=4n^{2}-2n--n^{3}-n^{2}+n^{3}-n^{2}-2n^{2}+2n=0.$$

Shunday qilib, 1 soni haqiqatdan ham $x^{2n}-nx^{n-1}+nx^{n-1}-1$ kKo'phadning uch karrali ildizi ekan. ■

12-m i s o l. Nayti kotorыy imeyet te je korni, chto i $f(x)=x^{6}-6x^{4}-4x^{3}+9x^{2}+12x+4,$ ko'phadning ildizlariga ega bo'ladigan, lekin karali ildizlarga ega bo'lmaydigan $\varphi(x),$ ko'phadni toping ($f(x)$), ko'phadning karali ildizlarini ajratingva $f(x)$ ko'phadni ***S*** maydon ustida chiziqli ko'paytuvchilarga ajrating).

Yechish. Berilgan ko'phadning hosilasini topamiz $f'(x)$:

$f'(x)=6x^{5}-24x^{3}-12x^{2}+18x+12$.

Yevklid algoritmidan foydalanib $f(x)$ va $f'(x)$ ko'phadlarning EKUBini topamiz:

$(f(x),f'(x))=x^{4}+x^{3}-3x^{2}-5x-2.$ Bu holda

$$\varphi(x)=\frac{f(x)}{(f(x),\ f'(x))}=x^{2}-x-2=(x+1)(x-2)$$

ko'phad $f(x)$ ko'phadning ildizlariga ega bo'ladi va $x_1=-1,\quad x_2=2$.

Gorner sxemasidan foydalanib $x_1=-1$ ildiz $f(x)$ ko'phadning 4 karrali ildizi ekanligini topamiz, $x_2=2$ ildiz esa $f(x)$ ning 2 karrali ildizi bo'ladi. Demak, $f(x)$ quyidagicha chiziqli ko'paytuvchilarga ajraladi: $f(x)=(x+1)^{4}(x-2)$. ■

13-m i s o l. a ning qiymatini shunday aniqlangki, $x^{3}-21x+a$ ko'phadning bir ildizi ikkinchisining ikkkilanganligiga teng bo'lsin.

Yechish. $\alpha_1,\alpha_2,\alpha_3-$ lar $x^{3}-21x+a.$ ko'phadning ildizlari bo'lsin.

$\alpha_1=2\alpha_2$. bo'lsin. U holda Viyet formulalariga asosan:

$$\left.\begin{array}{l}\alpha_1+\alpha_2+\alpha_3=0\\ \alpha_1\alpha_2+\alpha_1\alpha_3+\alpha_2\alpha_3=-21\\ \alpha_1\alpha_2\alpha_3=-a\end{array}\right\},\qquad \left.\begin{array}{l}2\alpha_2+\alpha_2+\alpha_3=0\\ 2\alpha_2^{2}+3\alpha_2\alpha_3=-21\\ 2\alpha_2^{2}\alpha_3=-a\end{array}\right\},$$

$$\left.\begin{array}{l}3\alpha_2+\alpha_3=0\\ 2\alpha_2+3\alpha_2\alpha_3=-21\\ 2\alpha_2^2\alpha_3=-a\end{array}\right\},\qquad \left.\begin{array}{l}\alpha_3=-3\alpha_2\\ 2\alpha_2^2-9\alpha_2^2=-21\\ -6\alpha_2^3=-a\end{array}\right\},\quad \alpha_2^2=3,\quad \alpha_2=\pm\sqrt{3}.$$

Bu yerdan $a=6\cdot(\pm3\sqrt{3})=\pm18\sqrt{3}.$

Shunday qilib, $a=\pm18\sqrt{3}$ bo'lganda $x^3-21x+a$ ko'phadning bir ildizi ikkinchisining ikkkilanganligiga teng ekan. ■

14-m i s o l. 1, -1, 3 ildizlarga ega bo'lgan bosh koeffisiyenti birga teng bo'lgan uchinchi darajali ko'phad quring.

Yechish. Izlanayotgan ko'phad quyidagicha bo'lsin:
$f(x)=x^3+a_1x^2+a_2x+a_3.$

U holda Viyet formulalariga asosan:

$$a_1=-(\alpha_1+\alpha_2+\alpha_3)=-(1-1+3)=-3$$

$$a_2=\alpha_1\alpha_2+\alpha_1\alpha_3+\alpha_2\alpha_3=1\cdot(-1)+1\cdot3+(-1)\cdot3=-1$$

$$a_3=-\alpha_1\alpha_2\alpha_3)=-1\cdot(-1)\cdot3=3.$$

Demak, izlanayotgan ko'phad $f(x)=x^3-3x^2-x+3$ dan iborat. ■

15-m i s o l. $f(x)=5x^4-3x^3+2x-1.$ ko'phadning kompleks ildizla-riga teskari bo'lgan sonlarning yig'indisini toping.

Yechish. $\alpha_1,\alpha_2,\alpha_3,\alpha_4-$ lar $f(x).$ ning ildizilari bo'lsin.

$\frac{1}{\alpha_1}+\frac{1}{\alpha_2}+\frac{1}{\alpha_3}+\frac{1}{\alpha_4}$ yig'inidini topish kerak. Bu holda

$$\frac{1}{\alpha_1}+\frac{1}{\alpha_2}+\frac{1}{\alpha_3}+\frac{1}{\alpha_4}==\frac{\alpha_2\alpha_3\alpha_4+\alpha_1\alpha_3\alpha_4+\alpha_1\alpha_2\alpha_4+\alpha_1\alpha_2\alpha_3}{\alpha_1\alpha_2\alpha_3\alpha_4}.$$

Viyet formulalariga asosan oxirgi kasrning maxraji $\left(-\frac{a_3}{a_0}\right)=-\frac{2}{5}$ ga teng, surati esa $\frac{a_4}{a_0}=-\frac{1}{5}.$ ga teng. Demak,

$$\frac{1}{\alpha_1}+\frac{1}{\alpha_2}+\frac{1}{\alpha_3}+\frac{1}{\alpha_4}=\left(-\frac{2}{5}\right):\left(-\frac{1}{5}\right)=2.\ ■$$

16-m i s o l. Lagranj interpolyasion formulsidan foydalanib qiymatlari quyidagi jadval bo'yicha berilgan ko'phadni tuzing:

x	1	3	4
$f(x)$	2	-2	-1

Yechish. Izlanayotgan ko'phadni ko'rinishi quyidagicha bo'ladi:
$f(x)=2\cdot\frac{(x-3)(x-4)}{(1-3)(1-4)}-2\frac{(x-1)(x-4)}{(3-1)(3-4)}-\frac{(x-1)(x-3)}{(4-1)(4-3)}=x^2-6x+7.$ ■

17-m i s o l. Nyuton interpolyasion formulasidan foydalanib qiymatlari quyidagi jadval bo'yicha berilgan eng kichik darajali ko'phadni tuzing:

x	1	-2	3

$f(x)$	4	7	12

Yechish. Nyuton formulasidan foydalanib:
$f(x)=4+\lambda_1(x-1)+\lambda_2(x-1)(x+2)$.

$x=-2$, deb olib $7=4+\lambda_1(-3)$, $\lambda_1=-1$ larga ega bo'lamiz. $x=3$, da esa $12=4-(3-1)+\lambda_2(3-1)(3+2)$, $\lambda_2=1$. Izlanayotgan ko'phad $f(x)=4-(x-1)+(x-1)(x+2)=x^2+3$ bo'ladi. ■

MAShQLAR

16. $f(x)\in \boldsymbol{Q}[x]$ ko'phadning koeffisiyentlari yig'indisini toping:

a) $f(x)=(2-5x+x^3)^{211}(3-7x+9x^2-5x^3)^{135}$;

b) $f(x)=(7-3x-3x^5)^{100}(5-x^2-5x^7)^{1000}$.

17. $\boldsymbol{C}[x]$ xalqada $q(x)$ bo'linma va $r(x)$ qoldiqni toping:

a) $x^4-2x^3+4x^2-6x+8$ на $x-1$;

b) $2x^5-5x^3-8x$ на $x+3$;

s) $4x^3+x^2$ на $x+1+i$;

d) x^3-x^2-x на $x-1+2i$.

18. Gorner sxemasidan foydalanib $f(\alpha)$, ni hisoblang:

a) $f(x)=x^4-3x^3+6x^2-10x+16$, $\alpha=4$;

b) $f(x)=5x^4-7x^3+8x^2-3x+7$, $\alpha=3$;

s) $f(x)=2x^5+2x^4-3x^3+4x^2-6x+5$, $\alpha=-\dfrac{1}{2}$;

d) $f(x)=x^5+(1+2i)x^4-(1+3i)x^2+7$, $\alpha=-2-i$;

ye) $f(x)=x^5+(1-2i)x^4-(3+i)x^2+7$, $\alpha=-1+2i$.

19. Gorner sxemasidan foydalanib $f(x)$ ko'phadni $x-\alpha$ ning darajalari bo'yicha yoying:

a) $f(x)=x^4+2x^3-3x^2-4x+1$, $\alpha=-1$;

b) $f(x)=x^5$, $\alpha=1$;

s) $f(x)=x^4-8x^3+24x^2-50x+90$, $\alpha=2$;

d) $f(x)=x^4+2ix^3-(1+i)x^2-3x+7+i$, $\alpha=-i$;

e) $f(x)=x^4+(3-8i)x^3-(21+18i)x^2-(33-20i)x+7+18i$, $\alpha=-1+2i$.

20. Gorner sxemasidan foydalanib x ning darajalari bo'yicha yoying*:*

a) $f(x+3)$, $f(x)=x^4-x^3+1$;

b) $f(x+2)$**,** $f(x)=2x^4-3x^3+5x^2+6x-1$**;**

s) $f(x)=(x-2)^4+4(x-2)^3+6(x-2)^2+10(x-2)+20$;

d) $f(x)=(x+3)^5-2(x+3)^3+3(x+3)^2+7(x+3)-8$.

21. Gorner sxemasidan foydalanib ildizning necha karrali ekanligini aniqlang:

a) $x^5-5x^4+7x^3-2x^2+4x-8$; ko'phad uchun 2 soni;

b) $x^5+7x^4+16x^3+8x^2-16x-16$; ko'phad uchun -2 soni;

c) $x^4-6x^3+10x^2-6x+9.$ ko'phad uchun 3 soni.

22. $f(x)$ ko'phad $x-1$ va $x-2$ larga bo'linganda qoldiqlar mos ravishda bir va ikkiga teng. $f(x)$ ni $(x-1)(x-2)$ ga bo'lgandagi qoldiqni toping.

23. $f(x)$ ko'phad $x+1,$ $x-1$ va $x+3$ Larga bo'linganida qoldiqlar mos ravishda 5, - 4 va 6 ga teng. $f(x)$ ni $(x^2-1)(x+3).$ ga bo'lgandagi qoldiqni toping.

24. $f(x)$ ko'phad $x-1$, $x-2$, $x-3$ va $x-4$ Larga bo'linganida qoldiqlar mos ravishda 1, 3, 5, va 6 ge teng. $f(x)$ ni $(x-1)(x-2)(x-3)(x-4).$ ga bo'lgandagi qoldiqni toping.

25. Qoldiqli bo'lish algoritmini qo'llamasdan $f(x)=x^{243}+x^{81}+x^{27}+x^9+x^3+x+1$ ko'phadni: a) $x^2-1;$ b) $x^2+1;$ c) $x^4-1.$ larga bo'lgandagi qoldiqni toping.

26. $f(x)\in P[x],$ $char\,P=0,$ ko'phadning α ildizi faqat va faqat $f'(\alpha)=...=f''(\alpha)=...=f^{(k-1)}(\alpha)=0,$ bo'lib, lekin $f^{(k)}(\alpha)\neq 0.$ bo'lgadagina k karrali bo'lishini isbotlang.

27. 1 soni quyidagi ko'phadlarningn uch karali ildizi ekanligini ko'rsating:

a) $x^{2n+1}-(2n+1)x^{n+1}+(2n+1)x^n-1;$

b) $(n-2m)x^n-nx^{n-m}+nx^m-(n-2m).$

28. $x^5+10ax^3+5bx+c$ ko'phadni noldan farqli uch karrali ildizga ega bo'lishi shartlarini toping.

29. a koeffisiyentni shunday aniqlangki, x^5-ax^2-ax+1 ko'phad uchun (-1) soni ikkidan kam bo'lmagan karralikga ega bo'lsin.

30. a va b larni shunday aniqlangki, ax^4+bx^3+1 ko'phad $\boldsymbol{R}[x].$ xalqada $(x-1)^2$ ga qoldiqsiz bo'linsin.

31. Qanday p,q,r larda quyidagi ko'phadlarning har biri $\boldsymbol{R}[x]$**:** xalqada $(x-1)^3$ ga qoldiqsiz bo'linadi:

a) $f(x)=x^4+px^3+qx^2+r;$

b) $f(x)=px^4+qx^2+rx+1?$

32. a ning qanday qiymatlarida $f(x)$ ko'phad karali bo'lgan 1ildizga ega bo'ladi va uning karraligi nechaga teng bo'ladi:

a) $f(x)=x^3+ax^2+3x-1;$

b) $f(x)=2x^3-x^2+ax+3;$

s) $f(x)=3x^4-6x^3+ax^2-2x+1?$

33.

$$f(x)=x^{2n+1}-\frac{n(n+1)(2n+1)}{6}x^{n+2}+\frac{(n-1)(n+2)(2n+1)}{2}x^{n+1}-$$
$$-\frac{(n-1)(n+2)(2n+1)}{2}x^n+\frac{n(n+1)(2n+1)}{6}x^{n-1}-1$$

ko'phadni $(x-1)^5$ ga bo'linishini, $(x-1)^6$ ga esa bo'linmasligini ko'rsating.

34*. $f(x)=a_0x^n+a_1x^{n-1}+...+a_n$ ko'phad $(x-1)^{k+1},$ ga bo'linishi uchun

$a_0 + a_1 + a_2 + \cdots + a_n = 0,$

$a_1 + 2a_2 + \cdots + na_n = 0,$

$a_1 + 4a_2 + \cdots + n^2 a_n = 0,$

....

$a_1 + 2^k a_2 + \cdots + n^k a_n = 0.$

shartning bajarilishi zarur va yetarli ekanligini ko'rsating.

35*. $x^n + ax^{n-m} + b$ ko'phad noldan farqli karraliligi ikkidan yuqori bo'lmagan ildizlarga ega bo'lmaslagini isbotlang.

36*. Qanday shartlarda $x^n + ax^{n-m} + b$ ko'phad noldan farqli ikki karali ildizga ega bo'ladi.

37*. $a_1 x^{p_1} + a_2 x^{p_2} + \cdots + a_k x^{p_k}$ $k-$hadli ko'phadni $(k-1)-$dan yuqori karali ildizlarga ega emasligini isbotlang.

38*. $a_1 x^{m_1} + a_2 x^{m_2} + \cdots + a_k x^{m_k}$ ko'phadning har bir noldan farqli $(k-1)-$karali ildizi

$a_1 x^{m_1} \varphi'(m_1) = a_2 x^{m_2} \varphi'(m_2) = \cdots = a_k x^{m_k} \varphi'(m_k),$

tenglamalarni qanoatlantirishini isbotlang, bu yerda $\varphi(t) = (t - m_1)(t - m_2)\cdots(t - m_k).$

39*. Ko'phad faqat va faqat u $a_0(x - x_0)^n$. ko'phaddan iborat bo'lgandagina o'zining hosilasiga bo'linishini isbotlang.

40*.

$$1 + \frac{x}{1} + \frac{x^2}{1 \cdot 2} + \cdots + \frac{x^n}{n!}$$

ko'phadni karali ildizlarga ega emasligini isbotlang.

41*. $f(x) = \dfrac{\varphi(x)}{\omega(x)},$ kasr-rasional funksiyaning maxraji $\omega(x)$ nolga aylanmaydigan x_0 soni uning suratining k karrali ildizi bo'lishi uchun

$f(x_0) = f'(x_0) = \cdots = f^{(k-1)}(x_0) = 0, \quad f^k(x_0) \neq 0.$ tengshliklarning bajarilishi zarur va yetarli ekanligini isbotlang.

42*. $f(x) = \dfrac{\varphi(x)}{\omega(x)}$ kasr rasional funksiya

$$f(x) = f(x_0) + \frac{f'(x_0)}{1}(x - x_0) + \cdots + \frac{f^{(n)}(x_0)}{n!}(x - x_0)^n + \frac{F(x)}{\omega(x)}(x - x_0)^{n+1},$$ bu yerda

$F(x)$ - ko'phad, ko'rinishda tasvirlanmasligini isbotlang. $\omega(x_0) \neq 0$ deb olinadi (kasr rasional funksiya uchun uchun *Teylor formulsi).*

43*. Agar x_0 $f_1(x) f_2'(x) - f_2(x) f_1'(x),$ ko'phadning k karrali ildizi bo'lsa, u holda x_0 aynan nolga teng bo'lmagan $f_1(x) f_2(x_0) - f_2(x) f_1(x_0),$ ko'phadning $k+1$ karrali ildizi bo'ladi va aksincha. Shu tasdiqni isbotlang.

44*. Agar $f(x)$ karrali ildizlarga ega bo'lmasa, u holda $[f'(x)]^2 - f(x)f''(x)$ ko'phad $n-1$, dan yuqori karrali ildizlarga ega bo'lmasligini isbotlang, bu yerda $n-$ $f(x)$ ko'phadning darajasi.

45*. n, darajali shunday $f(x)$ ko'phadni quringki, uning uchun $[f'(x)]^2 - f(x)f''(x)$ ko'phad $n-1$, karrali x_0 ildizga ega bo'lsin, lekin $f(x)$ ning ildizi bo'lmasin.

46*. $f(z)-$ kompleks koeffisiyentli ko'phad bo'lsin va $f(x+yi) = u(x,y) + iv(x,y)$, bu yerda $u(x,y)$ va $v(x,y)-$ko'phadlar haqiqiy koeffisiyentli. $u(x,y)=0$, $v(x,y)=0$ tenglamalar sistemasining barcha yechimlarini (haqiqiy va kompleks) $f(z)$ ko'phadning ildizlari orqali ifodalang.

47. $C[x]$ xalqada EKUB $(f(x)f'(x))$, ni toping, agar $f(x)$ quyidagicha bo'lsa:

a) $(x-1)(x^2-1)(x^3-1)(x^4-1)$; b) $(x^2-4)^3(x^2+4)^2(x^4-16)$;

s) $(x^2+1)^2(x^4+1)^4(x^6+1)^6(x^8+1)^8$; d) $x^{k+1} - x^k - x^l + 1$.

48. $f(x) = 3x^4 + x^3 + 2x^2 - x + 1$, $g(x) = 3x^4 - 5x^3 + 6x^2 - 3x + 1$ ko'phadlarning kompleks ildizlarini ularning EKUBini topish va kvadrat tenglamani yechish usuli orqali toping.

49. Agar $f(x)$ quyidagi ko'phadlardan iborat bo'lsa, $f(x)$ ko'phadning ildizlariga ega bo'lgan, lekin karrali ildizlarga ega bo'lmagan $\varphi(x)$ ko'phadni toping (ya'ni, $f(x)$ ko'phadning karrali ildizlarini ajrating) va $f(x)$ni C, maydon ustida chiziqli ko'paytuvchilarga ajrating:

a) $x^6 - 2x^5 - 9x^4 + 4x^3 + 31x^2 + 30x + 9$;

b) $x^6 - 2x^5 - x^4 + 4x^3 - x^2 - 2x + 1$;

c) $x^6 - 4x^5 - 7x^4 - 8x^3 + 7x^2 - 4x + 1$;

d) $x^5 - 8x^4 + 25x^3 - 38x^2 + 28x - 8$.

50. Bosh koeffisiyenti birga teng bo'lgan va quyidagi ildizlarga ega bo'lgan to'rtinchi darajali ko'phadni tuzing:

a) ildizlari 1, 2, -3, -4 sonlardan iborat;

b) (-1) uch karrali ildiz, i esa tub ildiz;

c) ildizlari 2, -1, $1+i$ va $1-i$;

d) 3 soni ikki karrali ildiz, –2 va –4 lar esa tub ildiz.

51. Quyidagi ko'phadning barcha kompleks ildizlarining kvadratlari yig'indisini va ularning ko'paytmasini toping:

a) $3x^5 - x^3 + x + 2$; b) $x^n - ax^{n-1} + b$ $(n \geq 3)$.

1. **52**. Birning n darajali kompleks ildizlarining yig'indisini va ko'paytmasini toping.

53. a ni shunday aniqlangki,

a) $x^3 + 12x^2 + a$ ko'phadning ikkita ildizining yig'indisi uning uchinchi ildiziga teng bo'lsin;

b) $x^3-20x+a$ ko'phadning ikkita ildizining ko'paytmasi uning uchinchi ildiziga teng bo'lsin.

54. $x^4+px^3+qx^2+rx+s$ ko'phadning ikkita ildizining yig'indisi uning boshqa ikkita ildizlarining yig'indisiga teng bo'lsa, berilgan ko'phadning koeffisiyentlari qanday shartni qanoatlantirishini toping.

55. $x^4+px^3+qx^2+rx+s$ ko'phadning ikkita ildizining ko'paytmasi uning boshqa ikkita ildizlarining ko'paytmasiga teng bo'lsa, berilgan ko'phadning koeffisiyentlari qanday shartni qanoatlantirishini toping.

56. x^3+ax^2+bx+c ko'phadning ildizlari faqat $b^3=a^3c.$ shart bajarilgandagina geometrik progressiya tashkil qilishini ko'rsating.

57. Agar $x^4-16x^3+86x^2-176x+105,$ ko'phadning ildizlari arifmetik progressiyani tashkil etsa, uning barcha ildizlarini toping.

58. $f(x)=x^n+ax^{n-1}+\cdots+a_n.$ ko'phad ildizlarining kvadratlari yig'indisini va kublari yig'indisini toping.

59. Ildizlari $\alpha_1^2,\alpha_2^2,\alpha_3^2,$ sonlardan iborat bo'lgan uchinchi darajali ko'phad tuzing, bu yerda $\alpha_1,\alpha_2,\alpha_3$ $-$lar $3x^3-4x^2+6x+10.$ko'phadning ildizlaridan iborat.

60. Agar $f(x)=a_0x^n+a_1x^{n-1}+\cdots+a_n$ ko'phad $\alpha_1,...,\alpha_n.$ildizlarga ega bo'lsa, quyidagi ko'phadlarning ildizlarini toping:

a) $a_0x^n-a_1x^{n-1}+a_2x^{n-2}-\cdots+(-1)^n a_n;$

b) $a_nx^n+a_{n-1}x^{n-1}+\cdots+a_0;$

c) $f(a)+\dfrac{f'(a)}{1!}x+\dfrac{f''(a)}{2!}x^2+\cdots+\dfrac{f^{(n)}(a)}{n!}x^n;$

d) $a_0x^n+a_1bx^{n-1}+a_2b^2x^{n-2}+\cdots+a_nb^n$?

61. Agar $2x^3-x^2-7x+\lambda=0$ tenglamaning ikkita ildizining yig'indisi 1 ga teng bo'lsa, λ ni toping.

62. Agar $x^3+px+q=0,$ tenglamaning ildizlari $x_3=\dfrac{1}{x_1}+\dfrac{1}{x_3}.$ shartni qanoatlantirsa, uning koeffisiyentlari orasidagi munosabatni toping.

63*. Ildizlari $\alpha,$ $\dfrac{1}{\alpha},$ $-\alpha,$ $-\dfrac{1}{\alpha}.$ sonlardan iborat bo'lgan to'rtinchi darajali tenglama tuzing.

64*. Ildizlari $\alpha,$ $\dfrac{1}{\alpha},$ $1-\alpha,$ $\dfrac{1}{1-\alpha},$ $1-\dfrac{1}{\alpha},$ $\dfrac{1}{1-\frac{1}{\alpha}}.$ sonlardan iborat bo'lgan oltinchi darajali tenglama tuzing.

65. Agar $\alpha_1,$ α_2 $-$lar butun koeffisiyentli $g(x)=x^2+ax+b$ ko'phadning ildizlari bo'lsa, $f(\alpha_1)+f(\alpha_2)-$ butun sondan iborat ekanligini isbot qiling.

66. Lagranjning interpolyasion formulasidan foydalanib qiymatlari quyidagi jadval bilan berilgan ko'phad tuzing:

a)

x	1	2	3	4
$f(x)$	2	1	4	3

b)

x	1	i	-1	$-i$
$f(x)$	1	2	3	4

c)

x	0	1	2	3
$f(x)$	1	-1	-3	1

d)

x	-2	-1	0	1	2
$f(x)$	13	1	1	1	13

67. Nyutonning interpolyasion formulasidan foydalanib, qiymatlari quyidagi jadval bilan berilgan eng kichik darajali ko'phad tuzing:

a)

x	0	1	2	3	4
$f(x)$	1	2	3	4	6

b)

x	-1	0	1	2	3
$f(x)$	6	5	0	3	2

c)

x	1	$\frac{9}{4}$	4	$\frac{25}{4}$
$f(x)$	1	$\frac{3}{2}$	2	$\frac{5}{2}$

$f(2)$ топинг;

d)

x	1	2	3	4	6
$f(x)$	5	6	1	-4	10

68*. Qiymatlari quyidagi jadval bilan berilgan $f(x)$ ko'phadni toping:

x	1	ε_1	ε_2	$\dots$	ε_{n-1}
$f(x)$	1	2	3	$\dots$	n

bu yerda $\varepsilon_k = \cos\frac{2\pi k}{n} + i\sin\frac{2\pi k}{n}.$

69. Darajasi n-1 dan oshmaydigan $f(x)$ ko'phad birning n-darajali ildizlariga teng bo'lgan $y_1, y_2, \dots, y_n$ qiymatlarni qabul qilsa, $f(0)$. ni toping.

70*. Agar $\varphi(x)$ ko'phadning ildizlari $x_1, x_2, \dots, x_n$ hammasi har xil bo'lsa, $0 \le s \le n-2$. bo'lganda $\sum_{i=1}^{n} \frac{x_i^s}{\varphi'(x_i)} = 0$ tenglikni isbot qiling.

71*. $\sum_{i=1}^{n} \frac{x_i^{n-1}}{\varphi'(x_i)}$ yig'indini toping (70 masalaning belgilashlariga qarang).

72*. Quyidagi jadvallar bilan berilgan eng kichik darajali ko'phad tuzing:

a)

x	0	1	2	…	n
y	1	2	4	…	2^n

b)

x	0	1	2	…	n
y	1	a	a^2	…	a^n

73*. Darajasi $2p$ teng bo'lgan $x(x-2)\cdots(x-2n)$ ko'phadga bo'linganida 1 qoldiq qoladigan va $(x-1)(x-3)\cdots[x-(2n-1)]$ ko'phadga bo'linganida esa (-1) qoldiq qoladigan ko'phadni toping.

74*. Quyidagi jadval bilan berilgan eng kichik darajali ko'phad tuzing:

x	1	2	3	…	n
y	1	$\frac{1}{2}$	$\frac{1}{3}$	…	$\frac{1}{n}$

75*. Darajasi $(n-1)-$dan oshmaydigan va $x_1, x_2, ..., x_n$, $x_i \neq a_i$, $i=\overline{1,n}$. nuqtalarda $f(x)=\dfrac{1}{x-a}$ shartni qanoatlantiradigan ko'phadni toping.

76*. Darajasi $k \leq n$, bo'lgan erkli o'zgaruvchining $n+1$ ta ketma-ket butun qiymatlarida butun qiymatlarni qabul qiladigan ko'phad erkli o'zgaruvchining barcha butun qiymatlarida butun qiymatlarni qabul qilishini isbotlang.

77*. Darajasi p ga teng bo'lgan va $x=0, 1, 4, 9, ..., n^2$, qiymatlarda butun qiymatlarni qabul qiladigan ko'phad barcha natural sonlarning kvadratlarida ham butun qiymatlarni qabul qilishini isbotlang.

§ 3. Keltirilmaydigan ko'paytuvchilarga yoyish. *C, R* va *Q* maydonlar ustidagi ko'phadlar

Darajasi $n \geq 1$ bo'lgan va koeffisiyentlari P maydonga tegishli bo'lgan $f(x)$ ko'phad P maydonda keltirilmaydigan deyiladi, agar u darajasi n dan kichik bo'lgan va koeffisiyentlari P maydonga tegishli bo'lgan ko'phadlarning ko'paytmasi shaklida tasvirlanmasa. Aks holda $f(x)$ ko'phad P maydonda keltiriladigan yoki ko'paytuvchilarga ajraladigan ko'phad deyiladi. O'zgarmas ko'phadlar ta'rif bo'yicha keltiriladigan ko'phadlarga ham keltirilmaydigan ko'phadlarga kiritilmaydi. Birinchi darajali ko'phadlar ixtiyoriy maydonda keltirilmaydigan ko'phadlardan iborat. Ko'phadning keltirilmasligi uning koeffisiyentlari tegishli bo'lgan maydonga bog'liq. Biror maydonda keltirilmaydigan ko'phad shu maydonning chekli algebraik kengaytmalarida keltiriladigan ko'phaddan iborat bo'lishi mumkin.

Koeffisiyentlari P maydonga tegishli bo'lgan har qanday o'zgarmasdan farqli ko'phad shu maydonda keltirilmaydigan ko'phadlarning ko'paytmasiga yoyiladi.

Agar o'zgarmas ko'paytuvchilarni va yoyilmadagi ko'paytuvchilarning joylashish tartibi e'tiborga olinmasa, bu yoyilma yagona bo'ladi.

$f(x) = a_0 x^n + a_1 x^{n-1} + \cdots + a_{n-1} x + a_n \in P[x], \quad n \ge 1$ bo'lsin. $f(x)$ ko'phadning

$$f(x) = a_0 p_1^{k_1}(x) p_2^{k_2}(x) \cdots p_s^{k_s}(x),$$

ko'rinishdagi yoyilmasi uning *P maydon ustidagi kanonik yoyilmasi deyilad,* bu yerda $p_1(x), p_2(x), \cdots, p_s(x) -$ bosh koeffisiyentlari 1 ga teng bo'lgan P maydonda keltirilmaydigan ko'phadlardan (*unitar ko'phadlar*) iborat. $f(x)$ ko'phadning kanonik yoyilmasi ko'paytuvchilarni joylashish tartibi aniqligida bir qiymatlidir.

$p(x) -$ P maydonda keltirilmaydigan ko'phad, $f(x) \in P[x]$ bo'lsin. Agar $p^k(x)$ ($k \in \boldsymbol{N}$) ko'phad $f(x)$ ningn bo'luvchisidan iborat bo'lib, $f(x)$ ko'phad $p^{k+1}(x)$ ga bo'linmasa, u holda $p(x)$ ko'phad $f(x)$ *ko'phadning $k-$karrali keltirilmaydigan ko'paytuvchisi deyiladi. Agar* $char\, P = 0$ bo'lsa, u holda $f(x)$ ko'phadning $k-$karrali keltirilmaydigan ko'pavytuvchisi uning hosilasi $f'(x)$ ko'phadning $(k-1)-$karrali ko'paytuvchisidan iborat bo'ladi. Xususiy holda, $k = 1$ bo'lsa $f'(x)$ ko'phad $p(x)$ ga bo'linmaydi.

$f(x) = a_0 p_1^{k_1}(x) p_2^{k_2}(x) \cdots p_s^{k_s}(x) -$ yoyilma $f(x)$ ko'phadning P, $char\, P = 0.$ maydondagi kanonik yoyilmasidan iborat bo'lsin.

U holda $(f(x), f'(x)) = p_1^{k_1-1}(x) p_2^{k_2-1}(x) \cdots p_s^{k_s-1}(x)$ va

$$f_1(x) = \frac{f(x)}{(f(x), f'(x))} = a_0 p_1(x) p_2(x) \cdots p_s(x)$$

ko'phadlar ham $f(x)$ yoyilmasidagi keltirilmaydigan ko'phadlarga ega bo'lishadi, lekin k $(k \ge 2)$ - karrali keltirilmaydigan ko'paytuvchilarga ega bo'lishmaydi

Darajasi $n \ge 1$ bo'lgan va koeffisiyentlari kompleks sonlardan iborat bo'lgan $f(x) = a_0 x^n + a_1 x^{n-1} + \cdots + a_n$ ko'phad $\boldsymbol{S}$ maydonda kamida bitta ildizga ega bo'ladi (algebraning asosiy teoremasi, *Gauss teoremasi*).

Agar $P[x]$ dan olingan darajasi $n \ge 1$ bo'lgan har bir $f(x)$ ko'phad P maydonda ildizga ega bo'lsa, u holda bu maydon algebraik yopiq maydon deyiladi.

Shunday qilib, algebraning asosiy teoremasi kompleks sonlar maydoni $\boldsymbol{S}$ ning algebraik yopiqligini anglatadi. Algebraik yopiq maydonda faqat birinchi darajali ko'phadlar keltirilmaydigan ko'phadlardan iboart bo'ladi.

$$f(x) = a_0 x^n + a_1 x^{n-1} + \cdots + a_n \quad (n \ge 1)$$

ko'phadning algebraik yopiq maydon P (*S maydon ustida ham*) kanonik yoyilmasi quyidagi ko'rinishda bo'ladi:

$$f(x) = a_0 (x - \alpha_1)^{k_1} (x - \alpha_2)^{k_2} \cdots (x - \alpha_s)^{k_s},$$

bu yerda $\alpha_1, \alpha_2, ..., \alpha_s -$ $f(x)$ ko'phadning har xil ildizlari, $k_1 + k_2 + \cdots + k_s = n.$

Haqiqiy sonlar maydoni $\boldsymbol{R}$ ning ustida faqat birinchi darajali ko'phadlar va diskriminanti manfiy bo'lgan ikkinchi darajali ko'phadlar keltirilmaydigan ko'phadlardan iborat.

Haqiqiy sonlar maydoni $\boldsymbol{R}$ ning ustida $n-$darajali $f(x)$ ko'phadning kanonik yoyilmasi quyidagi ko'rinishda bo'ladi:

$$f(x)=a_0(x-\alpha_1)^{k_1}\cdots(x-\alpha_s)^{k_s}(x^2+p_1x+q_1)^{l_1}\cdots(x^2+p_tx+q_t)^{l_t},$$

bu yerda $\alpha_1,...,\alpha_s$ – har xil haqiqiy sonlar; $x^2+p_1x+q_1,...,x^2+p_tx+q_t$ - $\boldsymbol{R}$ maydon ustidagi diskrminanti manfiy bo'lgan ikkinchi darajali har xil ko'phadlardan iborat.

$f(x)\in \boldsymbol{Q}[x]$ ko'phad $\boldsymbol{Q}$ maydon ustida keltirilmaydigan ko'phad bo'lishi uchun, uning koeffisiyentlari maxrajlarining eng kichik umumiy karralisiga ko'paytirish natijasida hosil bo'lgan butun koeffisiyentli ko'phad keltirilmaydigan ko'phaddan iborat bo'lishi kerak. Butun koeffisiyentli ko'phad rasional sonlar maydoni ustida keltirilmaydigan bo'lishi uchun u butun koeffisiyentli o'zgarmas bo'lmagan ikkita ko'phadning ko'paytmasi shaklida tasvirlanmasligi kerak. $\boldsymbol{Q}$ maydon ustida butun koeffisiyentli ko'phadning keltirilmasligini quyidagi *Eyzenshteyn alomati* bilan ham o'rnatish mumkin.

$f(x)=a_0x^n+a_1x^{n-1}+\cdots+a_{n-1}x+a_n$ – butun koeffisiyentli ko'phad bo'lsin. Agar a_0 bo'linmaydigan shunday p tub son mavjud bo'lib, $f(x)$ ko'phadning barcha boshqa koeffisiyentlari p ga bo'linsa, a_n esa p ga bo'linib, p^2 ga bo'linmasa, u holda $f(x)$ ko'phad rasional sonlar maydoni $\boldsymbol{Q}$ da keltirilmaydigan ko'phaddan iborat bo'ladi.

Misol 1. $f(x)=x^4+x^3+x+2$ ko'phadni $\boldsymbol{Z}_3$ maydon ustida keltirilmaydigan ko'phadlar ko'paytmasiga ajrating.

Yechish. $f(x)$ ni quyidagi ko'rinishda yozib olamiz:

$$f(x)=x^4+x^3+x-1=(x^4-1)+x(x^2+1)=(x^2+1)(x^2-1)+x(x^2+1)=$$
$$=(x^2+1)(x^2+x-1)=(x^2+1)(x^2+x+2).\ \blacksquare$$

M i s o l 2. $f(x)=x^6-27$ ko'phadni $\boldsymbol{C}$, $\boldsymbol{R}$ va $\boldsymbol{Q}$ maydonlar ustida keltirilmaydigan ko'phadlar ko'paytmasiga yoying.

Yechish. $f(x)$ ning $\boldsymbol{C}$ maydon ustida kanonik yoyilmasini quyidagicha topamiz:

$$f(x)=x^6-27=(x^2)^3-3^3=(x^2-3)(x^4+3x^3+9)=(x^2-3)((x^2+3)^2-(\sqrt{3}x)^2)=$$
$$=(x-\sqrt{3})(x+\sqrt{3})(x^2+\sqrt{3}x+3)(x^2-\sqrt{3}x+3)=$$
$$=(x-\sqrt{3})(x+\sqrt{3})\left(x-\frac{-\sqrt{3}+3i}{2}\right)\left(x-\frac{-\sqrt{3}-3i}{2}\right)\left(x-\frac{\sqrt{3}+3i}{2}\right)\left(x-\frac{\sqrt{3}-3i}{2}\right)$$

$f(x)$ ning $\boldsymbol{R}$ maydon ustidagi kanonik yoyilmasi quyidagicha bo'ladi:

$$f(x)=(x-\sqrt{3})(x+\sqrt{3})(x^2+\sqrt{3}x+3)(x^2-\sqrt{3}x+3).$$

$f(x)$ ning $\boldsymbol{Q}$ maydon ustidagi kanonik yoyilmasi esa quyidagicha bo'ladi: :

$$f(x)=(x^2-3)(x^4+3x^2+9).\ \blacksquare$$

M i s o l 3. Haqiqiy koeffisiyentlarga ega bo'lgan 1 soni ikki karrali ildizi, 2, 3 va $1+i$ lar esa tub ildizi bo'lgan eng kichik darajali ko'phad tuzing.

Yechish. Agar haqiqiy koeffisiyentli ko'phad kompleks ildizga ega bo'lsa, u holda bu kompleks sonning qo'shmasi ham shu ko'phadning ildizi bo'ladi. Shuninng uchun $1-i.$ soni ham ko'phadning ildizi bo'ladi. U holda izlanayotgan ko'phad quyidagi ko'paytmadan iborat bo'ladi:

$$f(x)=(x-1)^2(x-2)(x-3)(x-1-i)(x-1+i)=$$
$$=(x-1)^2(x-2)(x-3)(x^2-2x+2)=$$
$$=x^6-9x^5+33x^4-65x^3+74x^2-46x+12\,. \blacksquare$$

Agar $f(x)$ va $g(x)$ ko'phadlar chiziqli ko'paytuvchilarga ajratilgan bo'lib, ularning ko'rinishi quyidagicha bo'lsa,

$$f(x)=a_0(x-\alpha_1)^{k_1}\cdots(x-\alpha_p)^{k_p}(x-\beta_1)^{u_1}\cdots(x-\beta_t)^{u_t},$$
$$g(x)=b_0(x-\alpha_1)^{l_1}\cdots(x-\alpha_p)^{l_p}(x-\gamma_1)^{v_1}\cdots(x-\gamma_s)^{v_s}$$

($\alpha_i,\beta_q,\gamma_r$ sonlar o'zaro har xil), u holda bu ko'phadlarninng eng katta umumiy bo'luvchisi

$$(f(x),g(x))=(x-\alpha_1)^{m_1}\cdots(x-\alpha_p)^{m_p}$$

Formula bilan topiladi, bu yerda har bir $j\ (j=\overline{1,p})$ uchun m_j- k_j,l_j sonlarning eng kichigidan iborat.

Misol 4. $f(x)$ va $g(x)$ ko'phadlarning EKUBini toping.

$f(x)=(x-1)^3(x+2)^2(x-5),\quad g(x)=(x-1)(x+2)^4(x+7)(x+1)^2.$

Yechish. $(f(x),g(x))=(x-1)(x+2)^2$. ■

Misol 5. $\boldsymbol{R}[x]$ xalqada $x^{3m}+x^{3n+1}+x^{3p+2}$ ko'phadning x^2+x+1 ko'phadga qoldiqsiz bo'linishini ko'rsating

Yechish. Agar $\alpha-$ son x^2+x+1 ko'phadning ildizi bo'lsa, $\alpha^3=1$ bo'ladi. Demak, $\alpha^{3m}+\alpha^{3n+1}+\alpha^{3p+2}=1+\alpha+\alpha^2=0$. ■

M i s o l 6. Qanday shartlarda $x^{3m}-x^{3n+1}+x^{3p+2}$ ko'phad x^2-x+1ko'phadga qoldiqsiz bo'linadi?

Yechish. Agar α soni x^2-x+1 ko'phadning ildizi bo'lsa, $\alpha^3=-1$ bo'ladi. Demak,

$\alpha^{3m}-\alpha^{3n+1}+\alpha^{3p+2}=(-1)^m-(-1)^n\alpha+(-1)^p\alpha^2=(-1)^m-(-1)^p+\alpha[(-1)^p-(-1)^n].$

Oxirgi ifoda $(-1)^m=(-1)^p=(-1)^n$ holdagina nolga teng bo'ladi, ya'ni $m,n,p-$bir vaqtda juft yoki bir vaqtda toq sonlardan iborat bo'lishi kerak. ■

Endi quyidagi masalani qaraymiz: rasional koeffisiyentli $f(x)$ ko'phadning rasional ildizlari topilsin. Ma'lumki, agar $f(x)$- ko'phadning koeffisiyentlari rasional sonlardan iborat bo'lsa, u holda shunday butun λ soni mavjudki, $\lambda f(x)$ ko'phadning koeffisiyentlari butun sonlardan iboart bo'ladi. $f(x)$ va $\lambda f(x)$ ko'phadlar bir xil ildizlarga ega bo'ladi. Shuning uchun faqat koeffisiyentlari butun sonlardan iborat bo'lgan $f(x)=a_0x^n+a_1x^{n-1}+\cdots+a_{n-1}x+a_n$ ko'phadni qarash yetarli.

Agar qisqarmaydigan $\dfrac{k}{l}$ kasr $f(x)$ ko'phadning ildizi bo'lsa, u holda

$$a_0k^n+a_1k^{n-1}l+\cdots+a_{n-1}kl^{n-1}+a_nl^n=0.$$

Bu yerdan *ko'phadning rasional ildizlari to'g'risidagi birinchi teorema* kelib chiqadi*: agar qisqarmaydigan* $\dfrac{k}{l}$ *kasr koeffisiyentlari butun sonlardan iborat* *bo'lgan $f(x)$ ko'phadning ildizidan iborat bo'lsa, u holda $k-$ko'phad ozod hadining bo'luvchisidan, $l-$ ko'phad bosh koeffisiyentining bo'luvchisidan iborat bo'ladi.*

Shunday qilib, $f(x)$ ko'phadning rasional ildizlarini topish masalasi a_n ning barcha bo'luvchilari k-larda va a_0 ning barcha bo'luvchilari l ning qiymatlarida $f\left(\frac{k}{l}\right)$ ning chekli qiymatlari to'plamini tekshirishga keltiriladi.

Misol 7.

$$f(x)=2x^3-3x^2+3x-1$$

ko'phadning barcha rasional ildizlari topilsin.

Yechish. k ning mumkin bo'lgan qiymatlari $1,-1$. l ning mumkin bo'lgan qiymatlari $1,2$ lardan iborat (ishorani suratga yozamiz). Bu yerdan $\frac{k}{l}:1,-1,\frac{1}{2},-\frac{1}{2}$ qiymatlardan iborat ekanligini hosil qilamiz. Bu qiymatlarda ko'phadning qiymatlarini tekshiramiz:

$$f(1)=1,\quad f(-1)=-9,\quad f\left(\frac{1}{2}\right)=0,\quad f\left(-\frac{1}{2}\right)=-\frac{7}{2},$$

demak, $\frac{1}{2}-$ $f(x)$ko'phadning ildizidan iborat ekan.

Shunday qilib, $\frac{1}{2}-$son $f(x)$ ko'phadning tub ildizidan iborat ekan.■

Ildizlarni hisoblash jarayonini ko'phadning rasional ildizlari haqidagi ikkinchi teoremadan foydalanib, ancha soddalashtirish mumkin*: Agar qisqarmaydigan $\frac{k}{l}$ kasr koeffisiyentlari butun sonlardan iborat bo'lgan $f(x)$ ko'phadning ildizidan iborat bo'lsa, u holda $\frac{k}{l}$ ga teng bo'lmagan har qanday butun m da $f(m)$ soni $k-ml$ ga qoldiqsiz bo'linadi.*

Misol 8. $f(x)=12x^4+32x^3+23x^2+15x+18$ ko'phadning rasional ildizlari topilsin.

Yechish. Agar $\frac{k}{l}$ – kasr $f(x)$ ko'phadning ilidizdan iborat bo'lsa, u holda k va l sonlar quyidagi qiymatlarni qabul qilishi mumkin:

$$k:\pm1,\pm2,\pm3,\pm6,\pm9,\pm18;\quad l:1,2,3,4,6,12.$$

Yuqoridagi teoremaga ko'ra $f(1)=100$ $(k-l)$ ga $f(-1)=6$ esa $(k+l)$ga qoldiqsiz bo'linadi. Bu shartni faqat quyidagi sonlar $\frac{1}{2},-\frac{1}{3},-\frac{1}{4},2,-\frac{2}{3},-3,-\frac{3}{2}$ qanoatlantiradi. $f(x)$ ko'phadning ildizlarini shu sonlar ichidan izlash kerak.

Gorner sxemasidan foydalanib, $\frac{1}{2},-\frac{1}{3},-\frac{1}{4},2,-\frac{2}{3},-3,-\frac{3}{2}$ sonlardan qaysi biri $f(x)$ ko'phadning ildizlarini ekanligini tekshiramiz (ushbu holda ildiz faqat manfiy son bo'lishi mumkinligini hisobga olamiz):

	12	32	23	15	18
$-\frac{1}{3}$	12	28	$\frac{41}{3}$	$\frac{94}{9}$	$\frac{392}{27}$
$-\frac{1}{4}$	12	29	$\frac{63}{4}$	$\frac{177}{16}$	$\frac{211}{6}$
$-\frac{2}{3}$	12	24	7	$\frac{31}{3}$	$\frac{100}{9}$
-3	12	- 4	35	- 90	288
$-\frac{3}{2}$	12	14	2	12	0
$-\frac{3}{2}$	12	- 4	8	0	
$-\frac{3}{2}$	12	-22	41		

Demak, $-\frac{3}{2}$- $f(x)$ ko'phadning ikki karrali ildizi ekan va bu ko'phad boshqa rasional ildizlarga ega emas. ■

Misol 9. Eyzenshteyn alomatidan foydalanib quyidagi ko'phadlarninng $\boldsymbol{Q}$ rasional sonlar maydoni ustida keltirilmaydigan ko'phadlar ekanligini ko'rsating:

a) $f(x)=3x^7-4x^6+2x^5-6x^3-8x-2;$

b) $f(x)=x^4+x^3+x^2+x+1.$

Yechish. a) Eyzenshteyn alomatiga asosan $p=2$ bo'lganda $f(x)$ ning $\boldsymbol{Q}$ maydonda keltirilmasligini hosil qilamiz.

b) Berilgan ko'phadga Eyzenshteyn alomatini bevosita qo'llab bo'lmaydi, shuning uchun quyidagi almashtirishni olamiz $x=y+1$, natijada

$$h(y)=y^4+5y^3+10y^2+10y+5,$$

ko'phadni hosil qilamiz. Endi Eyzenshteyn alomatiga asosan $p=5$ da $f(x)$ ko'phadning $\boldsymbol{Q}$ maydonda keltirilmasligini hosil qilamiz. ■

$\boldsymbol{Q}$ maydon ustida ko'phadlarning keltirilmasligi yoki ko'paytuvchilarga ajratilishinig yana bir usulini keltiramiz. Ukajem yeщye odin sposob ustanovleniya privodimosti ili neprivodimosti ko'phadning nad polem peremennomu x o'zgaruvchiga t m ta butun $x_1,...,x_m$ qiymatlar beriladi, bu yerda, $m=\frac{n}{2}+1$ deb, n toq bo'lganda esa $m=\frac{n+1}{2}$ deb olinadi. So'ngra $c_1,c_2,...,c_m$, bu yerda $c_i(i=\overline{1,m})$ yest $f(x_i)$ning bo'luvchisi, sonlarning mumkin bo'lgan to'plamlari tuziladi (hammasi bo'lib $S=2^m s_1,...,s_m$ ta to'plam hosil bo'ladi, bu yerda s_i – $f(x_i)$ning barcha musbat bo'luvchilari soni). Ana shunday tuzilgan har bir to'plam uchun darajasi $m-1$ ga teng yoki kichik bo'lgan $h_j(x)$ $(j=\overline{1;S})$ ko'phad tuziladi. Bu ko'phad quyidagi xossaga ega bo'ladi: $h_j(x_i)=c_i$, bu yerda $i=\overline{1,m}$, c_i sonlar esa

$j-$to'plamdan olingan. Shundan so'ng darajasi musbat va koeffisiyentlari buitun sonlardan iborat bo'lgan $h_j(x)$ ko'phadlar uchun bevosita $f(x)$ ni $h_j(x)$ ga bo'linishi tekshirib ko'riladi. Agar $f(x)$ ko'phad hosil qilingan $h_j(x)$ ko'phadlardan birortasiga ham bo'linmasa, u holda $f(x)$ $\boldsymbol{Q}$ maydon ustida keltirilmaydigan ko'phaddan iborat bo'ladi. Aks holda esa $f(x)$ ko'phadning darajasi uning darajasidan kichik bo'lgan rasional koeffisiyentli ko'phadlarga yoyilmasini hosil qilamiz.

Yuqoridagi usulni $f(x)$ ko'phadga qo'lanilishida quyidagi soddalashtirishlarga erishish mumkin:

Agar sonlarning ikkita $c_1, c_2, ..., c_m$ va $c'_1, c'_2, ..., c'_m$ to'plamlari bir-biridan faqat ishoralarga farq qilsa, u holda $h_j(x)$ ko'phad faqat bitta to'plam uchun tuziladi.

O'zgaruvchining yana bir qancha butun qiymatlari $x_{m+1}, x_{m+2}, ..., x_{m+l}$ ham olish mumkin, agar ularning birortasi uchun $f(x_{m+k})$ qiymat $h_j(x_{m+k})$ ga bo'linmasa, u holda $f(x)$ ni $h_j(x)$ bo'linishini tekshirish shart emas.

M i s o l 10. Yuqorida keltirilgan usul bilan quyidagi ko'phadlarni $\boldsymbol{Q}$ maydon ustida keltirmasligini ko'rsating yoki ularning keltirilmaydigan ko'phadlarga yoyilmasini toping:

a) $f(x) = x^5 - 2x^4 - 3x^3 + 6x^2 + x - 1;$

b) $f(x) = x^4 + x^3 - 6x^2 - 3x + 9.$

Yechish. a) Bu yerda $f(0) = -1,$ $f(1) = 2,$ $f(-1) = 4,$ $f(2) = 1.$ Ular orasidan bo'luvchilari soni kam bo'lgan $f(0),$ $f(1),$ $f(2)$ qiymatlarni tanlab olamiz va quyidagi to'plamlarni tuzamiz:

$c_1 = 1$	1	1	-1	1	1	1	-1
$c_2 = 1$	1	-1	1	2	2	-2	2
$c_3 = 1$	-1	1	1	1	-1	1	1

(keltirilgan to'plamlardan faqat ishorasi bilan farq qiladiganlarini olmaymiz).

Bu to'plamlar uchun Nyutonning interpolyasion formulasiga asosan $h_j(x)$ $(j = \overline{1,8})$ ko'phadlarni tuzamiz:

$$h_1(x) = 1, \quad h_2(x) = -x^2 + x + 1, \quad h_3(x) = 2x^2 - 4x + 1,$$
$$h_4(x) = -x^2 + 3x - 1, \quad h_5(x) = -x^2 + 2x + 1, \quad h_6(x) = -2x^2 + 3x + 1,$$
$$h_7(x) = 3x^2 - 6x + 1, \quad h_8(x) = -2x^2 + 5x - 1,$$

Bu yerda $h_j(x) -$ $x = 0,$ 1, 2 larda qiymat qabul qiladigan va yuqoridagi jadvalning $j-$nchi ustunida joylashgan ko'phad.

$h_3(-1) = 7,$ $h_4(-1) = -5,$ $h_7(-1) = 10$ va $h_8(-1) = -8$ sonlar $f(-1) = 4$ ning bo'luvchilari emas, shuning uchun $h_3(x),$ $h_4(x),$ $h_7(x)$ va $h_8(x)$ ko'phadlarni qaramaymiz. Yana qo'shimcha ravishda $f(-2) = -19$ qiymatni olamiz. Bu qiymat

$h_2(-2) = -5,\quad h_5(-2) = -7$ va $h_6(-2) = -13$ sonlarga bo'linmaydi. Shunday qilib, $f(x)$ ko'phad $\boldsymbol{Q}$ maydonda keltirilmaydi.

b) $f(-2) = -1,\quad f(1) = 2,\quad f(2) = 3.$

Bu qiymatlarning bo'luvchilari to'plamini tuzamiz:

$c_1 = 1$	-1	1	1	1	-1	1	1	1	-1	1	1	1	1	-1	1	1
$c_2 = 1$	1	-1	1	2	2	-2	2	1	1	-1	-1	1	2	2	-2	2
$c_3 = 1$	1	1	-1	1	1	1	-1	3	3	3	3	-3	3	3	3	-3

(faqat ishorasi bilan farq qiladigan to'plamlarni qaramaymiz).

Bu to'plamlarga mos keladigan ko'phadlarni Nyutonning interpolyasion formulasi bilan tuzamiz:

$h_1(x) = 1;\quad h_2(x),\quad h_3(x),\quad h_4(x),\quad h_5(x),\quad h_6(x) -$ kasr koeffisiyentli ko'phadlar;

$h_7(x) = x^2 - 3 -$ esa $f(x)$ ning bo'luvchisi, ya'ni $f(x) = (x^2 - 3)(x^2 + x - 3)$, bu yerda $x^2 - 3$ va $x^2 + x - 3$ ko'phadlar $\boldsymbol{Q}$ maydonda keltirilmaydigan ko'phadlardan iborat (chunki ular $\boldsymbol{Q}$ da ildizga ega emas). ■

MAShQLAR

78. Quyidagi ko'phadlarni keltirilmaydigan ko'phadlar ko'paytmasiga yoying:

a) $x^3 + x + 1$ ni $\boldsymbol{Z_2}$ maydonda;

b) $x^5 + x^3 + x^2 + 1$ ni $\boldsymbol{Z_3}$ maydonda;

c) $x^3 + 2x^2 + 4x + 1$ ni $\boldsymbol{Z_5}$ maydonda;

d) $x^4 + 3x^3 + 2x^2 + x + 4$ ni $\boldsymbol{Z_5}$ maydonda;

79. $\boldsymbol{Z_3}$ maydonda bosh koeffisiyenti birga teng bo'lgan ikkinchi darajali barcha keltirilmaydigan ko'phadlarni toping.

80. $\boldsymbol{Z_3}$ maydonda bosh koeffisiyenti birga teng bo'lgan uchinchi darajali barcha keltirilmaydigan ko'phadlarni toping.

81. ***S*** va ***R*** maydonlar ustida quyidagi ko'phadlarni keltirilmaydigan ko'phadlar ko'paytmasiga yoying.

a) $x^3 - 8$; b) $x^3 + 8$; c) $x^4 - 16$; d) $x^4 + 16$; ye) $x^6 + 27$;

f) $x^8 - 6x^4 + 9$; g) $x^{2n} - 2x^n + 2$; h) $x^{2n} + x^n + 1$;

i) $x^{2n} - 1$; j) $x^{2n+1} - 1$.

82. Ildizlari birning barcha n-darajali ildizining qiymatlaridan iborat bo'lgan eng kichik darajali ko'phadni tuzing.

83. ***C*** va ***R*** maydonlar ustida berilgan ildizlar bo'yicha eng kichik darajali ko'phad tuzing:

a) 1 - ikki karrali ildiz, i va –1 lar tub ildizlar;

b) 1-2i uch karrali ildiz;

c) i ikki karrali ildiz va -1-i tub ildiz;

d) –1–i va -2+i lar ikki karrali ildizlar;

ye) 1, -1 va i tub ildizlar.

84. Quyidagi ko'phadlarning EKUBini toping:

a) $(x-1)^3(x+2)^2(x-3)(x-4)$ и $(x-1)^2(x+2)(x+5)$;

b) $(x-1)(x^2-1)(x^3-1)(x^4-1)$ и $(x+1)(x^2+1)(x^3+1)(x^4+1)$;

c) $(x^3-1)(x^2-2x+1)$ и $(x^2-1)^2$.

85*. Quyidagi ko'phadlarning EKUBini toping: x^m-1 va x^n-1.

86. Quyidagi ko'phadlarning EKUBini toping: x^m+a^m va x^n+a^n.

87. m ning qanday qiymatlarida $\boldsymbol{R}[x]$ xalqada quyidagi ko'phadlar x^2+x+1 ko'phadga qoldiqsiz bo'linadi:

a) $x^{2m}+x^m+1$; b) $(x+1)^m-x^m-1$; c) $(x+1)^m+x^m+1$.

88. m ning qanday qiymatlarida $\boldsymbol{R}[x]$ xalqada quyidagi ko'phadlar $(x^2+x+1)^2$ ko'phadga qoldiqsiz bo'linadi:

a) $(x+1)^m-x^m-1$; b) $(x+1)^m+x^m+1$.

89. $(x+1)^m+x^m+1$ va $(x+1)^m-x^m-1$ ko'phadlar $(x^2+x+1)^3$ ko'phadga qoldiqsiz bo'linishi mumkinmi?

90. Agar α soni butun koeffisiyentli $f(x)=a_0x^n+a_1x^{n-1}+\cdots+a_{n-1}x+a_n$ ko'phadning ildizi bo'lsa, quyidagilarni isbot qiling: a) $\alpha-$ soni a_n ning bo'luvchisidan iborat; b) $\alpha-m$ - soni ixtiyoriy butun m da $f(m)$ ning bo'luvchisidan iborat. Xususiy holda, $\alpha-1$ - $f(1)$ ning, $\alpha+1-$esa $f(-1)$ ning bo'luvchisidan iborat.

91. Quyidagi ko'phadlarning butun ildizlarini toping:

a) $6x^4+19x^3-7x^2-26x+12$;

b) $3x^4+5x^3+x^2+x-2$;

c) $2x^5+7x^4+3x^3-11x^2-16x-12$;

d) $3x^6-5x^4-10x^3-8x^2+x-2$;

ye) $6x^4+x^3+2x^2-4x+1$.

92*. Agar $\dfrac{k}{l}-$qisqarmaydigan rasional kasr koeffisiyentlari butun sonlardan iborat bo'lgan $f(x)=a_0x^n+a_1x^{n-1}+\cdots+a_n$ ko'phadning ildizi bo'lsa, quyidagilsharni isbot qiling:

a) $l-$ a_0 ning bo'luvchisi;

b) $k-$ a_n ning bo'luvchisi;

s) $k-ml-$ ixtiyoriy butun m da $f(m)$ ning bo'luvchisidan iborat. Xususiy holda, $k-l-$ $f(1)$ ning, $k+l-$esa $f(-1)$ ning bo'luvchisidan iborat.

93. Quyidagi ko'phadlarning rasional ildizlarini toping:

a) $x^3-6x^2+15x-14$; b) $x^4-2x^3-8x^2+13x-24$;

c) $x^5-7x^3-12x^2+6x+36$; d) $6x^4+19x^3-7x^2-26x+12$;

ye) $24x^4-42x^3-77x^2+56x+60$; f) $x^5-2x^4-4x^3+4x^2-5x+6$;

g) $10x^4-13x^3+15x^2-18x-24$; h) $3x^4-2x^3+4x^2-x+2$;

l) $4x^4-7x^2-5x-1$.

94. $\boldsymbol{Q}$ maydon ustida quyidagi ko'phadlarni keltirilmaydigan ko'phadlar ko'paytmasiga yoying:

a) $x^3+6x^2-8x+12$; b) $3x^3+5x^2+5x+2$; c) $30x^3+19x^2-1$;

d) $3x^3+4x^2+4x+4$.

95*. Eyzenshteyn alomatidan foydalanib, quyidagi ko'phadlarni $\boldsymbol{Q}$ maydon ustida keltirilmasligini isbot qiling:

a) $x^4-8x^3+12x^2-6x+2$; b) $x^5-12x^3+36x-12$; c) x^4-x^3+2x+2;

d) x^n+p, bu yerda $p-$tub son; ye) $\dfrac{x^p-1}{x-1}$, bu yerda $p-$tub son.

96*. Agar $f(0)$ va $f(1)-$toq sonlardan iborat bo'lsa, butun koeffisiyentli $f(x)$ ko'phad butun ildizlarga ega emasligini isbot qiling.

97*. Agar koeffisiyentlari butun sonlardan iborat bo'lgan $f(x)$ ko'phad erkli o'zgaruvchining ikkita x_1 va x_2 qiymatlarida ± 1 qiymatlarni qabul qilsa, u holda $|x_1-x_2|>2$ bo'lganda $f(x)$ ko'phad rasional ildizlarga ega emasligini isbot qiling. Agarda $|x_1-x_2|\le 2$ bo'lsa, u holda $f(x)$ ning rasional ilidizi faqat $\frac{1}{2}(x_1+x_2)$ dan iborat ekanligini ko'rsating.

98. $\boldsymbol{Q}$ maydon ustida $x^5+2x^3+3x^2-6x-5$ ko'phadning keltirilmasligini 2 modul bo'yicha reduksiya yordamida isbot qiling.

99*. $\boldsymbol{Q}$ maydon ustida $x^5-6x^3+2x^2-4x+5$ ko'phadning keltirilmasligini 2 va 3 modullar bo'yicha reduksiya yordamida isbot qiling.

100*. $f(x)$-- $\boldsymbol{Z}_p$ maydon ustida keltirilmaydigan ko'phad bo'lsin. $f(x)$, $f(x+1),\ldots,f(x+p-1)$ ko'phadlarning yo juft-jufti bilan har xil ekanligini, yoki ularning hammasi ustma-ust tushishini isbot qiling.

101*. $f(x)=x^p-x-a$ ko'phadni $\boldsymbol{Z}_p$ maydonda a soni p ga bo'linmagan holda keltirilmasligini isbot qiling.

102. Quyidagi ko'phadlarni $\boldsymbol{Q}$ maydon ustida o'zgaruvchining butun qiymatlarini ko'paytuvchilarga ajratish usuli bilan ko'paytuvchilarga ajrating yoki ularning keltirilmasligini isbot qiling:

a) x^4-3x^2+1; b) $x^4+5x^3-3x^2-5x+1$;

c) $x^4+3x^3-2x^2-2x+1$; d) $x^4-x^3-3x^2+2x+2$.

103*. Butun koeffisiyentli to'rtinchi darajali $x^4+ax^3+bx^2+cx+d$ ko'phadni, agar u $x^2+\dfrac{cm-am^2}{d-m^2}\cdot x+m$, bu yerda $m-$soni d ning bo'luvchisi, ko'rinishdagi ko'phadlardan birortasiga bo'linmasa, $\boldsymbol{Q}$ maydon ustida keltirilmasligini sboott qiling. Kasr koeffisiyentli ko'phadlar e'tiborga olinmasa ham bo'ladi. Bu holda cheklashlar faqat koeffisiyentlari $d=k^2$, $c=ak$ shartlarni qanoatlantiradigan ko'phadlar uchun o'rinli.

104*. Butun koeffisiyentli beshinchi darajali

$$x^5+ax^4+bx^3+cx^2+dx+e \text{ s}$$

ko'phad butun ildizlraga ega bo'lmasa va u quyidagi ko'rinishdagi butun koeffisiyentli ko'phadlardan

$$x^2+\frac{am^3-cm^2-dn+be}{m^3-n^2+ae-dm}\cdot x+m,$$

bu yerda $m-$soni e ning bo'luvchisi va $n=\frac{e}{m}$, birortasiga bo'linmasa, uni Q maydon ustida keltirilmasligini isbot qiling:

105. Quyidagi ko'phadlarni 103, 104 masalalardan foydalanib Q maydon ustida ko'paytuvchilarga ajrating yoki ularning keltirilmasligini isbot qiling:

a) $x^4-3x^3+2x^2+3x-9$; b) $x^4-3x^3+2x^2+2x-6$;

c) $x^4+4x^3-6x^2-23x-12$; d) $x^5+x^4-4x^3+9x^2-6x+6$.

106. a va b butun sonlar bo'lganda Q maydon ustida x^5+ax^3+bx+1 ko'rinishdagi barcha keltiriladigan ko'phadlarni toping.

107. Rasional koeffisiyentli x^4+px^2+q ko'phadning Q maydon ustida keltiriladigan ko'phaddan iborat bo'lishi uchun zaruriy va yetarli shartlarni toping.

108. Q maydon ustida
$f(x)=(x-a_1)(x-a_2)...(x-a_n)-1$; $a_1,a_2,...,a_n-$ o'zaro har xil butun sonlar, ko'phadning keltirilmasligini isbot qiling.

109*. Agar $n-$darajali butun koeffisiyentli ko'phad o'zgaruvchining $2m$ dan $(n=2m$ yoki $2m+1)$ ortiq butun qiymatlarida ±1qiymatni qabul qilsa, uning Q maydonda keltirilmasligini isbot qiling.

110*. Agar $a_1,a_2,...,a_n-$ o'zaro har xil butun sonlar bo'lsa, $f(x)=(x-a_1)^2(x-a_2)^2...(x-a_n)^2+1$ ko'phadning Q maydon ustida keltirilmasligini isbot qiling.

111*. Agar butun koeffisiyentli ax^2+bx+1 ko'phad Q maydon ustida keltirilmaydigan ko'phad bo'lsa, u holda $a[\varphi(x)]^2+b\varphi(x)+1$ ko'phad ham keltirilmasligini isbot qiling, bu yerda $\varphi(x)=(x-a_1)(x-a_2)...(x-a_n)$, $n\geq7$ bo'lganda va $a_1,a_2,...,a_n-$ o'zaro har xil butun sonlar.

§ 4. Rasional kasrlar

P maydonda r*asional kasr yoki* kasr-*rasional funksiya* deb $\frac{f(x)}{g(x)}$ ko'rinishdagi ifodaga aytiladi, bu yerda *f(x)* va *g(x)* ($g(x)\neq0$) koeffisiyentlari P maydondan olingan ko'phadlardan iborat. Agar $P[x]$ xalqada $f(x)\psi(x)=g(x)\psi(x)$ tenglik o'rinli bo'lsa, P maydon ustidagi $\frac{f(x)}{g(x)}$ va $\frac{\varphi(x)}{\psi(x)}$ rasional kasrlar teng deyiladi.

P maydon ustidagi barcha rasional kasrlar to'plamida *algebraik amallarni (qo'shish va ko'paytirish)* quyidagi tengliklar bilan berish mumkin

$$\frac{f(x)}{g(x)}+\frac{\varphi(x)}{\psi(x)}=\frac{f(x)\psi(x)+g(x)\varphi(x)}{g(x)\psi(x)},$$

$$\frac{f(x)}{g(x)}\cdot\frac{\varphi(x)}{\psi(x)}=\frac{f(x)\varphi(x)}{g(x)\psi(x)}.$$

Bu amallarga nisbatan P maydon ustidagi barcha rasional kasrlar to'plami maydon tashkil qiladi, bu maydon rasional kasrlar maydoni yoki P maydon ustidagi kasr-rasional funksiyalar maydoni deyiladi va $P(x)$ bilan belgilanadi.

Agar $f_1(x)$ va $g_1(x)$ ko'phadlar o'zaro tub bo'lsa, $\frac{f_1(x)}{g_1(x)}$ rasional kasr qisqarmaydigan rasional kasr deyiladi. Har qanday rasional kasr uni surati $f(x)$ va maxraji $g(x)$ ning eng katta umumiy bo'luvchisiga qiqartirgandan so'ng qisqarmaydigan rasional kasrga teng bo'ladi.

Agar rasional kasrda suratining darajasi maxrajining darajasidan kichik bo'lsa, bunday rasional kasrga *to'g'ri rasional kasr* deyiladi.

M i s o l 1. $\frac{x^2-x}{x^3+2x^2-x}=\frac{x-1}{x^2+2x-1}$. ■

Ko'phaddan farqli har qanday rasional kasr (ya'ni, kasrning qisqarmaydigan shaklida maxrajning darajasi noldan katta), yagona ravishda biror ko'phad va to'g'ri rasional kasrning yig'indisi shaklida tasvirlanadi. $\frac{f(x)}{g(x)}$ kasrning ana shunday tasvirini hosil qilish uchun $f(x)$ ko'phadni $g(x)$ ga qoldiqli bo'lish kerak. Agar shundan keyin $q(x)$ bo'linma va $r(x)$ qoldiq hosil bo'lsa,

$$\frac{f(x)}{g(x)}=q(x)+\frac{r(x)}{g(x)}$$

bo'ladi, bu yerda $\frac{r(x)}{g(x)}-$to'g'ri kasrdan iborat.

Misol 2. $\frac{f(x)}{g(x)}=\frac{3x^4+2x^2-4x+6}{x^2-2x+3}$.

$3x^4+2x^2-4x+6=(x^2-2x+3)(3x^2+6x+5)-12x-9$, bu yerdan

$$\frac{f(x)}{g(x)}=3x^2+6x+5+\frac{-12x-9}{x^2-2x+3}.\ ■$$

Agar $g(x)$ ko'phad P maydonda keltirilmaydigan qandaydir $p(x)$ ko'phadning darajasidan iborat bo'lsa, $\frac{f(x)}{g(x)}$ to'g'ri rasional kasr sodda kasr deyiladi, bu yerda $f(x)$ ko'phadning darajasi $p(x)$ ning darajasidan kichik bo'ladi.

Rasional kasrlar to'g'risidagi asosiy teorema quyidagicha: $P(x)$ maydondan olingan har qanday to'g'ri rasional kasr yagona ravishda sodda rasional kasrlarning yig'indisi ko'rinishida tasvirlanishi mumkin.

$\dfrac{f(x)}{g(x)}$ rasional kasrni P maydon ustida yoyilmasini hosil qilish uchun dastavval $g(x)$ ko'phad P maydonda keltirilmaydigan ko'phadlar ko'paytmasiga yoyiladi va agar yoyilma $g(x)=p_1^{s_1}(x)...p_t^{s_t}(x)$ $(p_1(x),...,p_t(x)-$ keltirilmaydigan ko'phadlar) shaklda bo'lsa, u holda:

$$\frac{f(x)}{g(x)}=\frac{U_1^{(1)}(x)}{p_1^{s_1}(x)}+\frac{U_2^{(1)}(x)}{p_1^{s_1-1}(x)}+\cdots+\frac{U_{s_1}^{(1)}(x)}{p_1(x)}+\cdots+\frac{U_1^{(t)}(x)}{p_t^{s_t}(x)}+\frac{U_2^{(t)}(x)}{p_t^{s_t-1}(x)}+\cdots+\frac{U_{s_t}^{(t)}(x)}{p_t(x)},$$

bo'ladi, bu yerda $U_j^{(i)}(x)$ $(i=\overline{1,t};\ j=\overline{1,s_i}$ tayinlangan $i\,da)-$ aniqmas koeffisiyentli ko'phadning darajasi $p_i(x)$ko'phadning darajasidan kichik. So'ngar yuqoridagi tenglikning o'ng tomonidagi kasrlar umumiy maxraj $g(x)$ ga keltiriladi va suratda hosil bo'lgan yig'indi $f(x)$ ko'phadga tenglashtiriladi. Hosil bo'lgan tenglikning o'ng va chap qismidagi x ning bir xil darajalari oldidagi koeffisiyentlar tenglashtirilib (yoki x o'zgaruvchiga sonli qiymatlar berib), chiziqli tenglamalar sistemasi hosil qilinadi. Hosil qilingan sistema yechilib $U_j^{(i)}(x)$ ko'phadlarninng koeffisiyentlari topiladi.

Agar $P=\boldsymbol{C}$ – kompleks sonlar maydonidan iborat bo'lsa, u holda sodda kasrlar $\dfrac{\alpha}{(x-\beta)^k}$ ko'rinishda bo'ladi, bu yerda α va $\beta-$ kompleks sonlardan iborat va $k\geq 1$.

Agar $K=\boldsymbol{R}$ – haqiqiy sonlar maydonidan iborat bo'lsa, u holda sodda kasrlar $\dfrac{\alpha}{(x-\beta)^m}$ (α va $\beta-$haqiqiy sonlar, $m\geq 1$) va $\dfrac{\alpha_1 x+\alpha_2}{(x^2+\beta_1 x+\beta_2)^n}$, bu yerda $x^2+\beta_1 x+\beta_2-$ haqiqiy koeffisiyentli ko'phaddan iborat bo'lib, u haqiqiy ildizlarga ega emas, α_1,α_2- haqiqiy sonlar, $m\geq 1$.

Misol 3. $\boldsymbol{R}$ va $\boldsymbol{C}$ maydonlar ustida $\dfrac{f(x)}{g(x)}=\dfrac{2x^4-2x^3-6x^2+2x+7}{x^5+2x^4+2x^3+4x^2+x+2}$ kasrni sodda rasional kasrlar yoying.

Yechish. Dastavval $\boldsymbol{R}$ maydonni qaraymiz.

Bu holda $g(x)=(x+2)(x^2+1)^2$. Bu yerdan

$$\frac{f(x)}{g(x)}=\frac{a}{x+2}+\frac{a_1x+a_2}{(x^2+1)^2}+\frac{c_1x+c_2}{x^2+1}.$$

Tenglikning o'ng qismidagi kasrlarni umumiy maxraj $g(x)$ga keltirib, quyidagi tenglikni hosil qilamiz:

$$f(x)=a(x^2+1)^2+(a_1x+a_2)(x+2)+(c_1x+c_2)(x+2)(x^2+1), \text{ yoki}$$

$$2x^4-2x^3+6x^2+2x+7=(a+c_1)x^4+(2c_1+c_2)x^3+(2a+a_1+c_1+2c_2)x^2+$$
$$+(2a_1+a_2+2c_1+c_2)x+(a+2a_2+2c_2).$$

x ning bir xil darajalari oldidagi koeffisiyentlarni tenglashtirib, quyidagi tengamalar sistemasini hosil qilamiz:

$$\begin{cases} a+c_1=2 \\ 2c_1+c_2=-2 \\ 2a+a_1+c_1+2c_2=6 \\ 2a_1+a_2+2c_1+c_2=2 \\ a+2a_2+2c_2=7. \end{cases}$$

Bu sistemadan $a=3,\quad a_1=1,\quad a_2=2,\quad c_1=-1,\quad c_2=0$ larni topamiz. Shunday qilib,

$\frac{f(x)}{g(x)}=\frac{3}{x+2}+\frac{x+2}{(x^2+1)^2}-\frac{x}{x^2+1}-$ berligan kasrning $\boldsymbol{R}$ maydon ustida sodda kasrldarga yoyilmasidan iborat.

Kompleks sonlar mydonida esa quyidagilarga ega bo'lamiz:

$g(x)=(x+2)(x+i)^2(x-i)^2$. Bu yerdan

$\frac{f(x)}{g(x)}=\frac{a}{x+2}+\frac{b}{(x+i)^2}+\frac{c}{x+i}+\frac{d}{(x-i)^2}+\frac{e}{x-i}.$

Tenglikning o'ng tomonidagi kasrlarni umumiy maxraj $g(x)$ga keltirib, quyidagi tenglikni hosil qilamiz:

$$2x^4-2x^3+6x^2+2x+7=a(x+i)^2(x-i)^2+b(x+2)(x-i)^2+$$
$$+c(x+2)(x+i)(x-i)^2+d(x+2)(x+i)^2+e(x+2)(x+i)^2(x-i).$$

$x=-2$ da: $75=a\cdot 25$, ya'ni $a=3$. $x=-i$ da: $3-4i=b(2-i)(-4)$, bu yerdan $b=-\frac{1}{2}+\frac{1}{4}i.$ $x=i$ da: $3+4i=d(2+i)(-4)$, bu yerdan. $b=-\frac{1}{2}-\frac{1}{4}i.$ Endi ketma-ket $x=0$ va $x=-1$ deb olib, yuqorida topilgan a,b,d koeffisiyentlarning qiymatlarini hisobga olib,

$$\begin{cases} 7=3+\left(1-\frac{1}{2}i\right)-2ci+\left(1+\frac{1}{2}i\right)+2ei, \\ 15=12+\left(-\frac{1}{2}-i\right)+c(-2-2i)+\left(-\frac{1}{2}+i\right)+e(-2+2i) \end{cases}$$

yoki

$$\begin{cases} -2ci+2ei=2 \\ c(-2-2i)+e(-2+2i)=4, \end{cases}$$

bu yerdan $c=\frac{-1+i}{2},\quad e=\frac{-1-i}{2}.$

Shunday qilib,

$$\frac{f(x)}{g(x)}=\frac{3}{x+2}-\frac{2-i}{4(x+i)^2}-\frac{1-i}{2(x+i)}-\frac{2+i}{4(x-i)^2}-\frac{1+i}{2(x-i)}-$$

berilgan kasrning $\boldsymbol{C}$ maydonda sodda kasrlarga yoyilmasini hosil qilamiz. ■

Rasional kasrlarning maxrajlari o'zaro tub chiziqli ko'phadlar yoyilmasidan iborat bo'lgan holni qaraymiz.

Quyidagi rasional kasr berilgan bo'lsin:

$$\frac{f(x)}{(x-x_1)(x-x_2)\cdots(x-x_n)}, \quad x_i \neq x_j.$$

Uning sodda rasional kasrlarga yoyilmasi

$$\frac{f(x)}{(x-x_1)(x-x_2)\cdots(x-x_n)} = \frac{a_1}{x-x_1} + \frac{a_2}{x-x_2} + \cdots + \frac{a_n}{x-x_n}$$

ko'rinishda bo'ladi. Bu tenglikdagi koeffisiyentlarni topish uchun uning ikkala tomonini maxrajiga ko'paytirib yuboramiz:

$$\begin{gathered} f(x) = a_1(x-x_2)\cdots(x-x_n) + a_2(x-x_1)(x-x_3)\cdots(x-x_n) + \cdots + \\ + a_n(x-x_1)\cdots(x-x_{n-1}) \end{gathered}.$$

Endi ketma – ket $x = x_1, x = x_2, ..., x = x_n$ qiymatlarni berib:

$$\begin{aligned} &f(x_1) = a_1(x_1-x_2)(x_1-x_3)\cdots(x_1-x_n), \\ &f(x_2) = a_2(x_2-x_1)(x_2-x_3)\cdots(x_2-x_n), \\ &\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\ &f(x_n) = a_n(x_n-x_1)(x_n-x_2)\cdots(x_n-x_{n-1}). \end{aligned}$$

tengliklarni hosil qilamiz . Tengliklarning o'ng tomonlaridagi ko'paytuvchilarningn hammasi noldan farqli va ular $F(x) = (x-x_1)(x-x_2)\cdots(x-x_n)$ ko'phadning hosilalari orqali osongina topiladi. Haqiqatdan ham,

$$\begin{gathered} F'(x) = (x-x_2)...(x-x_n) + (x-x_1)(x-x_3)...(x-x_n) + ... + \\ + (x-x_1)(x-x_2)...(x-x_{n-1}). \end{gathered}$$

Bu tenglikkka ketma-ket $x = x_1, x = x_2, ..., x = x_n$ qiymatlarni berib, quyidagilarni hosil qilamiz:

$$\begin{aligned} &F'(x_1) = (x_1-x_2)(x_1-x_3)\cdots(x_1-x_n), \\ &F'(x_2) = (x_2-x_1)(x_2-x_3)\cdots(x_2-x_n), \\ &\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\ &F'(x_n) = (x_n-x_1)(x_n-x_2)\cdots(x_n-x_{n-1}). \end{aligned}$$

Bu yerdan $a_1 = \frac{f(x_1)}{F'(x_1)}, \quad a_2 = \frac{f(x_2)}{F'(x_2)}, ..., a_n = \frac{f(x_n)}{F'(x_n)}$, va berilgan kasrni sodda kasrlarga yoyilmasi uchun quyidagi formulani hosil qilamiz

$$\frac{f(x)}{F(x)} = \sum_{k=1}^{n} \frac{f(x_k)}{F'(x_k)(x-x_k)} \quad \textit{(Lagranj formulasi).}$$

M i s o l 4. Quyidagi kasrni sodda kasrlarga yoying

$$\frac{x^3+5x+7}{(x+2)(x+1)x(x-1)(x-2)}.$$

Yechish. Bu yerda

$$F(x) = (x+2)(x+1)x(x-1)(x-2),$$

$$F'(-2)=(-2+1)(-2)(-2-1)(-2-2)=24,$$
$$F'(-1)=(-1+2)(-1)(-1-1)(-1-2)=-6,$$
$$F'(0)=(0+2)(0+1)(0-1)(0-2)=4,$$
$$F'(1)=-6,\quad F'(2)=24.$$

Demak,

$$\frac{x^3+5x+7}{(x+2)(x+1)x(x-1)(x-2)}=-\frac{11}{24(x+2)}-\frac{1}{6(x+1)}+\frac{7}{4x}-\frac{13}{6(x-1)}+\frac{25}{24(x-2)}.$$

Miso l 5. $\boldsymbol{R}$ maydon ustida drob $\frac{1}{x^{2n}+1}$ kasrni sodda kasrlarga yoying.

Yechish. Dastavval berilgan kasrning $\boldsymbol{C}$ maydon ustida yoyilmasini topamiz. $F(x)=x^{2n}+1$ ko'phadning ildizlari birlik aylanada yotadi va juft-jufti bilan o'zaro qo'shma kompleks sonlardan iborat. Ya'ni

$$x_k=\cos\frac{(2k-1)\pi}{2n}+i\sin\frac{(2k-1)\pi}{2n},\quad k=\overline{1,n},$$

ildizlar $\bar{x}_k=x_{2n+1-k}$ ildizlar bilan o'zaro qo'shma. Ildizlar juft-jufti bilan har xil bo'lganligi uchun Lagranj formulasinin qo'llash mumkin. Endi $F'(x)=2nx^{2n-1}$, bu yerdan

$F'(x_k)=2nx_k^{2n-1}=2nx_k^{-1}x_k^{2n}=-2nx_k^{-1}.$

Lagranj formulasiga asosan

$$\frac{1}{x^{2n}+1}=-\frac{1}{2n}\sum_{k=1}^{n}\frac{x_k}{x-x_k}-\frac{1}{2n}\sum_{k=1}^{n}\frac{\bar{x}_k}{x-\bar{x}_k}.$$

Endi kompleks qo'shma qo'shiluvchilarni birlashtirib, quyidagini hosil qilamiz

$$\frac{1}{x^{2n}+1}=-\frac{1}{2n}\sum_{k=1}^{n}\left(\frac{x_k}{x-x_k}+\frac{\bar{x}_k}{x-\bar{x}_k}\right)=$$

$$=-\frac{1}{2n}\sum_{k=1}^{n}\frac{(x_k+\bar{x}_k)x-2}{x^2-(x_k+\bar{x}_k)x+1}=\frac{1}{n}\sum_{k=1}^{n}\frac{1-x\cos\frac{(2k-1)\pi}{2n}}{x^2-2x\cos\frac{(2k-1)\pi}{2n}+1}.\ ■$$

Misol 6. $\boldsymbol{Z}_\mathbf{p}$ - p modul bo'yicha chegirmalar maydoni ustida $\frac{1}{x^p-x}$ kasrni sodda kasrlarga yoying.

Yechish. $\boldsymbol{Z}_\mathbf{p}$ maydoning elementlari $0,1,...,p-1$ lar $F(x)=x^p-x$ ko'phadning ildizxlari bo'lganligi uchun $x^p-x=x(x-1)\cdots(x-p-1)$. U holda $F'(x)=px^{p-1}-1=-1$. Demak, $\frac{1}{x^p-x}=-\sum_{k=0}^{p-1}\frac{1}{x-k}$. ■

MAShQLAR

112*. Gorner sxemasidan foydalanib, quyidagi kasrlarni sodda kasrlar yig'indisiga yoying:

a) $\frac{x^2+x+1}{(x+2)}$; b) $\frac{x^3-x+1}{(x-2)^5}$; c) $\frac{x^4-2x^2+3}{(x+1)^5}$.

113*. ***C*** maydon ustida sodda kasrlar yig'indisiga yoying:

a) $\frac{x^2}{(x-1)(x+2)(x+3)}$; b) $\frac{1}{(x-1)(x-2)(x-3)(x-4)}$;

c) $\frac{3+x}{(x-1)(x^2+1)}$; d) $\frac{x^2}{x^4-1}$; e) $\frac{1}{x^3-1}$; f) $\frac{1}{x^4+4}$;

g) $\frac{1}{x^n-1}$; h) $\frac{1}{x^n+1}$; i) $\frac{n!}{x(x-1)(x-2)\cdots(x-n)}$;

j) $\frac{(2n)!}{x(x^2-1)(x^2-4)\cdots(x^2-n^2)}$.

114*. ***C*** maydon ustida sodda kasrlar yig'indisiga yoying:

a) $\frac{x}{(x^2-1)^2}$; b) $\frac{1}{(x^2-1)^2}$; c) $\frac{5x^2+6x-23}{(x-1)^3(x+1)^2(x-2)}$;

d) $\frac{1}{(x^n-1)^2}$; e) $\frac{1}{x^m(1-x)^n}$; f) $\frac{1}{(x^2-a^2)^n}$, $a\neq 0$; g) $\frac{1}{(x^2+a^2)^n}$.

115*. ***R.*** maydon ustida sodda kasrlar yig'indisiga yoying:

a) $\frac{1}{x^3-1}$; b) $\frac{x^2}{x^4-16}$; c) $\frac{1}{x^4+4}$; d) $\frac{x^2}{x^6+27}$;

e) $\frac{x^m}{x^{2n+1}-1}$, $m<2n+1$; f) $\frac{x^m}{x^{2n+1}+1}$, $m<2n+1$; g) $\frac{1}{x^{2n}-1}$;

h) $\frac{x^{2m}}{x^{2n}+1}$, $m<n$; i) $\frac{1}{x(x^2+1)(x^2+4)\cdots(x^2+n^2)}$.

116. ***R.*** maydon ustida sodda kasrlar yig'indisiga yoying:

a) $\frac{x}{(x+1)(x^2+1)^2}$; b) $\frac{2x-1}{x(x+1)^2(x^2+x+1)^2}$;

c) $\frac{1}{(x^4-1)^2}$; d) $\frac{1}{(x^{2n}-1)^2}$.

117. $\varphi(x)=(x-x_1)(x-x_2)\cdots(x-x_n)$. bo'lsin. $\varphi(x)$ ni quyidagi summalar orqali ifodasini toping: a) $\sum\frac{1}{x-x_i}$; b) $\sum\frac{x_i}{x-x_i}$; c) $\sum\frac{1}{(x-x_i)^2}$.

118*. x_1,x_2,x_3 –lar $\varphi(x)$ ko'phadning ildizlari ekanligini bilgan holda quyidagi yig'indilarni hisoblang: :

a) $\frac{1}{2-x_1}+\frac{1}{2-x_2}+\frac{1}{2-x_3}$, $\varphi(x)=x^3-3x-1$;

b) $\frac{1}{x_1^2-3x_1+2}+\frac{1}{x_2^2-3x_2+2}+\frac{1}{x_3^2-3x_3+2}$, $\varphi(x)=x^3+x^2-4x+1$;

s) $\dfrac{1}{x_1^2-2x_1+1}+\dfrac{1}{x_2^2-2x_2+1}+\dfrac{1}{x_3^2-2x_3+1}, \quad \varphi(x)=x^3+x^2-1.$

§ 5. Bir necha o'zgaruvchili ko'phadlar

P maydon ustida $x_1, x_2, ..., x_n$ o'zgaruvchilardan bog'liq bo'lgan $f(x_1, x_2, ..., x_n)$ ko'phad deb

$$a\, x_1^{k_1} x_2^{k_2} ... x_n^{k_n}, \qquad (*)$$

ko'rinishdagi hadlarning chekli sondagi yig'indisiga aytiladi, bu yerda $k_i \geq 0$ $(i=\overline{1,n})$, $a - P$ maydonning elementidan iborat bo'lib, (*) hadning koeffisiyenti deb yuritiladi. $f(x_1, x_2, ..., x_n)$ ko'phadda o'xshash hadlar keltirilgan hisoblanadi va koeffisiyenti nolga teng hadlar yozilmaydi.

Ikkita $f(x_1, ..., x_n)$ va $g(x_1, ..., x_n)$ ko'phadlar teng deyiladi, agar ularning bir xil hadlari oldidagi koeffisiyentlar teng bo'lsa.

$k_1 + k_2 + ... + k_n$ yig'indi $a\, x_1^{k_1} x_2^{k_2} ... x_n^{k_n}$. hadning darajasi hisoblanadi.

$f(x_1, x_2, ..., x_n)$ *ko'phadning barcha o'zgaruvchilari bo'yicha darajasi deb* uning hadlarining eng bqori darajasiga aytiladi. Nolinchi darajali ko'phadlar – bu eto P sonlar maydoning noldan farqli elementlaridan iborat. Barcha koeffisiyentlari nolbga teng bo'lgan ko'phad nol ko'phad deb yuritiladi. Nol ko'phadning darajasi aniqlanmagan hisoblanadi. Agar $f(x_1, ..., x_n)$ ko'phadning hadlari barcha o'zgaruvchilar bo'yicha bir xil m, darajali bo'lsa, bunday ko'phad bir jinsli *ko'phad* yoki $m-$darjali n o'zgaruvchili *forma* deb yuritiladi.

$f(x_1, ..., x_n)$ *ko'phadning bita o'zgaruvchi* x_i $(i=\overline{1,n})$ ga nisbatan darajasi deb, bu ko'phadning hadlariga kirgan x_i ning eng yuqori darajasiga aytiladi (bu daraja nolga teng bo'lishi ham mumkin).

$f(x_1, ..., x_n)$ va $g(x_1, ..., x_n)$ *ko'phadlarning yig'indisi deb,* koeffisiyentlari f va g; ko'phadlarning mos darajali hadlari koeffisiyentlarining yig'indisidan iborat bo'lgan ko'phadga aytiladi..

$f(x_1, ..., x_n)$ va $g(x_1, ..., x_n)$ ko'phadalarning *ko'paytmasi deb,* f ni g ga hadma-had kshpaytirib, so'ngra o'xshash hadlari ixchamlangan *ko'phadga* aytiladi. Yuqorida kiritilgan ko'phadlarni qo'shish va ko'paytirish amallariga nisbatan P maydon ustidagi $x_1, x_2, ..., x_n$ o'zgaruvchilardan bog'liq barcha ko'phadlar to'plami kommutativ xalqa tashkil etadi va bu xalqa $P[x_1, x_2, ..., x_n]$. orqali belgilanadi.

$\alpha = a\, x_1^{k_1} x_2^{k_2} ... x_n^{k_n}$ va $\beta = b\, x_1^{l_1} x_2^{l_2} ... x_n^{l_n}$ $-$lar $f(x_1, ..., x_n) \in P[x_1, x_2, ..., x_n]$, $a \neq 0$, $b \neq 0$. ko'phadning ikkita har xil hadlari bo'lsin. α had β haddan yuqori (β had esa α haddan quyi) deyiladi, agar shunday i, $1 \leq i \leq n$, mavjud bo'lib, $k_1 = l_1, k_2 = l_2, ..., k_{i-1} = l_{i-1}$, va $k_i > l_i$. bo'lsa.

Agar $f(x_1, ..., x_n)$ ko'phadning hamma hadlari shunday tuzilgan bo'lsaki, har bir keyingi had o'zidan oldingi haddan quyi bo'lsa, u holda bu ko'phadning hadlari *leksikografik yoki lug'at bo'yicha yozilgan deyiladi* (yoki $f(x_1,...,x_n)$ ko'phad leksikografik (lug'atiy) ko'phad deyiladi).

Ko'phadning leksikografik yozuvida birinchi o'rinda turgan hadi ko'phadning yuqori hadi deyiladi. Ko'phadlar ko'paytmasining yuqori hadi ular yuqori hadlarining ko'paytmasiga teng.

1 - M i s o l. a) $3x_1^6x_2^2x_3^5$ hadning darajasi 13 ga teng;

b) $f = 3x_1\,x_2^5x_3^3 + x_2^3x_3^8 - 4x_1^6x_2^7x_3^8$ ko'phadning darajasi 21 ga teng;

s) $f = 2x_1^3x_2^2x_3^4 - 7x_1\,x_2^8 + 11x_2^3x_3^6$ - bir jinsli to'qqizinchi darajali ko'phaddan iborat. ■

$f(x_1, x_2,..., x_n)$ ko'phad $x_1, x_2,..., x_n$ o'zgaruvchilarning o'rinlarini almashtirganda ham o'zgarmasa unga *simmetrik ko'phad* deyiladi. Aniqroq qilib aytadigan bo'lsak, $\alpha -$ S_n;dan olingan o'rniga qo'yish bo'lsin, $f = f(x_1,...,x_n)$ ko'phad uchun $\alpha\ f(x_1,\dots x_n) = f\ (x_{\alpha(1)},\dots x_{\alpha(n)})$ deb olamiz. f ko'phad simmetrik ko'phad deyildai, agar barcha $\alpha \in S_n$. lar uchun $\alpha\ f = f$ tenglik o'rinli bo'lsa.

2-M i s o l. $\boldsymbol{R}[x_1, x_2, x_3, x_4]$ xalqaning quyidagi ko'phadlari simmetrik ko'phadlar bo'ladi:

a) $f = x_1 + x_2 + x_3 + x_4$; b) $g = x_1^3 + x_2^3 + x_3^3 + x_4^3$;

c) $h = x_1^2x_2^2 + x_1^2x_3^2 + x_1^2x_4^2 + x_2^2x_3^2 + x_2^2x_4^2 + x_3^2x_4^2$. ■

Quyidagi n o'zgaruvchili simmetrichek ko'phadlar

$$\sigma_1 = x_1 + x_2 + \cdots + x_n;$$
$$\sigma_2 = x_1x_2 + x_1x_3 + \cdots + x_1x_n + x_2x_3 + \cdots + x_{n-1}x_n;$$
$$\sigma_3 = x_1x_2x_3 + x_1x_2x_4 + \cdots + x_1x_2x_n + \cdots + x_{n-2}x_{n-1}x_n;$$
$$\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots$$
$$\sigma_{n-1} = x_1x_2\cdots x_{n-1} + x_1x_2\cdots x_{n-2}x_n + \cdots + x_2x_3\cdots x_n;$$
$$\sigma_n = x_1x_2\cdots x_n$$

elementar (yoki asosiy*) simmetrichek ko'phadlar deyiladi.*

Agar $g(x) = a_0x^n + a_1x^{n-1} + \cdots + a_{n-1}x + a_n -$ko'phad koeffisiyentlari P maydondan olingan bir o'zgaruvchili ko'phad bo'lsa, u holda n o'zgaruvchili elementar simmetrik ko'phadlarning qiymatlari o'zgaruvchilar $g(x)$, ko'phadning ildizlariga teng bo'lgan qiymatlarni qabul qilganda mos ravishda $-\dfrac{a_1}{a_0}, \dfrac{a_2}{a_0},...,(-1)^n\dfrac{a_n}{a_0}$. larga teng bo'ladi.

$x_1, x_2,..., x_n$ o'zgaruvchili simmetrik ko'phadning barcha hadlari ulardan bittasining o'zgaruvchilarini o'rnini almashtirish yordamida hosil qilingan bo'lsa, unga *monoge* ko'phad deyiladi. Agar $a\ x_1^{k_1}x_2^{k_2}...x_n^{k_n} -$ monogen ko'phadning yuqori hadi bo'lsa, u holda bu ko'phad $S(a\ x_1^{k_1}...x_n^{k_n})$. bilan belgilanadi.

Simmetrik ko'phadlar to'g'risida asosiy teorema

P maydon ustidagi har qanday simmmetrik ko'phadni yagona ravishda koeffisiyentlari P maydon elementlari bo'lgan elementar simmetrik ko'phadlar $\sigma_1, \sigma_2, ..., \sigma_n$ ning ko'phadi ko'rinishida tasvirlash mumkin.

Berilgan simmetrik ko'phadning elementar ko'phadlar orqali ifodasini topish uchun dastlab bu ko'phadning barcha o'zgaruvchilari bo'yicha bir xil darajaga ega bo'lgan hadlarini yig'ib bir jinsli qismlarga ajaratish kerak, so'ngra esa hosil bo'lgan har bir bir jinsli qimni alohida elementar simmetrik ko'phadlar orqali ifodalash kerak.. Bir jinsli simmetrik $f(x_1,...,x_n)$ ko'phadni elementarныye simmetrik ko'phadlar orqali ifodalash uchun uning yuqori hadi $a\, x_1^{k_1} ... x_n^{k_n}$, ni olib, bu hadning ko'rsatkichlari $k_1, k_2, ..., k_n$ larni yozib chiqish kerak, so'ngra quyidagi xossalarga ega bo'lgan $l_1, l_2, ..., l_n$ sonlarning mumkin bo'lgan majmualarini yozib chiqish kerak:

1) Har bir majmuada $l_1 + l_2 + ... + l_n$ yig'indi bir xil bo'lib, u $k_1 + k_2 + ... + k_n$; ga teng bo'lishi kerak.

2) har bir majmuaning sonlari quyidagi tartibda joylashadi $l_1 \geq l_2 \geq ... \geq l_n$

3) $x_1^{l_1} x_2^{l_2} ... x_n^{l_n}$ had $x_1^{k_1} x_2^{k_2} ... x_n^{k_n}$. haddan yuqori emas.

Shundan so'ng har bir $l_1, l_2, ..., l_n$ majmua uchun $\sigma_1^{l_1 - l_2} \sigma_2^{l_2 - l_3} \cdots \sigma_{n-1}^{l_{n-1} - l_n} \sigma_n^{l_n}$ ko'paytmalarni tuzib chiqiladi va $f(x_1, x_2, ..., x_n)$ ko'phad aniqmas koeffisiyentlar orqali tuzilgan ko'paytmalarning yig'indisiga tenglashtiriladi ($\sigma_1^{k_1 - k_2} \cdots \sigma_{n-1}^{k_{n-1} - k_n} \sigma_n^{k_n}$ hadning koeffisiyenti a_0 ga teng qilib olinadi). Hosil bo'lgan tenglikning ikkala tomonidagi $x_1, x_2, ..., x_n$ o'zgaruvchilarga har xil usullar bilan qiymatlar berilib aniqmas koeffisiyentlar topiladi, natijada $f(x_1, x_2, ..., x_n)$ ko'phadning elementar simmetrik ko'phadlar orqali ifodasi topiladi.

3 - M i s o l.

$$f(x_1, x_2, x_3) = x_1^4 x_2 + x_1^4 x_3 + x_1 x_2^4 + x_1 x_3^4 + x_2^4 x_3 + x_2 x_3^4$$

ko'phadni asosiy simmetrik ko'phadlar orqali ifodasini toping.

Yechish. Bu yerda $f(x_1, x_2, x_3) -$ bir jinsli simmetrik ko'phaddan iborat.

1) f ko'phadning yuqori hadi $x_1^4 x_2$; ga teng.

2) ko'rsatkichlarning mumkin bo'lgan barcha majmualari uchun va ularga mos keladigan $\sigma_1^{l_1 - l_2} \sigma_2^{l_2 - l_3} \cdots \sigma_{n-1}^{l_{n-1} - l_n} \sigma_n^{l_n}$ ko'paytmalar uchun quyidagi jadvalni tuzamiz:

Ko'rsatkichlar majmuasi	$\sigma_1^{l_1 - l_2} \cdots \sigma_n^{l_n}$
4 1 0	$\sigma_1^3 \sigma_2$
3 2 0	$A\,\sigma_1 \sigma_2^2$
3 1 1	$B\,\sigma_1^2 \sigma_3$
2 2 1	$C\,\sigma_2 \sigma_3$

3) $f=\sigma_1^3\sigma_2+A\,\sigma_1\sigma_2^2+B\,\sigma_1^2\sigma_3+C\,\sigma_2\sigma_3$, bu yerda *A,V,S*—noma'lum koeffisiyentlar

4) $x_1, x_2,...x_n$ o'zgaruvchilarga har xil qiymatlar berib, hosil bo'lgan qiymalrani quyidagi jadvalga kiritamiz:

x_1	x_2	x_3	f	σ_1	σ_2	σ_3
1	1	1	6	3	3	1
1	1	0	2	2	1	0
1	1	-1	2	1	-1	-1

5) 4 va 3 bandlardan quyidagi sistemani hosil qilamiz:

$$\begin{cases} 6=81+27A+9B+3C \\ 2=8+2A \\ 2=-1+A-B+C \end{cases};$$

6) Bu sistemani yechib, $A=-3,\quad B=-1,\quad C=5;$ larni hosil qilamiz.

7) 3 banddagi ko'phadga koeffisiyentlarning topilgan qiymatlarini qo'yib:

$$f=\sigma_1^3\sigma_2-3\sigma_1\sigma_2^2-\sigma_1^2\sigma_3+5\sigma_2\sigma_3.$$

ni hosil qilamiz.■

M i s o l 4.

$f(x_1,x_2,...,x_n)=S(x_1^3)+2x_1^2+2x_2^2+\cdots+2x_n^2$

ko'phadni asosiy simmetrik ko'phadlar orqali ifodasini toping.

Yechish. $f(x_1,x_2,...,x_n)$ ko'phadni bir jinsli qimlarga ajratamizx:

$$f_1(x_1,...,x_n)=S(x_1^3) \text{ va } f_2(x_1,...,x_n)=S(2x_1^2).$$

$f_1=x_1^3+x_2^3+\cdots+x_n^3$ ni elementar simmetrik ko'phadlar orqali ifodasini topamiz:

1) f_1 ko'phadning yuqori hadi x_1^3;

2) ko'rsatkichlarning mumkin bo'lgan barcha majmualari uchun va ularga mos keladigan ko'paytmalar uchun quyidagi jadvalni tuzamiz:

Ko'rsatkichlar majmuasi	$\sigma_1^{l_1-l_2}\cdots\sigma_n^{l_n}$
3 0 0	σ_1^3
2 1 0	$A\,\sigma_1\sigma_2$
1 1 1	$B\,\sigma_3$

3) $f_1=\sigma_1^3+A\,\sigma_1\sigma_2+B\,\sigma_3$, bu yerda *A va V* – noma'lum koeffisiyentlar;

4) $x_1, x_2,...x_n$ o'zgaruvchilarga har xil qiymatlar berib quyidagi jadvalni tuzamiz:

x_1	x_2	x_3	…	x_n	f_1	σ_1	σ_2	σ_3
1	1	1	…	1	n	n	C_n^2	C_n^3
1	1	0	…	0	2	2	1	0

5) 4 va 3 bandlardan quyidagi sistemani hosil qilamiz:

$$\begin{cases} n = n^3 + nC_n^2 A + C_n^3 B \\ 2 = 8 + 2A + 0 \cdot B \end{cases};$$

6) 5 banddagi sistemani yechib: $A = -3, \quad B = 3;$ larni topamiz.

7) $f_1 = \sigma_1^3 - 3\sigma_1 \sigma_2 + 3\sigma_3$.

Xuddi shunga o'xshash $f_2(x_1,...,x_n) = 2\sigma_1^2 - 4\sigma_2$. ni topamiz.

Shunday qilib, $f(x_1,...x_n) = \sigma_1^3 - 3\sigma_1 \sigma_2 + 3\sigma_3 + 2\sigma_1^2 - 4\sigma_2$. ■

Simmetrik ko'phadlar to'g'risidagi asosiy teoremadan foydalanib, $g(x)$ ko'phadning ildizlarini hisoblamasdan turib, ulardan tuzilgan ixtiyoriy simmetrik ko'phadning qiymatini hisoblash mumkin.

M i s o l 5. $g(x) = 7x^4 - 14x^3 - 7x + 2.$ ko'phadning ildizlari $\alpha_1, \alpha_2, \alpha_3, \alpha_4$ larning kublari yig'inidisini simmetrik ko'phadlar orqali ifodalab, uning qiymatini toping.

Yechish. Quyidagi simmetrik ko'phad $f(x_1, x_2, x_3, x_4) = x_1^3 + x_2^3 + x_3^3 + x_4^3$ ni elementar simmetrik ko'phadlar orqali ifodasini topamiz:

$$f(x_1, x_2, x_3, x_4) = \sigma_1^3 - 3\sigma_1\sigma_2 + 3\sigma_3.$$

O'zgaruvchilarning quyidagi qiymatlari $x_1 = \alpha_1, \quad x_2 = \alpha_2, \quad x_3 = \alpha_3, \quad x_4 = \alpha_4$ -ni elnementar simmetrik ko'phadlarga qo'yib $\sigma_1 = 2, \quad \sigma_2 = 0, \quad \sigma_3 = 1.$ ni hosil qilamiz. Bu yerdan $f(\alpha_1, \alpha_2, \alpha_3, \alpha_4) = \alpha_1^3 + \alpha_2^3 + \alpha_3^3 + \alpha_4^3 = 8 - 0 + 3 = 11$. ■

$S_k = x_1^k + x_2^k + \cdots + x_n^k, \quad k = 1, 2,....$ simmetrik ko'phadlar darajali yig'indilar deyiladi. Elementar simmetrik ko'phadlar *Nyuton formulalari* bilan quyidagicha bohlangan *:*

$$S_k - S_{k-1}\sigma_1 + S_{k-2}\sigma_2 - \cdots + (-1)^{k-1} S_1 \sigma_{k-1} + (-1)^k k\sigma_k = 0, \quad k \le n,$$

$$S_k - S_{k-1}\sigma_1 + S_{k-2}\sigma_2 - \cdots + (-1)^n S_{k-n}\sigma_n = 0, \quad k > n.$$

Bu formulalardan $S_1, S_2,...$ ifodalarni ketmag'ket ravishda $\sigma_1, \sigma_2, ..., \sigma_n$ lar orqali topish mumkin va aksincha.

6 - M i s o l . $n \ge 3$ bo'lsin. U holda $S_1 = \sigma_1; \quad S_2 - S_1\sigma_1 + 2\sigma_2 = 0,$
bu yerdan $S_2 = \sigma_1^2 - 2\sigma_2; \quad S_3 - S_2\sigma_1 + S_1\sigma_2 - 3\sigma_3 = 0,$ bu yerdan esa $S_3 = \sigma_1^3 - 3\sigma_1\sigma_2 + 3\sigma_3$. ■

M A Sh Q L A R

119. Quyidagi simmetrik ko'phadlarni elementar simmetrik ko'phadlar orqali ifodalang:

a) $x_1^3 + x_2^3 + x_3^3 + 3x_1 x_2 x_3$;

b) $x_1^2 x_2 + x_1 x_2^2 + x_1^2 x_3 + x_1 x_3^2 + x_2^2 x_3 + x_2 x_3^2$;

c) $x_1^4 + x_2^4 + x_3^4 - 2x_1^2 x_2^2 - 2x_2^2 x_3^2 - 2x_3^2 x_1^2$;

d) $x_1^5 x_2^2 + x_1^2 x_2^5 + x_1^5 x_3^2 + x_1^2 x_3^5 + x_2^5 x_3^2 + x_2^2 x_3^5$;

e) $(x_1 + x_2)(x_1 + x_3)(x_2 + x_3)$;

f) $(x_1^2+x_2^2)(x_1^2+x_3^2)(x_2^2+x_3^2)$;

g) $(2x_1-x_2-x_3)(2x_2-x_1-x_3)(2x_3-x_1-x_2)$;

h) $(x_1-x_2)^2(x_1-x_3)^2(x_2-x_3)^2$.

120. Quyidagi ko'phadlarni elementar simmetrik ko'phadlar orqali ifodalang:

a) $(x_1+x_2)(x_1+x_3)(x_1+x_4)(x_2+x_3)(x_2+x_4)(x_3+x_4)$;

b) $(x_1x_2+x_3x_4)(x_1x_3+x_2x_4)(x_1x_4+x_2x_3)$;

c) $(x_1+x_2-x_3-x_4)(x_1-x_2+x_3-x_4)(x_1-x_2-x_3+x_4)$.

121. *p* o'zgaruvchili monogen ko'phadlarni elementar simmetrik ko'phadlar orqali ifodalang:

a) $S(x_1^2)$; b) $S(x_1^2x_2x_3)$; c) $S(x_1^2x_2^2)$; d) $S(x_1^3x_2)$;

e) $S(x_1^4)$; f) $S(x_1^2x_2^2x_3)$; g) $S(x_1^3x_2x_3)$; h) $S(x_1^3x_2^2)$;

i) $S(x_1^4\,x_2)$; j) $S(x_1^4)$; k) $S(x_1^2x_2^2x_3x_4)$; l) $S(x_1^2x_2^2x_3^2)$;

m) $S(x_1^3x_2x_3x_4)$; n) $S(x_1^3x_2^2x_3)$; o) $S(x_1^3x_2^3)$; p) $S(x_1^4x_2x_3)$;

q) $S(x_1^4x_2^2)$; r) $S(x_1^5x_2)$; s) $S(x_1^6)$.

122. $S(x_1^2x_2^2\cdots x_k^2)$. monogen ko'phadni elementar simmetrik ko'phadlar orqali ifodalang.

123. Quyidagi kasrlarni elementar simmetrik ko'phadlar orqali ifodalang:

a) $\dfrac{x_1}{x_2}+\dfrac{x_2}{x_3}+\dfrac{x_3}{x_1}+\dfrac{x_2}{x_1}+\dfrac{x_3}{x_2}+\dfrac{x_1}{x_3}$; b) $\dfrac{(x_1-x_2)^2}{x_1+x_2}+\dfrac{(x_2-x_3)^2}{x_2+x_3}+\dfrac{(x_3-x_1)^2}{x_3+x_1}$;

c) $\left(\dfrac{x_2}{x_1}+\dfrac{x_3}{x_2}+\dfrac{x_1}{x_3}\right)\left(\dfrac{x_1}{x_2}+\dfrac{x_2}{x_3}+\dfrac{x_3}{x_1}\right)$.

124. Quyidagi yig'indilarni elementar simmetris ko'phadlar orqali ifodaldang: a) $\sum\dfrac{1}{x_i}$; b) $\sum\dfrac{1}{x_i^2}$; c) $\sum_{i\neq j}\dfrac{x_i}{x_j}$.

125. Quyidagi tenglama ildizlari kvadratlari yig'indisini hisoblang:

$$x^3+2x-3=0.$$

126. $x_1^3x_2+x_1x_2^3+x_2^3x_3+x_2x_3^3+x_3^3x_1+x_3x_1^3$ ifodaning qiymatini hisoblang, bu yerda o'zgaruvchilar $x^3-x^2-4x+1=0.$ tenglamaning ildizlaridan iborat.

127. $S(x_1^3x_2x_3)$ monogen ko'phadning o'zgaruvchilari $f(x)=x^4+x^2-2x^2-3x+1=0.$ ko'phadning ildizlariga teng bo'lganda, uning qiymatini hisoblang.

128. Simmetrik $S(x_1,x_2,...,x_n)$ ko'phadning $g(x)$ ko'phad ildizlari orqali qiymatini hisoblang:

a) $f=S(x_1^4x_2)$, $g(x)=3x^3-5x^2+1$;

b) $f=S(x_1^3x_2^3)$, $g(x)=3x^4-2x^2+x-1$;

c) $f=(x_1^2+x_1x_2+x_2^2)(x_2^2+x_2x_3+x_3^2)(x_3^2+x_1x_3+x_1^2)$, $g(x)=5x^3-6x^2+7x-8$.

129. Quyidagi simmetrik fnuksiyalarni $a_0x^3+a_1x^2+a_2x+a_3=0$ tenglamaning koeffisiyentlari orqali ifodalang;

a) $a_0^4(x_1-x_2)^2(x_1-x_2)^2(x_2-x_3)^2$;

b) $a_0^4(x_1^2-x_2x_3)(x_2^2-x_1x_3)(x_3^2-x_1x_2)$;

c) $\dfrac{(x_1-x_2)^2}{x_1x_2}+\dfrac{(x_1-x_3)^2}{x_1x_3}+\dfrac{(x_2-x_3)^2}{x_2x_3}$;

d) $a_0^4(x_1^2+x_1x_2+x_2^2)(x_2^2+x_2x_3+x_3^2)(x_3^2+x_3x_1+x_1^2)$.

130. Nyuton formulalaridan foydalanib S_4, S_5, S_6 larni elementar simmetrik ko'phadlar orqali ifodalang.

131. $\sigma_2, \sigma_3, \sigma_4, \sigma_5, \sigma_6$ larni Nyuton formulalaridan foydalanib $S_1, S_2, ...,$ darajali ifodalar orqali ifodalang.

132. Quyidagi ko'phadlar ildizlarining $k-x$ darajalari yig'indisin hisoblang;

a) $x^6-4x^5+3x^3-4x^2+x+1, \quad k=5$; b) $x^4-x^3-1, \quad k=8$;

c) $x^3-3x+1, \quad k=10$; d) $x^5-x^4-x-1, \quad k=5$;

e) $x^5-x^4-x^3-1, \quad k=6$.

133. $S_1, S_2, ..., S_n$ larni

$$x^n+\frac{x^{n-1}}{1!}+\frac{x^{n-2}}{2!}+\cdots+\frac{1}{n!}=0.$$

tenglamaning ildizlaridan foydalanib toping.

134. Agar x_1, x_2, x_3 $-$lar $3x^3-4x^2+6x+10$. ko'phadning ildizlari bulsa, ildizlari $x_1^2, x_2^2, x_3^2,$ lardan iborat bulgan kupxadni tuzing.

135. Agar uchinchi darajali ko'phadning ildizlari $x_1^3+x_1, \quad x_2^3+x_2, \quad x_3^3+x_3,$ ifodalardan iborat bo'lsa, bu ko'phadni tuzing, bu yerda x_1, x_2, x_3 $-$lar $2x^3+6x^2+6x-3$. ko'phadning ildizlaridan iborat.

136. $S_1=S_2=\cdots=S_{n-1}=0$. shartni qanoatlantiradigan $n-$darajali tenglamani toping.

137. $S_2=S_3=\cdots=S_n=0$. shartni qanoatlantiradigan $n-$darajali tenglamani toping.

138*. $S_k=\begin{vmatrix} \sigma_1 & 1 & 0 & \cdots & 0 \\ 2\sigma_2 & \sigma_1 & 1 & \cdots & 0 \\ 3\sigma_3 & \sigma_2 & \sigma_1 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ k\sigma_k & \sigma_{k-1} & \sigma_{k-2} & \cdots & \sigma_1. \end{vmatrix}$ ekanligini isbotlang.

139. $\sigma_k = \dfrac{1}{k!}\begin{vmatrix} S_1 & 1 & 0 & \cdots & 0 \\ S_2 & S_1 & 2 & \cdots & 0 \\ S_3 & S_2 & S_1 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ S_k & S_{k-1} & S_{k-2} & \cdots & S_1. \end{vmatrix}$ tenglikni isbotlang

140. $\begin{vmatrix} x^n & x^{n-1} & x^{n-2} & \cdots & 1 \\ S_1 & 1 & 0 & \cdots & 0 \\ S_2 & S_1 & 2 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ S_n & S_{n-1} & S_{n-2} & \cdots & n. \end{vmatrix}$ determinantni hisoblang.

JAVOBLAR va KO'RSATMALAR

§ 1

1. a) $q(x)=x^2-2x+4$, $r(x)=3x+11$;

b) $q(x)=5x^2-15x+34$, $r(x)=-72x-62$;

s) $q(x)=0$, $r(x)=2x^2-3x+1$;

d) $q(x)=2x^2+3x+11$, $r(x)=25x-5$.

2. a) $a=0, \quad q=-1$; b) $a=-3$, $q=8$; c) $p=a^3-2a, \quad q=a^2-1$;

d) $a=\pm 3, \quad q=1; \quad a=0, \quad q=-8$.

3. a) $r(x)=0, \quad q(x)=x^2+2 \quad \boldsymbol{Z}_3[x]$; $r(x)=3x-3, q(x)=x^2+2 \quad \boldsymbol{Z}_5[x]$ da va $\boldsymbol{Q}[x]$ da;

b) $r(x)=2x+1, \quad q(x)=2x+2$ v $\boldsymbol{Z}_3[x]$; $r(x)=2x, q(x)=2x^2$ v $\boldsymbol{Z}_5[x]$; $r(x)=2x+1, q(x)=2x^2+5$ v $\boldsymbol{Q}[x]$.

5. EKUB: a) $x+1$; b) x^2+1; c) x^3+1; d) 1; e) 1. EKUK: a) $\dfrac{f(x)g(x)}{x+1}$; b) $\dfrac{f(x)g(x)}{3(x^2+1)}$; s) $\dfrac{f(x)g(x)}{3(x^3+1)}$; d) $f(x)g(x)$; e) $f(x)g(x)$.

6. a) $x^2-2=(-x-1)f(x)+(x+2)g(x)$;

b) $x^3+1=-f(x)+(x+1)g(x)$;

c) $x-1=-\dfrac{x-1}{3}f(x)+\dfrac{2x^2-2x-3}{3}g(x)$;

d) $1=xf(x)+(-3x^3-x+1)g(x)$;

e) $1=(-x-1)f(x)+(x^3+x^2-3x-2)g(x)$;

f) $1=\dfrac{-x^2+3}{2}f(x)+\dfrac{x^4-2x^2-2}{2}g(x)$.

9. a) $\varphi(x)=4-3x, \quad \psi(x)=1+2x+3x^2$;

b) $\varphi(x)=\dfrac{1}{16}(x^2-6x+9), \quad \psi(x)=-\dfrac{1}{16}(x^3-3x^2-4)$;

c) $\varphi(x)=\dfrac{1}{17}(6x-11), \psi(x)=-\dfrac{1}{17}(6x^2-5x+25)$;

d) $\varphi(x)=\dfrac{-16x^2+37x+26}{3}, \quad \psi(x)=\dfrac{16x^3-53x^2-37x-23}{3}$.

10. a) x^3-3x+3; b) $4x^4-27x^3+66x^2-65x+24$; c) x^4-3x^2+1.

11. a) $\dfrac{-3+7\sqrt[3]{2}-\sqrt[3]{4}}{23}$; b) $1+3\sqrt[4]{2}+2\sqrt{2}-\sqrt[4]{8}$;

c) $\dfrac{1}{5}(1+\sqrt{2}-\sqrt[3]{2})(1+\sqrt[3]{4})$.

12. a) $\varphi(x)=9x^2-26x-21, \quad \psi(x)=-9x^3+44x^2-39x-7$;

b) $\varphi(x)=3x^3+3x^2-7x+2, \quad \psi(x)=-3x^3-6x^2+x+2.$

13. $\varphi(x)=1+\frac{n}{1}x+\frac{n(n+1)}{1\cdot 2}x^2+\dots+\frac{n(n+1)\cdots(n+m-2)}{1\cdot 2\cdots(m-1)}x^{m-1}$,

$$\psi(x)=1+\frac{m}{1}(1-x)+\frac{m(m+1)}{1\cdot 2}(1-x)^2+\dots+\frac{m(m+1)\cdots(m+n-2)}{1\cdot 2\cdots(n-1)}(1-x)^{n-1}=$$

$$=\frac{(m+1)(m+2)\cdots(m+n-1)}{(n-1)!}-\frac{m}{1}\frac{(m+2)\cdots(m+n-1)}{(n-2)!}x+$$

$$+\frac{m(m+1)}{1\cdot 2}\frac{(m+3)\cdots(m+n-1)}{(n-3)!}x^2-\dots+(-1)^{n-1}\frac{m(m+1)\cdots(m+n-2)}{(n-1)!}x^{n-1}.$$

Ko’rsatma. $(1-x)^n$ ga bo’linsin va $m-1$ marta differensiallab, har bir differensiallashdan so’ng $x=0$ deb olinsin. $\psi(x)$ darajasi m dan kichikligi, $\varphi(x)$ ning darajasi n dan kichikligidan foydalanish lozim.

14. a) $x+2$ $\boldsymbol{Z}_3[x]$ da , bir $\boldsymbol{Z}_5[x]$ va $\boldsymbol{Q}[x]$ da.

b) bir $\boldsymbol{Z}_3[x]$ da, x^3+3x+2 $\boldsymbol{Z}_5[x]$ da, $x+1$ $\boldsymbol{Q}[x]$ da.

s) bir $\boldsymbol{Z}_3[x]$ da, $x-2$ $\boldsymbol{Z}_5[x]$ da, bir $\boldsymbol{Q}[x]$ da.

15. a) $(f(x),g(x))=x^2+x+1, \quad \varphi(x)=x+1, \quad \psi(x)=x^2;$

b) $(f(x),g(x))=x+1, \quad \varphi(x)=x, \quad \psi(x)=x^2+1;$

s) $(f(x),g(x))=1, \quad \varphi(x)=x+1, \quad \psi(x)=x^2;$

d) $(f(x),g(x))=1, \quad \varphi(x)=x^3+x, \psi(x)=x^4+x+1.$

§ 2

16. a) 0; b) 1.

17. a) $q(x)=x^3-x^2+3x-3, \quad r(x)=5;$

b) $q(x)=2x^4-6x^2+13x^2-39x+109, \quad r(x)=-327;$

c) $q(x)=4x^2-(3+4i)x+7i-1, \quad r(x)=8-6i;$

d) $q(x)=x^2-2ix-5-2i, \quad r(x)=8i-9.$

18. a) 136; b) 286; c) $\frac{151}{16}$; d) -1-44i ; e) $5+22i$.

19. a) $(x+1)^4-2(x+1)^3-3(x+1)^2+4(x+1)+1;$

b) $(x-1)^5+5(x-1)^4+10(x-1)^3+10(x-1)^2+5(x-1)+1;$

c) $(x-2)^4-18(x-2)+38;$

d) $(x+i)^4-2i(x+i)^3-(1+i)(x+i)^2-5(x+i)+7+5i;$

e) $(x+1-2i)^4-(x+1-2i)^3+2(x+1-2i)+1.$

20. a) $x^4+11x^3+45x^2+81x+55$; b) $2x^4+13x^3+35x^2+54x+39$;

c) $x^4-4x^3+6x^2+2x+8$; d) $x^5+15x^4+88x^3+255x^2+376x+229$.

21. a) 3; b) 4; c) 2.

22. x.

23. $-x^2-\frac{9}{2}x+\frac{3}{2}.$

24. $-\frac{1}{6}x^3+x^2+\frac{1}{6}x.$

25. a) $6x+1;$ b) 1; c) $3x^3+3x+1.$

28. $b=9a^2,\quad 1728a^5+c^2=0.$

29. $a=-5.$

30. $a=3,\quad b=-4.$

31. a) $p=-\frac{8}{3},\ q=2,\ r=-\frac{1}{3};$ b) $p=-\frac{1}{3},\ q=2,\ r=-\frac{8}{3}.$

32. a) $a=-3,\quad k=3;$ b) $a=-4,\quad k=2;$ c) $a=4,\quad k=2.$

34. *Yechilishi.* $f(x)$ ko'phad $(x-1)^{k+1}$ ga bo'linishi uchun $f(1)=a_0+a_1+\dots+a_n=0$ tenglikning o'rinli bo'liishi zarur va yetarlidir, $f'(x)$ ning $(x-1)^k$ ga bo'linishi uchun $f(1)=0$ shart bajarilganda $f_1(x)=nf(x)-xf'(x)$ ning $(x-1)^k$ ga bo'linishi zarur va yeatrlidir. $f_1(x)$ ni formal ravishda n-darajali ko'phad deb qarab yuqoridagi mulohazalarni k marta takrorlaymiz.

35. *Yechilishi*. Berilgan ko'phadning hosialsi $x^{n-m-1}[nx^m+(n-m)a]$ noldan farqli karrali ildizlarga ega emas.

36. *Yechilishi*. $(m,n)=d,\quad m=dm_1,\quad n=dn_1$ deb olib, $(-1)^{n_1}(n_1-m_1)^{n_1-m_1}m_1^{m_1}a^{n_1}=b^{m_1}n_1^{n_1}$ shartni hosil qilamiz.

37. *Ko'rsatma.* Matematik induksiya metodi bilan isbot qilinadi .

38. *Ko'rsatma.* $f(x)$ ko'phadning noldan farqli $(k-1)-$karrali ildizi $xf'(x)$ ko'phadning $(k-2)-$karrali ildizidan iborat va hakoza. Agar $x_1\neq 0-$ $a_1x^{m_1}+\dots+a_kx^{m_k}$, ko'phadning $(k-1)-$karrali ildizidan ibyuorat bo'lsa, u holda $a_1x^{m_1},\dots,a_kx^{m_k}$ sonlar quyidagi bir jinsli sistemaning yechimidan iborat bo'ladi:

$$\left.\begin{array}{l} z_1+z_2+\dots+z_k=0 \\ m_1z_1+m_2z_2+\dots+m_kz_k=0 \\ \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \\ m_1^{k-2}z_1+m_2^{k-2}z_2+\dots+m_k^{k-2}z_k=0 \end{array}\right\}$$

va demak, ular $\frac{\Delta}{\varphi'(m_1)},\dots,\frac{\Delta}{\varphi'(m_k)}$ - sonlarga proporsional bo'ladi, bu Δ- Vandermond determinantidan iborat.

39. *Yechilishi*. Agar $f(x)$ ko'phad $f'(x)$ ga bo'linsa, u holda bo'linma ko'phad bosh koeffisiyenti $\frac{1}{n}$ ga teng bo'lgan chiziqli ko'phaddan iborat, bu yerda $n-$ $f(x)$ ning darajasidan iboat. Shuning uchun $n\,f(x)=(x-x_0)\,f'(x)$. Bu tenglikni differensiallab, $(n-1)\,f'(x)=(x-x_0)\,f''(x)$ ni topamiz, va hakoza, bu yerdan $f(x)=\frac{(x-x_0)^n}{n!}f^{(n)}(x)=a_0(x-x_0)^n.$ Teskarisi ko'0rinib turibdi.

40. *Yechilishi.* $f(x)=1+\frac{x}{1}+\cdots+\frac{x^n}{n!}$ ko'phadning karrali ildizi

$$f'(x)=1+\frac{x}{1}+\cdots+\frac{x^{n-1}}{(n-1)!}=f(x)-\frac{x^n}{n!}$$

Ko'phadning ildizidan iborat. Demak, agar $f(x_0)=f'(x_0)=0$ bo'lsa, u holda $x_0=0$, ammo nol $f(x)$ ko'phadning ildizi emas.

41. *Yechilishi. Agar* $f(x)=(x-x_0)^k\ f_1(x)$, bu yerda $f_1(x)-$kasr–rasional funksiya, ko'phad $x=x_0$ da nolga aylanmasa, u holda bevosita differensiallash natijasida quyidagini hosil qilamiz:

$$f(x_0)=f'(x_0)=\cdots=f^{(k-1)}(x_0)=0,\quad f^{(k)}(x_0)\neq 0.$$

42. *Yechilishi.*

$$g(x)=\frac{\psi(x)}{\omega(x)}=f(x)-f(x_0)-\frac{f'(x_0)}{1!}(x-x_0)-\cdots-\frac{f^{(n)}(x_0)}{n!}(x-x_0)^n$$

funksiya $g(x_0)=g'(x_0)=\cdots=g^{(n)}(x_0)=0$ shartni qanoatlantiradi. Demak, $\psi(x)=(x-x_0)^{n+1}\ F(x)$, bu yerda $F(x)-$ko'phad, shuni isbotlash talab etilgan edi.

43. *Yechilishi. Agar* $f_1(x)f_2(x_0)-f_2(x)f_1(x_0)$ ko'phad aynan nolga teng bo'lmasa, u holda $f_1(x_0)\neq 0$ deb hisoblash mumkin. $\frac{f_2(x)}{f_1(x)}-\frac{f_2(x_0)}{f_1(x_0)}$. kasr-rasional funksiyani qaraymiz. U aynan nolga teng emas va x_0 - uning ildizidan iborat. Uning karraligi $\frac{f_1(x)f_2'(x\ )-f_2(x)f_1'(x\ )}{[f_1(x)]^2}$ hosila ildizi x_0 ning karraliligidan bir birlikka katta. Bu yerdan berilgan tasdiqning isboti kelib chiqadi.

44. *Yechilishi.* x_0 -- $[f'(x)]^2-f(x)f''(x)$ ko'phadning k karrali ildizi bo'lsin. U holda $f(x_0)\neq 0,$ chunki, aks holda x_0 $f(x)$ va $f'(x)$ larningn umumiy ildizidan iborat bo'lar edi. Oldingi masalaga asosan x_0 -- darajasi n dan oshmaydigan $f(x)f'(x_0)-f(x_0)f'(x)$ ko'phadning $k+1$ karrali ildizidan iborat. Demak, $k+1\leq n,\quad k\leq n-1.$

45. *Yechilishi.* $f(x)f'(x_0)-f(x_0)f'(x)$ ko'phad $n-$karrali x_0 ildizga ega bo'lishi kerak, ya'ni u $A(x-x_0)^n$ ga teng bo'lishi lozim, bu yerda $A-$o'zgarmas son. $x-x_0$ ning darajalari bo'yicha yoyib, $x-x_0=z$ almashtirishdan so'ng $(a_0+a_1z+a_2z^2+\cdots+a_nz^n)a_1-(a_1+2a_2z+3a_3z^2+\cdots+na_nz^{n-1})a_0=Az^n$ ni hosil qilamiz, bu yerda $a_0=f(x_0)\neq 0.$ Bu yerdan esa $a_2=\frac{a_1^2}{2a_0},\quad a_3=\frac{a_1^3}{a_0^2 3!},\ldots,a_n=\frac{a_1^n}{a_0^{n-1}n!}$ kelib chiqadi. $\frac{a_1}{a_0}=\alpha$ almashtirish olib,

$$f(x)=a_0\left[1+\frac{\alpha(x-x_0)}{1!}+\frac{\alpha^2(x-x_0)^2}{2!}+\cdots+\frac{\alpha^n(x-x_0)^n}{n!}\right]$$

ni hosil qilamiz.

46. *Yechilishi.* $u(x,y)=\frac{1}{2}(f(x+yi)+\bar{f}(x-yi)$ va $\upsilon(x,y)=\frac{1}{2i}(f(x+yi)-\bar{f}(x-yi))$ bo'lganligi uchun $u=0,\quad \upsilon=0$ sistema $f(x+yi)=0,\ \bar{f}(x-yi)=0$ sistemaga teng kuchli, bu yerdan $x+yi=z_k,\quad x-yi=\bar{z}_m$ и $x=\frac{z_k+\bar{z}_m}{2},\quad y=\frac{z_k-\bar{z}_m}{2i}$. Bu yerda $z_k,\quad k=\overline{1,n}-$ sonlar $f(z)$ ko'phadning ildizlari, k va m indekslar esa o'zaro bog'liqsiz ravishda 1 dan n gacha bo'lgan qiymatlarni qabul qiladi.

47. a) $(x-1)^3(x+1)$;

b) $(x^2-4)^3(x^2+4)^2$;

s) $(x^2+1)(x^4+1)^3\cdot(x^6+1)^5(x^8+1)^7$.

48. $\frac{1}{2}(-1\pm i\sqrt{3}),\quad \frac{1}{3}(1\pm i\sqrt{2})-\quad f(x)$ ninng ildizlari, $\frac{1}{2}(1\pm i\sqrt{3}),\quad \frac{1}{3}(1\pm i\sqrt{2})$- $g(x)$ ning ildizlaridan iborat.

49. a) $\varphi(x)=(x+1)(x-3),\quad f(x)=(x+1)^4(x-3)^2$;

b) $\varphi(x)=x^2-1,\quad f(x)=(x-1)^4(x+1)^2$;

s) $\varphi(x)=(x-1)(x^2+1),\quad f(x)=(x-1)^4(x+i)(x-i)$;

d) $\varphi(x)=(x-1)(x-2),\quad f(x)=(x-1)^2(x-2)^3$.

50. a) $x^4+4x^3-7x^2-22x+24$;

b) $x^4+(3-i)x^3+(3-3i)x^2+(1-3i)x-i$;

s) $x^4-3x^3+2x^2+2x-4$;

d) $x^4-19x^2-6x+72$.

51. a) $\frac{2}{3}$ va -$\frac{2}{3}$; b) a^2 va $(-1)^n b$.

52. –1 va $(-1)^{n-1}$.

53. a) –216; b) –25 yoki –16.

54. $p^3+4pq+8r=0$.

55. $p^2s=r$.

57. $\alpha_1=1,\quad \alpha_2=3,\quad \alpha_3=5,\quad \alpha_4=7$.

58. $a_1^2-2a_2,\quad -a_1^3+3a_1a_2-3a_3$.

59. $9x^3+20x^2+116x-100$.

60. a) $-\alpha_1,\quad \alpha_2,...,\quad -\alpha_n$; b) $\frac{1}{\alpha_1},\quad \frac{1}{\alpha_2},...,\quad \frac{1}{\alpha_n}$;

c) $\alpha_1-a,\quad \alpha_2-a,...,\quad \alpha_n-a$; d) $b\alpha_1,\quad b\alpha_2,...,\quad b\alpha_n$.

61. $\lambda=-3$.

62. $q^3+pq+q=0$.

63. $x^4-ax^2+1=0$, bu yerda $a=\frac{\alpha^4+1}{\alpha^2}$. *Ko'rsatma.* Berilgan tenglama x ni $-x$ ga va x ni $\frac{1}{x}$ ga almashtirish natijasida o'zgarmasligidan foydalanish kerak.

64. $(x^2-x+1)^3-a(x^2-x)^2=0,$ bu yerda $a=\dfrac{(\alpha^2-\alpha+1)^3}{(\alpha^2-\alpha)^2}$. *Ko'rsatma.* Berilgan tenglama x ni $\dfrac{1}{x}$ ga va x ni $1-x$ ga almashtirish natijasida o'zgarmasligidan foydalanish kerak.

66. a) $-\frac{1}{3}(x-2)(x-3)(x-4)+\frac{1}{2}(x-1)(x-3)(x-4)-2(x-1)(x-2)(x-4)+$
$+\frac{1}{2}(x-1)(x-2)(x-3)=-\frac{4}{3}x^3+10x^2-\frac{65}{3}x+15$;

b) $\frac{1}{2}(5-(1-i)x-x^2-(1+i)x^3)$; c) x^3-3x+1; d) x^4-x^2+1.

67. a) $x+1+\dfrac{1}{24}x(x-1)(x-2)(x-3)$;

b) $-x^4+4x^3-x^2-7x+5$;

c) $1+\dfrac{2}{5}(x-1)-\dfrac{1}{105}(x-1)(4x-9)+\dfrac{1}{945}(x-1)(4x-9)(x-4)$,
$f(2)=1\dfrac{389}{945}$;

d) $x^3-9x^2+21x-8$.

68. $f(x)=\dfrac{n+1}{2}-\dfrac{1}{2}\sum_{k=1}^{n-1}(1-i\,ctg\dfrac{k\pi}{n})x^k$. *Ko'rsatma.* Lagranj formulasidan foydalaniladi. Natijaning har bir qo'shiluvchisida bo'lishni bajarib, o'xshash hadlarni ixchamlash kerak.

69. $f(x)=\sum_{k=1}^{n}\dfrac{y_k(x^n-1)}{(x-\varepsilon_k)n\varepsilon_k^{n-1}}=\dfrac{1}{n}\sum_{k=1}^{n}\dfrac{y_k(1-x^n)}{1-x\varepsilon_k^{-1}}$, $f(0)=\dfrac{1}{n}\sum_{k=1}^{n}y_k$.

70. *Yechilishi.* x^s ko'phad o'zining qiymatlari orqali Lagranjning interpolyasion formulasi orqali ifodalanadi: $x^s=\sum_{i=1}^{n}\dfrac{x_i^s\varphi(x)}{(x-x_i)\varphi'(x_i)}$. x^{n-1} ning koeffisiyentlarini taqqoslab: $\sum_{i=1}^{n}\dfrac{x_i^s}{\varphi'(x_i)}=0$ ni hosil qilamiz.

71. *Yechilishi.* $x^{n-1}=\sum_{i=1}^{n}\dfrac{x_i^{n-1}\varphi(x)}{(x-x_i)\varphi'(x_i)}$. x^{n-1} ning koeffisiyentlarini taqqoslab: $\sum_{i=1}^{n}\dfrac{x_i^{n-1}}{\varphi'(x_i)}=1$.

72. a) $f(x)=1+\dfrac{x}{1!}+\dfrac{x(x-1)}{2!}+\cdots+\dfrac{x(x-1)\cdots(x-n+1)}{n!}$;

b) $f(x)=1+\dfrac{(a-1)x}{1!}+\dfrac{(a-1)^2x(x-1)}{2!}+\cdots+\dfrac{(a-1)^nx(x-1)\cdots(x-n+1)}{n!}$.

Ko'rsatma. Nyuton usuli bo'yicha interpolyasion ko'phad tuzish lozim.

73. $f(x)=1-\frac{2x}{1!}+\frac{2x(2x-2)}{2!}+\cdots+\frac{2x(2x-2)\cdots(2x-4n+2)}{(2n)!}$.

Ko'rsatma. Izlanayotgan ko'phadning $x=0,1,2,3,...,2n$ lardagi qiymatlarini topish kerak.

74. $f(x)=1-\frac{x-1}{2!}+\frac{(x-1)(x-2)}{3!}-\cdots+(-1)^n\frac{(x-1)(x-2)\cdots(x-n+1)}{n!}=$

$=\frac{n!-(1-x)(2-x)\ldots(n-x)}{n!\,x.}$ *Ko'rsatma.* Masalani Nyuton usulidan foydalanib ham yechish mumkin. $F(x)=x\,f(x)-1$ ko'phadni qarash lozim, bu yerda $f(x)-$ izlanayotgan ko'phad.

75. $f(x)=\frac{\varphi(a)-\varphi(x)}{\varphi(a)(x-a)}$,, bu yerda $\varphi(x)=(x-x_1)(x-x_2)\cdots(x-x_n)$.. *Ko'rsatma.* $(x-a)f(x)-1$ ko'phadni qarash kerak.

76. *Yechilishi.* Nyuton usuli bo'yicha ko'phad tuziladi. $f(x)$ ni

$$f(x)=A_0+A_1\frac{x-m}{1!}+A_2\frac{(x-m)(x-m-1)}{2!}+\cdots+A_m\frac{(x-m)(x-m-1)\cdots(x-m-n+1)}{n!},$$

ko'irinshda izlaymiz, bu yerda $m,m+1,...,m+n-$ x ning $f(x)$ butun qiymat qabul qiladigan qiymatlaridan iborat.

Ketma-ket $x=m,m+1,...,m+n$ deb olib, $A_0,A_1,...,A_n:A_0=f(m)$ larni aniqlash uchun quyidagi tengliklarni hosil qilamiz:

$$A_k=f(m+k)-A_0-\frac{k}{1!}A_1-\frac{k(k-1)}{2!}A_2-\cdots-kA_{k-1},\quad k=\overline{1,n},$$

Bu yerdan hamma A_k- koeffisiyentlarning butunligi kelib chiqadi.

x ning butun qiymatlarida $f(x)$ ning barcha qo'shiluvchilari butun A_k ko'paytuchvilarga ega binomial koeffisiyentlarga aylanadi va shuning uchun ular butun sonlardan iborat bo'ladi. Demak, $f(x)$ ko'phad x ning butun qiymatlaridan butun qiymatlarni qabul qiladi.

77. *Yechilishi.* $F(x)=f(x^2)$, ko'phadni qaraymiz, bu yerda $f(x)-$ izlanayotgan ko'phad. Darajasi $2n$ bo'lgan $F(x)$ ko'phad $2n+1$ ta $x=-n,\ -(n-1),...,-1,0,1,...,n$ qitymatlarda butun qiymatlarni qabul qiladi va oldingi masalaga asosan bu ko'phad x ning qolgan qiymatlarida ham butun qiymatlarni qabul qiladi.

§ 3

78. a) x^3+x+1; b) $(x+1)^3(x^2+x+1)$; c) $(x+3)(x^2+4x+2)$;

d) $(x^2+x+1)(x^2+2x+4)$.

79. $f_1(x)=x^2+1$; $f_2(x)=x^2+x+2$, $f_3(x)=x^2+2x+2$.

80. $f_1(x)=x^3+2x-1$, $f_2(x)=x^3+2x+2$, $f_3(x)=x^3+x^2+2$, $f_4(x)=x^3+2x^2+1$, $f_5(x)=x^3+x^2+x+2$, $f_6(x)=x^3+x^2+2x+1$, $f_7(x)=x^3+2x^2+x+1$, $f_8(x)=x^3+2x^2+2x+2$.

81. a) $(x-2)(x+1+i\sqrt{3})(x+1-i\sqrt{3}),\quad (x-2)(x^2+2x+4);$

b) $(x+2)(x-1+i\sqrt{3})(x-1-i\sqrt{3}),\quad (x+2)(x^2-2x+4);$

c) $(x-2)(x+2)(x-2i)(x+2i),\quad (x-2)(x+2)(x^2+4);$

d)

$(x-\sqrt{2}-i\sqrt{2})(x-\sqrt{2}+i\sqrt{2})(x+\sqrt{2}-i\sqrt{2})(x+\sqrt{2}+i\sqrt{2}),\quad (x^2-2\sqrt{2}x+4)(x^2+2\sqrt{2}x+4)$**;**

e) $(x-i\sqrt{3})(x+i\sqrt{3})(x-\frac{3}{2}-i\frac{\sqrt{3}}{2})(x+\frac{3}{2}-i\frac{\sqrt{3}}{2})(x-\frac{3}{2}+i\frac{\sqrt{3}}{2})(x+\frac{3}{2}+i\frac{\sqrt{3}}{2}),$

$(x^2+3)(x^2-3x+3)(x^2+3x+3);$

f) $(x-\sqrt[4]{3})^2(x-i\sqrt[4]{3})^2(x+\sqrt[4]{3})(x+i\sqrt[4]{3})^2,\quad (x-\sqrt[4]{3})^2(x+\sqrt[4]{3})^2(x^2+\sqrt{3})^2;$

g) $\prod_{k=0}^{n-1}(x-(\mathbf{cos}\frac{1+8k}{4n}\pi\pm i\,\mathbf{sin}\frac{1+8k}{4n}\pi)),\quad \prod_{k=0}^{n-1}(x^2-2\sqrt[2n]{2}x\,\mathbf{cos}\frac{8k+1}{4n}\pi+\sqrt[n]{2})$**;**

h) $\prod_{k=0}^{n-1}(x-(cos\frac{2\pi(1+3k)}{3n}\pm isin\frac{2\pi(1+3k)}{3n})),\quad \prod_{k=0}^{n-1}(x^2-2xcos\frac{2\pi(3k+1)}{3n}+1)$**;**

i) $\prod_{k=0}^{2n-1}(x-(\cos\frac{k\pi}{n}\pm i\sin\frac{k\pi}{n})),\quad (x^2-1)\prod_{k=1}^{n-1}(x^2-2x\cos\frac{k\pi}{n}+1);$

j) $\prod_{k=0}^{2n}(x-(\cos\frac{3k\pi}{2n+1}+i\sin\frac{2k\pi}{2n+1})),\quad (x-1)\prod_{k=1}^{n}(x^2+2x\cos\frac{2k\pi}{2n+1}+1).$

83. a) $x^4-(1+i)x^3-(1-i)x^2+(1+i)x-i,\quad x^5-x^4-x+1;$

b) $x^3-3(1+2i)x^2-3(3+4i)x+(11-2i)$**,** $x^6-6x^5+27x^4-68x^3+135x^2-150x+125$;

c) $x^3+(1-i)x^2+(1-2i)x+1-i,\quad x^6+2x^5+4x^4+4x^3+5x^2+2x+2;$

d) $x^4+6x^3+(15+2i)x^2+(18+6i)x+8+6i$**,** $x^8+12x^7+66x^6+216x^5+461x^4+$ $+660x^3+624x^2+360x+100$ ye) $x^3-ix^2-x+i,\quad x^4-1.$

84. a) $(x-1)^2(x+2);$ b) $(x+1)^2(x^2+1);$ c) $(x-1)^3.$

85. x^d-1, bu yerda $d=(m,n)$. *Ko'rsatma.* Umumiy ildizlar topilsin.

86. x^d+a^d, agar $\frac{m}{d}$ va $\frac{n}{d}$ -- sonlar toq bo'lsa; 1, agar bu sonlardan kamida biri juft bo'lsa; $d=(m,n)$.

87. a) $m=3n+1$ va $m=3n+2$; b) $m=6n+1$ va $m=6n+5$;

s) $m=6n+2$ va $m=6n+4$.

88. a) $m=6k+1$; b) $m=6k+4$.

89. Yo'q, chunki birinchi va ikkinchi hosilalar bir vaqtda nolga aylanmaydi.

91. a) -3; b) -2; c) -2 – ikki karrali ildiz; d) 2; e) butun ildizlar yo'q.

92. *Yechilishi.* $\frac{k}{l}$ ni $f(x)$ ga qo'yib, l^n ga ko'paytirilgandan so'ng: $a_0k^n+a_1k^{n-1}l+\cdots+a_{n-1}kl^{n-1}+a_nl^n=0$ ni hosil qilamiz, bu yerdan a_0k^n ning l *ga*, a_nl^n ning esa k ga bo'linishi kelib chiqadi. k va l sonlar o'zaro tub. Demak, a_o koeffisiyent l ga, a_n esa k ga bo'linadi.

$f(x)$ ni $(x-m)$ ning darajalari bo'yicha yoyamiz:

$f(x)=a_0(x-m)^n+c_1(x-m)^{n-1}+\cdots+c_{n-1}(x-m)+c_n.$

$c_1,\dots,c_n$ – koeffisiyentlar butun sonlar, chunki m – butun son: $c_n = f(m)$. $x=\frac{k}{l}$ ni qo'yib

$$a_0(k-ml)^n + c_1(k-ml)^{n-1}l + \cdots + c_{n-1}(k-ml)l^{n-1} + c_n l^n = 0,$$

ni hosil qilamiz, bu yerdan $c_n\, l^n$ ning $k-ml$ ga bo'linishi kelib chiqadi. Demak, $c_n = f(m)$ ozod had $k-ml$ bo'linadi, chunki l va $k-ml$ lar o'zaro tub.

93. a) 2; b) -3; c) -2; d) -3, $\frac{1}{2}$; e) $\frac{5}{2}, -\frac{3}{4}$; f) 1, -2, 3; g), h) rasional ildizlar yo'q; i) $-\frac{1}{2}$ – ikki karrali ildiz.

94. a), d) keltirilmaydigan; b) $(3x+2)(x^2+x+1)$;

c) $(2x+1)(5x-1)(3x+1)$.

95. *Ko'rsatma.* c) berilgan ko'phadni $(x-1)$ ning darajalari bo'yicha yoyish kerak; e) berilgan ko'phadni $(x-1)$ ning darajalari bo'yicha yoyish kerak (yoki $x=y+1$ deb olish kerak).

96. *Yechilishi.* 92 masalaga asosan k va $k-l$ – bir vaqtda toq. Demak, l – juft son va birga teng bo'la olmaydi.

97. *Yechilishi.* 92 masalaga asosan $k-x_1\, l=\pm1$, $k-x_2\, l=\pm1$, bu yerdan $(x_2-x_1)l=\pm2$ yoki 0. 0 qiymat olinmaydi, chunki $q>0$, $x_2\neq x_1$. Aniqlik uchun $x_2>x_1$ deb olib, $(x_2-x_1)l=2$ ni hosil qilamiz. Bu tenglik $x_2-x_1>2$ da o'rinli emas. Endi $x_2-x_1=1$ yoki 2 deb olamiz. k va l ning $(x_2-x_1)\,q=2$ tenglik o'rinli bo'ladigan yagona mumkin bo'lgan qiymati $p=x_1\,q+1, q=\frac{2}{x_2-x_1}$, bu yerdan $\frac{p}{q}=x_1+\frac{1}{q}=\frac{x_1+x_2}{2}$ rasional ildizning mumkin bo'lgan yagona qiymatini hosil qilamiz.

99. *Yechilishi.* 2 va 3 modullar bo'yicha keltirilmaydigan ko'paytuvchilarga ajratamiz:

$x^5-6x^3+2x^2-4x+5 \equiv (x+1)(x^4+x^3+x^2+x+1)(\text{mod } 2)$,

$x^5-6x^3+2x^2-4x+5 \equiv (x^2+1)(x^3-x-1)(\text{mod } 3)$.

Ko'paytuvchilar mos modullar bo'yicha keltirilmaydigan ko'phadlar va ularning darajalari har xil.

100. *Yechilishi.* Agar $f(x+a)=f(x+b)$, u holda $f(x)=f(x+c)=f(x+2c)=\cdots=f(x+(p-1)c)$, bu yerda $c=b-a$. Agar $b\not\equiv a(modp)$ bo'lsa, u holda $0, c, 2c, \dots, (p-1)c$ ketma-ketlik chegirmalar maydonining barcha elementlarini tashkil qiladi, chunki $f(x)=f(x+1)=\cdots=f(x+p-1)$.

101. *Yechilishi.* $\varphi(x)$ – ko'phad $f(x)$ ning keltirilmaydigan ko'paytuvchisi bo'lsin. Uning darajasi 1 dan katta. $\varphi(x), \varphi(x+1), \dots, \varphi(x+p-1)$ ko'phadlarning hammasi keltirilmaydigan ko'phadlardan iborat bo'lib, ular $f(x)$ ning bo'luvchilaridan iborat. Ular juft-jufti bilan har xil bo'lishi mumkin emas, chunki

$f(x)$ ularning ko'paytmasi bo'lgan va darajasi $2p$ dan katta yoki teng bo'lgan ko'phadga bo'linmaydi. Demak, $\varphi(x)=\varphi(x+1)=...=\varphi(x+p-1)$. Shuning uchun $x=0,1,...,p-1$ larda $\varphi(x)-\varphi(0)=0$ bo'ladi. Shunday qilib, $\varphi(x)$ ning darajasi p dan kichik emas va $f(x)=\varphi(x)$.

102. a) $(x^2+x-1)(x^2-x-1)$; b), c) keltirilmaydi;

d) $(x^2-x-1)(x^2-2)$.

103. *Yechilishi.* Rasional ildizlarga ega bo'lmaydigan $x^4+ax^3+bx^2+cx+d$ ko'phad keltiriladigan holda faqat ikkinchi darajali butun koeffisiyentli ko'phadlar ko'paytmasiga yoyilishi mumkin:

$$x^4+ax^3+bx^2+cx+d=(x^2+\lambda x+m)(x^2+\mu x+n).$$

Ravshanki, m soni d ning bo'luvchisi bo'ladi: $mn=d$. x^3 va x larning koeffisiyentlarini taqqoslash natijasida $\lambda+\mu=a,\quad n\lambda+m\mu=c$ larni hosil qilamiz.

Agar $m\neq n$ bo'lsa, u holda $\lambda=\dfrac{c-am}{n-m}=\dfrac{cm-am^2}{d-m^2}$, shuni isbotlash talab etilgan edi.

Agarda $m=n$ bo'lsa, u holda $d=m^2,\quad c=am.$ Bu holda λ va μ sonlar $\lambda+\mu=a,\quad \lambda\mu+2m=b$ sistemadan topiladi.

104. *Yechilishi.* Berilgan ko'phad keltiriladigan holda $x^5+ax^4+bx^3+cx^2+dx+e=(x^2+\lambda x+m)(x^3+\lambda' x^2+\lambda'' x+n)$ bo'lishi kerak. Ko'paytuvchilarning koeffisiyentlari butun sonlar bo'lishi kerak.

Koeffisiyentlarni solishtirish $nm=e$ ni beradi, bu yerdan m ning e ning bo'luvchisi ekanligi kelib chiqadi. Endi

$$\lambda+\lambda'=a,$$
$$n\lambda+m\lambda''=d,$$
$$m+\lambda\lambda'+\lambda''=b,$$
$$n+\lambda\lambda''+m\lambda'=c,$$

bu yerdan

$$m\lambda''-n\lambda'=d-an,$$
$$\lambda(m\lambda''-n\lambda')+m^2\lambda'-n\lambda''=cm-bn$$

va demak, $(d-an)\lambda+m^2\lambda'-n\lambda''=cm-bn.$ Bu tenglamani $\lambda+\lambda'=a,\quad n\lambda+m\lambda''=d$ tenglamalar bilan birgalikda yechib, $\lambda=\dfrac{am^3-cm^2-dn+be}{m^3-n^2+ae-dm}$ ni hosil qilamiz.

105. a) $(x^2-2x+3)(x^2-2x-3)$; b) keltirilmaydi;

s) $(x^2-x-4)(x^2+5x+3)$; $(x^2-2x+2)(x^2+3x+3)$.

106. a) $x^5+mx^3-mx+1=(x+1)(x^4-x^3+(m+1)x^2-(m+1)x+1)$;

b) $x^5+mx^3-(m+2)x+1=(x-1)(x^4+x^3+(m+1)x^2+(m+1)x-1)$;

c) $x^5+x+1=(x^2+x+1)(x^3-x^2-1)$;

d) $x^5-2x^3-x+1=(x^2+x-1)(x^3-x^2-1)$;

e) $x^5+2x^3+x+1=(x^2-x+1)(x^3+x^2+2x+1)$;

f) $x^5-4x^3+3x+1=(x^2-x-1)(x^3+x^2-2x-1)$.

107. x^4+px^2+q ko'phadningn keltiriladigan ko'phad bo'lishi uchun quyidagi ikki shartdan kamida birining bajarilishi zarur va yetarlidir:

a) p^2-4q rasional sonning kvadratidan iborat bo'lishi;

b) q soni μ - rasional sonning kvadrati bo'lishi, $2\mu-p$ soni esa λ rasional soning kvadratidan iborat bo'lishi.

108. *Yechilishi.* $f(x)=\varphi(x)\psi(x)$ va $\varphi(x),\psi(x)$ lar butun koeffisiyentlarga ega bo'lsin. $f(a_i)=-1$ bo'lganligi uchun $\varphi(a_i)=1$, $\psi(a_i)=-1$, yoki $\varphi(a_i)=-1$, $\psi(a_i)=1$ bo'lishi kerak, demak,

$$\varphi(a_i)+\ \psi(a_i)=0,\quad i=\overline{1,n}.$$

Agar $\varphi(x)$ va $\psi(x)$ larning ikkalasi ham o'zgarmas bo'lmasa, u holda $\varphi(x)+\psi(x)$ ning darajasi n dan kichik, bu yerdan $\varphi(x)+\psi(x)$=0 aynan tenglik kelib chiqadi. Shunday qilib, $f(x)=-[\varphi(x)]^2$ bo'lishi kerak. Buning bo'lishi mumkin emas, chunki $f(x)$ ning bosh koeffisiyenti musbat.

109. *Yechilishi.* Agar n-darajali $f(x)$ ko'phad $n=2m$ yoki $n=2m+1$ da keltiriladigan bo'lsa, u holda uning biror ko'paytuvchisi $\varphi(x)$ ning darajasi m dan oshmaydi. Agar $f(x)$ ko'phad ±1 qiymatni o'zgaruvchining $2m$ tadan ortiq qiymatlarida qabul qilsa, u holda $\varphi(x)$ ham o'zgaruvchining shu qiymatlarida ±1 qiymatni qabul qiladi. $\varphi(x)$ ning bu qiymatlari orasida m tadan ko'p +1 yoki -1 tenglari uchraydi. Ammo bu holda $\varphi(x)=+1$ yoki -1 ga aynan teng bo'ladi.

110. *Yechilishi.* $f(x)$ ko'phad haqiqiy ildizlarga ega emas. Demak, agar u keltiriladigan ko'phad bo'lsa, uning ko'paytuvchilari $\varphi(x)+\psi(x)$ lar ham haqiqiy ildizlarga ega bo'lmaydi va shuning uchun ular x ning haqiqiy qiymatlarida ishoralarini o'zgartirmaydi. x ning barcha haqiqiy qiymatlarida $\varphi(x)>0,\quad \psi(x)>0$ deb hisoblash mumkin. $f(a_k)=1$ bo'lganligi uchun $\varphi(a_k)=\psi(a_k)=1,\quad k=\overline{1,n}.$ Agar $\varphi(x)$ (yoki $\psi(x)$) ning darajasi n dan kichik bo'lsa, $\varphi(x)$=1 (yoki $\psi(x)$=1) aynan tenglik o'rinli bo'ladi. Demak, $\varphi(x)$ va $\psi(x)$ ning darajalari n ga teng. Bu holda $\varphi(x)=1+\alpha(x-a_1)...(x-a_n)$, $\psi(x)=1+\beta(x-a_1)...(x-a_n)$, bu yerdaye α va β -- qandaydir butun sonlar. Ammo bu holda

$f(x)=(x-a_1)^2...(x-a_n)^2+1=1+(\alpha+\beta)(x-a_1)...(x-a_n)+\alpha\beta(x-a_1)^2...(x-a_n)^2.\ x^{2n}$

va x^n ning darajalarini solishtirish natijasida butun ildizlarga ega bo'lmaydigan $\alpha\beta=1,\quad \alpha+\beta=0,$ tenglamalar sistemasini hosil qilamiz. Demak, $f(x)$ keltirilmaydigan ko'phaddan iborat.

111. *Yechilishi.* $a[\varphi(x)]^2+b\varphi(x)+1=\psi(x)\omega(x)$ bo'lsin. Ko'paytuvchilardan birining darajasi $\leq n$; $\psi(x)$ esa $x=a_1,\ a_2,\ ...,\ a_n$ larda +1 qiymatni qabul qiladi. $n\geq 7$ bo'lganligi uchun $\psi(x)$ ning barcha qiymatlari bir xil ishorali bo'lishi kerak. Demak,

$\psi(x)=\pm1+\alpha(x-\alpha_1)(x-\alpha_2)\dots(x-\alpha_n)=\pm1+\alpha\varphi(x).$

Agar $\alpha\neq 0$ bo'lsa, u holda $\omega(x)$ ning ham darajasi n ga teng va $\omega(x)=\pm1+\beta\varphi(x)$. Ammo $a[\varphi(x)]^2+b\varphi(x)+1=[\pm1+\alpha\varphi(x)][\pm1+\beta\varphi(x)]$ tenglikning bajarilishi mumkin emas, chunki ax^2+bx+c ko'phad keltirilmaydigan ko'phad.

§ 4

112. a) $\frac{1}{12(x-1)}-\frac{4}{3(x+2)}+\frac{9}{4(x+3)}$;

b) $-\frac{1}{6(x-1)}+\frac{1}{2(x-2)}-\frac{9}{2(x-3)}+\frac{1}{6(x-4)}$;

s) $\frac{2}{x-1}+\frac{-2+i}{2(x-i)}+\frac{-2-i}{2(x+i)}$;

d) $\frac{1}{4(x-1)}-\frac{1}{4(x+1)}-\frac{i}{4(x-i)}+\frac{i}{4(x+i)}$;

e) $\frac{1}{3}\left(\frac{1}{x-1}+\frac{\varepsilon}{x-\varepsilon}+\frac{\varepsilon^2}{x-\varepsilon^2}\right),\varepsilon=-\frac{1}{3}+\frac{i\sqrt{3}}{2}$;

f) $-\frac{1}{16}\left(\frac{1+i}{x-1-i}+\frac{1-i}{x-1+i}+\frac{-1+i}{x+1-i}+\frac{-1+i}{x+1+i}\right)$;

g) $\frac{1}{n}\sum_{k=0}^{n-1}\frac{\varepsilon_k}{x-\varepsilon_k}$, $\varepsilon_k=\cos\frac{2k\pi}{n}+i\sin\frac{2k\pi}{n}$;

h) $-\frac{1}{m}\sum_{k=1}^{n}\frac{\eta_k}{x-\eta_k}$, $\eta_k=cos\frac{(2k-1)\pi}{n}+i\,sin\frac{(2k-1)\pi}{n}$;

i) $\sum_{k=0}^{n}\frac{C_n^k(-1)^{n-k}}{x-k}$;

j) $\sum_{k=-n}^{n}\frac{(-1)^{n-k}C_{2n}^{n+k}}{x-k}$. *Ko'rsatma.* Lagranj formulasi yordamida osonroq chiqariladi.

114. a) $\frac{1}{4(x-1)^2}-\frac{1}{4(x+1)}$;

b) $\frac{1}{4(x+1)}-\frac{1}{4(x-1)}+\frac{1}{4(x-1)^2}+\frac{1}{4(x+1)^2}$;

c) $\frac{3}{(x-1)^3}-\frac{4}{(x-1)^2}+\frac{1}{x-1}-\frac{1}{(x+1)^2}-\frac{2}{x+1}+\frac{1}{x+2}$;

d) $\frac{1}{n^2}\left(\sum_{k=0}^{n-1}\frac{\varepsilon_k}{(x-\varepsilon_k)^2}-(n-1)\sum_{k=0}^{n-1}\frac{\varepsilon_k}{x-\varepsilon_k}\right)$, $\varepsilon_k=\cos\frac{2k\pi}{n}+i\sin\frac{2k\pi}{n}$;

e) $\dfrac{1}{x^m}+\dfrac{\frac{n}{1!}}{x^{m-1}}+\dfrac{\frac{n(n+1)}{2!}}{x^{m-2}}+\cdots+\dfrac{\frac{n(n+1)\cdots(n+m-2)}{(m-1)!}}{x}+$

$+\dfrac{1}{(1-x)^n}+\dfrac{\frac{m}{1!}}{(1-x)^{n-1}}+\dfrac{\frac{m(m+1)}{2!}}{(1-x)^{n-2}}+\cdots+\dfrac{\frac{m(m+1)\cdots(m+n-2)}{(n-1)!}}{1-x};$

f) $\dfrac{1}{(-4a^2)^n}\sum_{k=0}^{n-1}(2a)^{n-k}\dfrac{n(n+1)\cdots(n+k-1)}{k!}\left(\dfrac{1}{(a-x)^{n-k}}+\dfrac{1}{(a+x)^{n-k}}\right);$

g) $\dfrac{1}{(4a^2)^n}\sum_{k=0}^{n-1}(2a)^{n-k}\dfrac{n(n+1)\cdots(n+k-1)}{k!}\left(\dfrac{1}{(a-ix)^{n-k}}+\dfrac{1}{(a+ix)^{n-k}}\right).$

Ko'rsatma. f) $\dfrac{a+x}{2a}=y$ deb olish kerak; d), h) Yoyilmani apniqmas koeffisiyentlar metodi bilan izlash kerak. Uning bir qismini umumiy maxrajga ko'paytirilgandan so'ng $x=x_1,x_2,...,x_n$ qiymatlar bilan topish kerak. So'ngra differensiallab yana $x=x_1,x_2,...,x_n$ qiymatlarni berish kerak.

115. a) $\dfrac{1}{3(x-1)}-\dfrac{x+2}{3(x^2+x+1)};$

b) $\dfrac{1}{8(x-2)}-\dfrac{1}{8(x+2)}+\dfrac{1}{2(x^2+4)};$

c) $\dfrac{1}{8}\cdot\dfrac{x+2}{x^2+2x+2}-\dfrac{1}{8}\cdot\dfrac{x-2}{x^2-2x+2};$

d) $\dfrac{1}{18}\cdot\left(\dfrac{1}{x^2+3x+3}+\dfrac{1}{x^2-3x+3}-\dfrac{2}{x^2+3}\right);$

ye) $\dfrac{1}{2n+1}\left(\dfrac{1}{x-1}+2\sum_{k=1}^{n}\dfrac{x\cos\frac{2k(m+1)\pi}{2n+1}-\cos\frac{2km\pi}{2n+1}}{x^2-2x\cos\frac{2k\pi}{2n+1}+1}\right);$

f) $\dfrac{(-1)^m}{2n+1}\left(\dfrac{1}{x+1}+2\sum_{k=1}^{n}\dfrac{x\cos\frac{2k(m+1)\pi}{2n+1}+\cos\frac{2km\pi}{2n+1}}{x^2+2x\cos\frac{2k\pi}{2n+1}+1}\right);$

g) $\dfrac{1}{2n}\left(\dfrac{1}{x-1}-\dfrac{1}{x+1}+2\sum_{n=1}^{n-1}\dfrac{x\cos\frac{k\pi}{n}-1}{x^2-2x\cos\frac{k\pi}{n}+1}\right);$

h) $\dfrac{1}{n}\sum_{k=1}^{n}\dfrac{\cos\frac{(2k-1)m\pi}{n}-x\cos\frac{(2k-1)(2m+1)}{2n}\pi}{x^2-2x\cos\frac{(2k-1)\pi}{2n}+1};$

i) $\dfrac{1}{(n!)^2 x}+2\sum_{k=1}^{n}\dfrac{(-1)^k x}{(n+k)!\,(n-k)!\,(x^2+k^2)}$.

Ko'rsatma. Lagranj formulasi yordamida yoyib, so'ngra qo'shma kompleks qo'shiluvchilarni birlashtirish kerak.

116. a) $-\dfrac{1}{4(x+1)}+\dfrac{x-1}{4(x^2+1)}+\dfrac{x+1}{2(x^2+1)^2}$;

b) $-\dfrac{1}{x}+\dfrac{7}{x+1}+\dfrac{3}{(x+1)^2}-\dfrac{6x+2}{x^2+x+1}-\dfrac{3x+2}{(x^2+x+1)^2}$;

c) $\dfrac{1}{16(x-1)^2}-\dfrac{3}{16(x-1)}+\dfrac{1}{16(x+1)^2}+\dfrac{3}{16(x+1)}+\dfrac{1}{4(x^2+1)}+\dfrac{1}{4(x^2+1)^2}$;

d) $\dfrac{1}{4n^2}\left(\dfrac{1}{(x-1)^2}+\dfrac{1}{(x+1)^2}-\dfrac{2n-1}{x-1}+\dfrac{2n-1}{x+1}\right)+$

$$+\frac{1}{n^2}\sum_{k=1}^{n-1}\frac{\sin^2\frac{k\pi}{n}\left(1-2x\cos\frac{k\pi}{n}\right)}{\left(x^2-2x\cos\frac{k\pi}{n}+1\right)^2}+\frac{1}{n^2}\sum_{k=1}^{n-1}\frac{n-\sin^2\frac{k\pi}{n}-\left(n-\frac{1}{2}\right)x\cos\frac{k\pi}{n}}{x^2-2x\cos\frac{k\pi}{n}+1}.$$

117. a) $\dfrac{\varphi'(x)}{\varphi(x)}$; b) $\dfrac{x\varphi'(x)-n\varphi(x)}{\varphi(x)}$; c) $\dfrac{(\varphi'(x))^2-\varphi(x)\varphi''(x)}{(\varphi(x))^2}$.

118. a) 9; b) $-\dfrac{\varphi'(2)}{\varphi(2)}+\dfrac{\varphi'(1)}{\varphi(1)}=-\dfrac{17}{5}$; c) 17. *Ko'rsatma. 117 masaladan foydalanilsin.* b) $\dfrac{1}{x^2-3x+2}$ ni sodda kasrlarga yoyish kerak.

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119. a) $\sigma_1^3-3\sigma_1\sigma_2$; b) $\sigma_1\sigma_2-3\sigma_3$; c) $\sigma_1^4-4\sigma_1^2\sigma_2+8\sigma_1\sigma_3$;

d) $\sigma_1^3\sigma_2^2-2\sigma_1^4\sigma_3-3\sigma_1\sigma_2^3+6\sigma_1^2\sigma_2\sigma_3+3\sigma_2^3\sigma_3-7\sigma_1\sigma_3^2$;

e) $\sigma_1\sigma_2-\sigma_3$; f) $\sigma_1^2\sigma_2^2-2\sigma_1^3\sigma_3-2\sigma_2^3+4\sigma_1\sigma_2\sigma_3-\sigma_3^2$;

g) $2\sigma_1^3-9\sigma_1\sigma_2+27\sigma_3$;

h) $\sigma_1^2\sigma_2^2-4\sigma_1^3\sigma_3-4\sigma_2^3+18\sigma_1\sigma_2\sigma_3-27\sigma_3^2$.

120. a) $\sigma_1\sigma_2\sigma_3-\sigma_1^2\sigma_4-\sigma_3^2$; b) $\sigma_1^2\sigma_4+\sigma_3^2-4\sigma_2\sigma_4$;

c) $\sigma_1^3-4\sigma_1\sigma_2+8\sigma_3$.

121. a) $\sigma_1^2-2\sigma_2$; b) $\sigma_1\sigma_3-4\sigma_4$; c) $\sigma_2^2-2\sigma_1\sigma_3+2\sigma_4$;

d) $\sigma_1^2\sigma_2-\sigma_1\sigma_3-2\sigma_2^2+4\sigma_4$; e) $\sigma_1^4-4\sigma_1^2\sigma_2+2\sigma_2^2+4\sigma_1\sigma_3-4\sigma_4$;

f) $\sigma_2\sigma_3-3\sigma_1\sigma_4+5\sigma_5$; g) $\sigma_1^2\sigma_3-2\sigma_2\sigma_3-\sigma_1\sigma_4+5\sigma_5$;

h) $\sigma_1\sigma_2^2-2\sigma_1^2\sigma_3-\sigma_2\sigma_3+5\sigma_1\sigma_4-5\sigma_5$;

i) $\sigma_1^3\sigma_2-3\sigma_1\sigma_2^2-\sigma_1^2\sigma_3+5\sigma_2\sigma_3+\sigma_1\sigma_4-5\sigma_5$;

j) $\sigma_1^5-5\sigma_1^3\sigma_2+5\sigma_1\sigma_2^2+5\sigma_1^2\sigma_3-5\sigma_2\sigma_3-5\sigma_1\sigma_4+5\sigma_5$;

k) $\sigma_2\sigma_4-4\sigma_1\sigma_5+9\sigma_6$; l) $\sigma_3^2-2\sigma_2\sigma_4+2\sigma_1\sigma_5-2\sigma_6$;

m) $\sigma_1^2\sigma_4-2\sigma_2\sigma_4-\sigma_1\sigma_5+6\sigma_6$;

n) $\sigma_1\sigma_2\sigma_3-3\sigma_1^2\sigma_4-3\sigma_1^2\sigma_4-3\sigma_3^2+4\sigma_2\sigma_4+7\sigma_1\sigma_5-12\sigma_6$;

o) $\sigma_3^2-3\sigma_1\sigma_2\sigma_3+3\sigma_1^2\sigma_4+3\sigma_3^2-3\sigma_2\sigma_4-3\sigma_1\sigma_5+3\sigma_6$;

p) $\sigma_1^3\sigma_3-3\sigma_1\sigma_2\sigma_3-\sigma_1^2\sigma_4+3\sigma_3^2+2\sigma_2\sigma_4+\sigma_1\sigma_5-6\sigma_6$;

q) $\sigma_1^2\sigma_2^2-2\sigma_1^3\sigma_3-2\sigma_2^3+4\sigma_1\sigma_2\sigma_3+2\sigma_1^2\sigma_4-3\sigma_3^2+2\sigma_2\sigma_4-6\sigma_1\sigma_5+6\sigma_6$;

r) $\sigma_1^4\sigma_2-4\sigma_1^2\sigma_2^2-\sigma_1^3\sigma_3+2\sigma_2^3+7\sigma_1\sigma_2\sigma_3+\sigma_1^2\sigma_4-3\sigma_3^2+6\sigma_2\sigma_4-\sigma_1\sigma_5+6\sigma_6$;

s) $\sigma_1^6-6\sigma_1^4\sigma_2+9\sigma_1^2\sigma_2^2+6\sigma_1^3\sigma_3-2\sigma_2^3-12\sigma_1\sigma_2\sigma_3-6\sigma_1^2\sigma_4-3\sigma_3^2+6\sigma_2\sigma_4+6\sigma_1\sigma_5-6\sigma_6$

122. $\sigma_k^2-2\sigma_{k-1}\sigma_{k+1}+2\sigma_{k-2}\sigma_{k+2}-2\sigma_{k-3}\sigma_{k+3}+\cdots$.

123. a) $\dfrac{\sigma_1\sigma_2-3\sigma_3}{\sigma_3}$; b) $\dfrac{2(\sigma_1^2\sigma_3-3\sigma_1\sigma_3-2\sigma_2^3)}{\sigma_1\sigma_2-\sigma_3}$;

c) $\dfrac{\sigma_2^3+\sigma_1^3\sigma_3-6\sigma_1\sigma_2\sigma_3+9\sigma_2^3}{\sigma_3^2}$.

124. a) $\dfrac{\sigma_{n-1}}{\sigma_n}$; b) $\dfrac{\sigma_{n-1}^2-2\sigma_{n-1}\sigma_n}{\sigma_n^2}$; c) $\dfrac{\sigma_1\sigma_{n-1}-n\sigma_n}{\sigma_n}$.

125. – 4.

126. –35.

127. 16.

128. a) $\dfrac{25}{27}$; b) $\dfrac{35}{27}$; c) $-\dfrac{1679}{625}$.

129. a) $a_1^2a_2^2-4a_1^3a_3-4a_2^3a_0+18a_0a_1a_2a_3-27a_0^2a_3^2$; b) $a_1^3a_3-a_2^3a_0$;

c) $\dfrac{a_1a_2}{a_0a_3}-9$; d) $a_1^2a_2^2-a_1^3a_3-a_2^3a_0$.

130. $S_4=\sigma_1^4-4\sigma_1^2\sigma_2+2\sigma_2^2+4\sigma_1\sigma_3-4\sigma_4$;

$S_5=\sigma_1^5-5\sigma_1^3\sigma_2+5\sigma_1\sigma_2^2+5\sigma_1^2\sigma_3-5\sigma_2\sigma_3-5\sigma_1\sigma_4+5\sigma_5$;

$S_6=\sigma_1^6-6\sigma_1^4\sigma_2+9\sigma_1^2\sigma_2^2+6\sigma_1^3\sigma_3-2\sigma_2^3-12\sigma_1\sigma_2\sigma_3-6\sigma_1^2\sigma_4+3\sigma_3^2+$

$+6\sigma_2\sigma_4+6\sigma_1\sigma_5-6\sigma_6$.

131. $2\sigma_2=S_1^2-S_2$; $6\sigma_3=S_1^3-3S_1S_2+2S_3$;

$24\sigma_4=S_1^4-6S_1^2S_2+8S_1S_3+3S_2^2-6S_4$;

$120\sigma_5=S_1^5-10S_1^3S_2+20S_1^2S_3+15S_1S_2^2-20S_2S_3-30S_1S_4+24S_5$;

$720\sigma_6=S_1^6-15S_1^4S_2+40S_1^3S_3+45S_1^2S_2^2-120S_1S_2S_3-15S_2^3-90S_1^2S_4+40S_3^2+$

$+90S_2S_4+144S_1S_5-120S_6$;

132. a) 859; b) 13; c) 621; d) 16; e) 24.

133. $S_1=-1, S_2=S_3=\cdots=S_n=0$.

134. $9x^3+20x^2+116x-100$.

135. $8x^3-12x^2+726x-291$.

136. $x^n-a=0$.

137. $x^n - \frac{a}{1}x^{n-1} + \frac{a^2}{1\cdot 2}x^{n-2} - \cdots + (-1)^n \frac{a^n}{n!} = 0.$

138. *Ko'rsatma.* Ikkinchi ustunni $-S_1$ ga, uchinchisini – S_2, ..., $k-$nchisini – $(-1)^{k-1}S_k$ ga ko'paytirib birinchi ustunga qo'shiladi, so'ngra Nyuton formulasidan foydalaniladi.

140. $n!(x^n - \sigma_1 x^{n-1} + \sigma_2 x^{n-2} + \cdots + (-1)^n \sigma_n).$

FOYDALANISH UCHUN TAVSIYA ETILGAN ADABIYOTLAR RO'YXATI

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